St Catherine's School

Year: 11
Subject: Mathematics – Extension 1.
Time Allowed: 90 minutes
Date : Sept 2001

Student Number:_______________

Directions to candidates:

- All questions are to be attempted.
- All necessary **working** must be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Each question attempted should be started on a **new page**.
- Write your **student number** on this cover sheet and on every sheet of paper you use.
- Approved calculators may be used.

Hand in your work in 3 bundles.
Section A Questions 1 and 2.
Section B. Questions 3, 4 and 5.
Examination paper.

TEACHER'S USE ONLY
Q.1
Q.2
Q.3
Q.4
Q.5
TOTAL

Section A

Question One

a) Find the co-ordinates of the point P(x, y) which divides the join of A(5,3) and B(1,-3) externally in the ratio 3:2. (2)

b) Solve for x: $\dfrac{2x-5}{x+3} \geq 1$ (3)

c) Find the acute angle (to the nearest degree) between the lines $2x+y=3$ and $\dfrac{x}{3}+\dfrac{y}{2}=1$ (3)

d) Find x if the first three terms of a geometric sequence are $x,\ x+3,\ x+2$ (2)

e) Express $\dfrac{1-x^{-1}}{x^{-1}-x^{-2}}$ in simplest form. (2)

Question Two

a) Solve for x: $3x+2=|1-2x|$ (3)

b) Consider the series - 7 - 2 + 3.......
   i) Which term of this series is 123 ? (2)
   ii) Evaluate the sum of - 7 - 2 + 3 + ...+ 123 (1)

c) Find the equation of the parabola whose focus is (1, 3), directrix is $y=-1$ and the axis of symmetry is parallel to the y axis. (2)

d) Solve $4\cos^2\vartheta+3\sin\vartheta=3$ for $0\leq\vartheta\leq 360$. Give your solution to the nearest minute. (4)

Section B

Question Three (Start a new page)

a) Solve for x: $(x+\frac{2}{x})^2 - 2(x+\frac{2}{x}) - 3 = 0$ (3)

b) The equation $kx^2 + 4x + k - 2 = 0$ has roots α and β such that the sum of the roots equals the difference of the roots. Find the value of k and the value of the roots, α and β (3)

c) The angle between the lines $y = mx$ and $y = \frac{1}{2}x$ is 45°. Find two possible values of m. (3)

d) Given the quadratic identity $a(x+1)^2 + b(x+1) + 1 \equiv 2x^2 + 5x + c$ find the values of a, b, c. (3)

Question Four

a) Find the locus of the point P(x, y) which moves so that it is always 2 units from the line $3x + 4y - 1 = 0$. (3)

b) If $P(x) = x^2 - 2kx - 4k$ find the possible values of k if the roots are real and different. (3)

c) A clever new toy comes onto the market, and sells 20 000 units in the first month. Popularity wanes and each month the sales are only 70% of the sales of the previous month.

i) If sales were to continue indefinitely what would be the number of toys eventually sold ? (1)

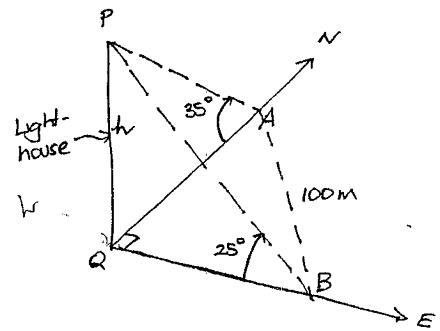
ii) What proportion of the total sales are sold in the first 6 months? (2)

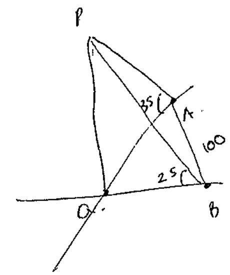
iii) In which month will sales eventually drop below 500 per month? (3)

