St. Catherine's School

Year 11 Extension 1 Mathematics
Preliminary Task #1
April 2008

Time allowed: 55 minutes

INSTRUCTIONS

- There are 3 questions of different values
- Marks for each part of a question are indicated
- All questions should be attempted.
- All necessary working should be shown
- Start each question on a new page
- Approved scientific calculators and drawing templates may be used

Question 1 (12 marks) **Marks**

(a) Solve for x: $x^2+5x+6<0$ **2**

(b) Solve for n: $|2r+1|=1-r$ **3**

(c) Solve for x: $\frac{3}{x-1}\le 2$ **3**

(d) If $g(x)=\begin{cases} x^2 \text{ for } x>2 \\ x \text{ for } -2\le x\le 2 \\ 4 \text{ for } x<-2 \end{cases}$

find $g(-3)+g(-2)+g(2)$. **2**

(e) Determine if $f(x)=x^4+5x^2-2$ is even, odd or neither. **2**

START A NEW PAGE
Question 2 (20 marks)

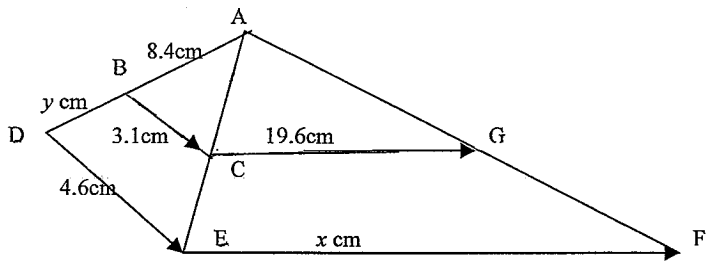
(a) Find the **domain & range** and **sketch** each the following showing all the important features:

(i) $f(x)=\frac{2}{x+1}$ **3**

(ii) $y=\sqrt{25-x^2}$ **3**

(iii) $y=-2x^2+9x+5$ **4**

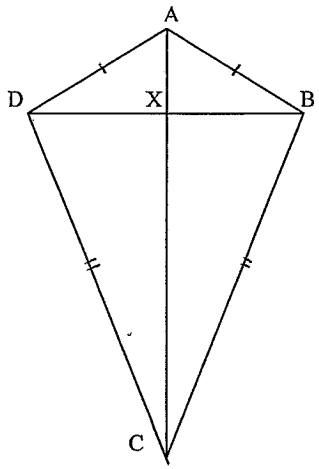
(b) Consider the diagram below:



(i) Prove $\Delta ABC \,|||\, \Delta ADE$ **3**

(ii) Hence or otherwise find the values of x and y to 1 decimal place (you may assume $\Delta ACG \,|||\, \Delta AEF$) **3**

(c) By considering the kite below, prove the diagonals in a kite are perpendicular. **4**



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Question 3 (12 marks) **Marks**

(a) Sketch $y = \dfrac{x^2+4x+3}{x+1}$ showing the point of discontinuity. **2**

(b) Solve for $0^o \le \theta \le 360^o$:

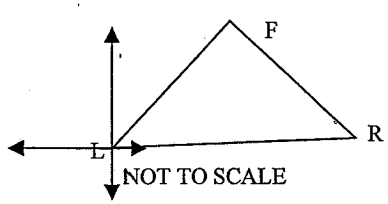
(i) $\cot\theta = -\sqrt{3}$ **2**

(ii) $\sin 2\theta = \dfrac{\sqrt{3}}{2}$ **2**

(c) Simplify: $\dfrac{\sin(90°-\alpha)}{\cos(180°-\alpha)}$ **2**

(d) A fisherman is fishing 1.2 km out at sea at a point F from a lighthouse at point L. Its bearing from the lighthouse is 46° and 316° from a reef R. The reef is due east of the lighthouse.

(i) Redraw the diagram below in your answer booklet and show the given length and angles to reflect the question above. Specify the size of $\angle LFR$. **2**



(ii) How far is the reef from the fisherman correct to 2 decimal places? **1**

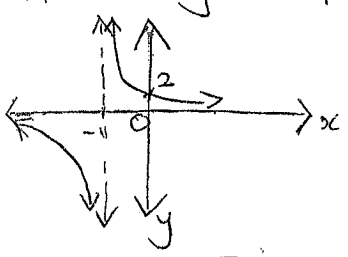
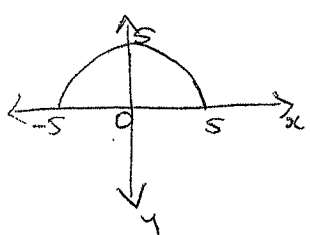
(iii) If the fisherman is drifting directly toward the reef at 2 km/hr, how long will it take to reach the reef? **1**

End of Task

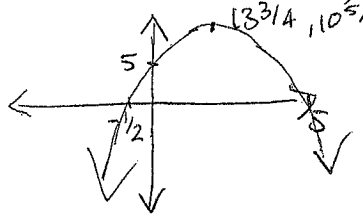
TASK: YR 11 Ext Task 1 . 2008

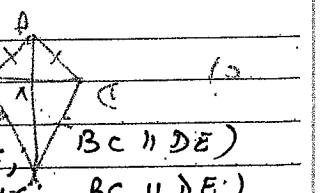
SOLUTIONS	COMMENTS	M's
Q1 a, $x^2+5x+6<0$		
$(x+2)(x+3)<0$		1
$-3<x<-2$		1
b, $\lvert 2r+1\rvert = 1-r$		
$2r+1=1-r$ $\quad -(2r+1)=1-r$		
$3r=0$ $\quad -2r-1=1-r$		
$r=0$ ✓ $\quad -r=2$		
$r=-2$ ✓		2