Question Five

a) Ryan borrows $177 000 over a 30 year period to buy a studio apartment in Randwick. He is charged interest at the end of each month at a rate of 0.5% per month reducible. Find the size of his monthly repayment, $M. Show each step in your working clearly. (5)

b) The angle of elevation of a lighthouse at a point A due North of it is 35° and at another point B due East of the lighthouse is 25°. The distance from A to B is 100 metres.





i) Show that in triangle PQA, $AQ = h \cot 35°$ (1)

ii) Show that in triangle PQB, $BQ = h \cot 25°$ $AQ^2 + BQ^2 = h^2 \cot^2 35 + h^2 \cot^2 25$ (1)

iii) Hence show that $h = \dfrac{100}{\sqrt{\cot^2 25 + \cot^2 35}}$ $= h^2(\cot^2 35 + \cot^2 25)$ $100^2 = h^2(\cot^2 35 + \cot^2 25)$ (2)

iv) Find the height, h, of the lighthouse to the nearest metre. $h^2 = \dfrac{10000}{\cot^2 35 + \cot^2 25}$ (1)

v) Find the bearing of A from B to the nearest minute. $h = \dfrac{100}{\sqrt{\cot^2 35 + \cot^2 25}}$ (2)

End of examination

(a) $M(x,y) = \left(\dfrac{mx_2 + nx_1}{m+n} , \dfrac{my_2 + ny_1}{m+n} \right)$

$m:n = -3:2$

$A(5,3)$

$B(1,-3)$

$= \dfrac{(-3)(1) + (2)(5)}{-3+2} , \dfrac{(-3)(-3) + (2)(3)}{-3+2}$

$= \dfrac{-3+10}{-1} , \dfrac{9+6}{-1}$

$= (-7, -15)$

2

b) $\dfrac{2x-5}{x+3} (x+3)^2 \geq 1 \cdot (x+3)^2$

(note $x \neq 3$)

$(2x-5)(x+3) \geq (x+3)^2$

$2x^2 + 6x - 5x - 15 \geq x^2 + 6x + 9$

$2x^2 + x - 15 \geq x^2 + 6x + 9$

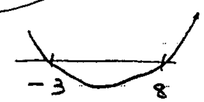
$x^2 - 5x - 24 \geq 0$

$(x-8)(x+3) \geq 0$

$x \geq 8$ or $x \leq -3$

but $x \neq 3$ so $x \geq 8$ or $x \leq -3$

$\frac{1}{2}$ only for $x \geq 8$



3

$-\frac{1}{2}$ for $x \leq -3$

c) $2x + y = 3$

$\boxed{m_1 = -2}$

$\dfrac{x}{3} + \dfrac{y}{2} = 1$

$2x + 3y = 6$

$\boxed{m_2 = \dfrac{-2}{3}}$

$\frac{1}{2}$

ignore $-30°$

if θ is acute

$\tan\theta = \left| \dfrac{m_1 - m_2}{1 + m_1 m_2} \right|$

wrong formula ($\pm$) $-\frac{1}{2}$

$= \left| \dfrac{-2 + \frac{2}{3}}{1 + (2)(\frac{2}{3})} \right|$

$= \left| \dfrac{-1\frac{1}{3}}{1\frac{4}{3}} \right|$

$= \left| -\dfrac{4}{3} \div \dfrac{7}{3} \right|$

$= \left| \dfrac{-4}{3} \times \dfrac{3}{7} \right| = +\dfrac{4}{7}$

$\theta = 29°45'$

$= 30°$ (nearest degree)

3

d) $x, x+3, x+2$

if in GP then $\dfrac{T_2}{T_1} = \dfrac{T_3}{T_2}$

$\dfrac{x+3}{x} = \dfrac{x+2}{x+3}$ → 1 mark.

$(x+3)^2 = x(x+2)$

$x^2 + 6x + 9 = x^2 + 2x$

$4x + 9 = 0$

$x = \dfrac{-9}{4}$

2

e) $\dfrac{1 - \frac{1}{x}}{\frac{1}{x} - \frac{1}{x^2}} \times \dfrac{x^2}{x^2} = \dfrac{x^2 - x}{x - 1}$ $1\frac{1}{2}$

$\frac{1}{2}$

$= \dfrac{x(x-1)}{x-1}$

$= x$

2

a) $3x+2 = |1-2x|$

$3x+2 = 1-2x$ or $3x+2 = -(1-2x)$

$5x = -1$ $\qquad$ $3x+2 = -1 + 2x$

$x = -1/5$ $\qquad$ $x = -3$

check at $x = -\frac{1}{5}$

$LHS = 3x+2$ $\qquad$ $RHS = |1-2x|$

$= -\frac{3}{5}+2$ $\qquad$ $= |1-2(-\frac{1}{5})|$

$= 1\frac{2}{5}$ $\qquad$ $= |1+\frac{2}{5}|$

$= RHS$ ✓✓

check at $x=-3$

$LHS = -7$ $\qquad$ $RHS = |+7|$

$\neq RHS$

3

$\therefore$ $x = -1/5$ is the only solution.

b) i) $-7 -2 + 3 \dots$

is an AP where $a = -7$ $\quad d = 5$

if $T_n = 123$

$T_n = a+(n-1)d$

$123 = -7 + (n-1)5$

$= -7 + 5n - 5$

$135 = 5n$

2

$n = 27$

$\therefore$ 123 is the 27th term

ii) $S_n = \frac{n}{2}(a+l)$

$S_{27} = \frac{27}{2}(-7+123)$

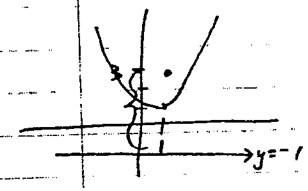
$= \frac{27}{2} \times 116$

$= 27 \times 58$

$= 1566$

1

4



dist from focus to directrix is 4 units

$\therefore$ $a = 2$

2

$\therefore$ ~~focus~~ $(1,1)$ is vertex

$(x-h)^2 = 4a(y-k)$

$(x-1)^2 = 8(y-1)$

2d) $4\cos^2\theta + 3\sin\theta - 3 = 0$

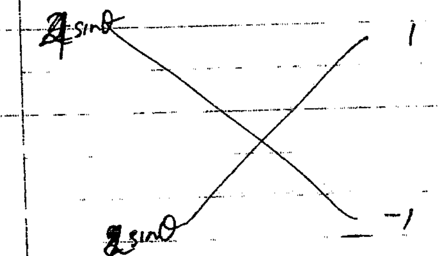
$4(1-\sin^2\theta) + 3\sin\theta - 3 = 0$ $\qquad \frac{1}{2}$

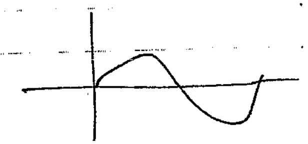
$4 - 4\sin^2\theta + 3\sin\theta - 3 = 0$

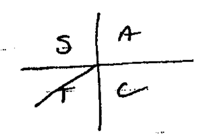
$-4\sin^2\theta + 3\sin\theta + 1 = 0$

$4\sin^2\theta - 3\sin\theta - 1 = 0$ $\qquad \frac{1}{2}$

$(4\sin\theta + 1)(\sin\theta - 1) = 0$ $\qquad 1$







$\therefore$ $4\sin\theta + 1 = 0$ $\qquad$ $\sin\theta - 1 = 0$

$\sin\theta = -\frac{1}{4}$ $\qquad$ $\sin\theta = 1$ $\quad \frac{1}{2}$

$\frac{1}{2}$ basic $\angle$ is $14°29$ $\frac{1}{2}$ $\qquad$ $\boxed{\theta = 90°}$

$\theta = 180 + 14°29'$, $360 - 14°29$

$\boxed{\theta = 194°29', 345°31'}$ $\frac{1}{2}$

a) $\left(x+\frac{2}{x}\right)^2 - 2\left(x+\frac{2}{x}\right) - 3 = 0$

let $v = x + \frac{2}{x}$

$v^2 - 2v - 3 = 0$

✓✓ $(v-3)(v+1) = 0$

✓ $v = 3$ or $v = -1$

$x + \frac{2}{x} = 3$ $\qquad$ $x + \frac{2}{x} = -1$

$x^2 + 2 - 3x = 0$ $\qquad$ $x^2 + 2 = -x$

$x^2 - 3x + 2 = 0$ $\qquad$ $x^2 + x + 2 = 0$ $\qquad$ 3

$(x-2)(x-1) = 0$ $\qquad$ $\Delta = b^2 - 4ac$

✓✓ $\boxed{x = 2, 1}$ $\qquad$ $= (1)^2 - 4(1)(2)$

✓ $\Delta = -7$

$\Delta < 0 \therefore$ no solut

b) $kx^2 + 4x + k - 2 = 0$

let roots be α, β.

$\alpha + \beta = \alpha - \beta$

$2\beta = 0$

$\boxed{\therefore \beta = 0}$ ✓✓ $\qquad$ (i) $\alpha\beta = 0$

$\alpha + \beta = \alpha\beta$

$\frac{4}{k} = \frac{-(k-2)}{k}$

$k \neq 0$

$4 = -k + 2$

$\boxed{k = -2}$

3

(ii) $\therefore \alpha + \beta = \frac{-b}{a}$ $\qquad$ $\alpha\beta = +\frac{c}{a}$

$\alpha + 0 = \frac{-4}{k}$ $\qquad$ $0 = +\frac{(k-2)}{k}$

$\boxed{\therefore k = +2}$ ✓✓

at $k = +2$

$\alpha = \frac{-4}{2}$

$0 = \frac{-k+2}{k}$

$\boxed{\therefore \alpha = -2}$ ✓✓

$k = 2$

$\boxed{\text{and } \beta = 0}$

c) $m_1 = m$ $\qquad$ $m_2 = \frac{1}{2}$

$\tan 45^\circ = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ ✓

✓

$1 = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}\right|$ ✓ ✓

$1 = \left|\frac{2m - 1}{2 + m}\right|$

$2 + m = 2m - 1$ $\qquad$ or $\qquad$ $-(2+m) = 2m - 1$

$3 = m$ $\qquad$ $-2 - m = 2m - 1$

$\boxed{m = 3}$ ✓ $\qquad$ $-1 = +3m$

$\boxed{m = -\frac{1}{3}}$ ✓ $\qquad$ 3

d) $a(x+1)^2 + b(x+1) + 1 \equiv 2x^2 + 5x + c$

let $x = -1$ $\qquad$ let $x = 0$

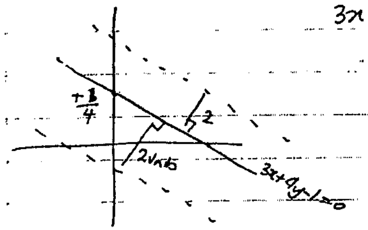
$1 = 2(-1)^2 + 5(-1) + c$ $\qquad$ $a + b + 1 \equiv c$ $\qquad$ 3

$1 = 2 - 5 + c$ $\qquad$ $\boxed{a = 2}$ $\boxed{c = 4}$

$4 = c$ ✓✓ $\qquad$ $\therefore 2 + b + 1 = 4$

$\boxed{b = 1}$

(a)



$$3x + 4y - 1 = 0$$
$$y = -\frac{3x}{4} + \frac{1}{4}$$

Condition : perp dist = 2 units

$$p = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}} \right| \qquad A=3 \quad B=4 \quad C=-1$$
$$x_1 = ? \quad y_1 = ?$$
$$p = 2$$

3

$$2 = \left| \frac{3x+4y-1}{\sqrt{3^2+4^2}} \right|$$

$$2 = \left| \frac{3x+4y-1}{\sqrt{25}} \right|$$

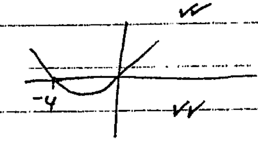
$$10 = 3x+4y-1 \qquad \text{or} \qquad -10 = 3x+4y-1$$
$$\boxed{3x+4y-11=0} \qquad \boxed{3x+4y+9=0}$$

b) $P(x) = x^2 - 2kx - 4k$

if roots are real & different $\Delta > 0$

ie $b^2 - 4ac > 0$

$$(-2k)^2 - 4(1)(-4k) > 0$$
$$4k^2 + 16k > 0$$
$$4k(k+4) > 0$$
$$k < -4 \quad \text{or} \quad k > 0$$



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$$S_\infty = \frac{a}{1-r}$$
$$= \frac{20000}{1-0.7} \qquad 20\,000 \times \frac{10}{3}$$
$$= 66,667$$

1

ii) $$S_6 = \frac{20000\left(1-(0.7)^6\right)}{0.3}$$

$$S_\infty = \frac{20000}{1-0.7}$$

$$\frac{S_6}{S_\infty} = \frac{20000\left(1-(0.7)^6\right)}{0.3} \div \frac{20000}{0.3}$$

$$= \frac{20000\left(1-(0.7)^6\right)}{0.3} \times \frac{0.3}{20000}$$

2

$$= \left[1-(0.7)^6\right] \cancel{0.3}$$
$$= 0.882351$$
$$\Rightarrow 0.88 \text{ of total}$$

iii) $$T_n < 5000$$
$$ar^{n-1} < 500$$
$$20\,000(0.7)^{n-1} < 500$$

$$40(0.7)^{n-1} < 1$$

3

$$(0.7)^{n-1} < \frac{1}{40}$$

$$(n-1)\log_{10}(0.7) < \log_{10}\frac{1}{40}$$
$$n-1 > \frac{\log_{10}\frac{1}{40}}{\log_{10}(0.7)}$$

$$n-1 > 10.34\ldots$$
$$n > 11.34\ldots$$

$\therefore$ The sales will ... 12 mths

Amou ...

rate : 0.5% p.m.

time : 30 yrs = 30×12 months

Let A_n be the amount owing at the end of n months.

$A_1 = 177000(1.005) - M$

$A_2 = A_1 \times 1.05 - M$

$= 177000(1.005)^2 - M(1.005) - M$

⋮

$A_{360} = 177000(1.05)^{360} - M(1 + 1.005 + \dots + 1.005^{359})$

$A_{360} = 0$

$\therefore 177000(1.005)^{360} = M(1 + 1.005 + \dots + 1.005^{359})$

$= M\left[\frac{1(1.005^{360} - 1)}{\cancel{1-}0.005}\right]$

$\therefore M = \frac{177000(1.005)^{360} \times \cancel{(-0.005)}}{\cancel{(1 - 1.005^{360})}}$

$= 177000(1.005)^{360} \times \frac{0.005}{(1.005)^{360} - 1}$

$= \$1061.20.$

Consider $177... as a loan to Ryan over 30yrs

Bank expects:

① $A = P(1 + \frac{r}{100})^n$

$= 177\,000(1 + 0.005)^{360}$

$=$

Consider Ryans payments as separate investments of $ M

$A_1 = M(1.005)^{359}$

⋮

$A_{360} = M(1.005)^0$

Sum of investments $= A_1 + A_2 + A_3 + \dots A_{360}$

$= M[1.005^0 + 1.005^1 + \dots + 1.005^{359}]$

$= M[\text{Sum of GP}]$ $S_{360} = \frac{a(1 - r^n)}{1 - r}$

② $= M\left[\frac{1.005^0(1 - 1.005^{360})}{1 - \cancel{1}.005}\right]$

① = ②

$177\,000(1.005)^{360} = M\left[\frac{1.005^0(1 - 1.005^{360})}{-0.005}\right]$

(1M)

$M =$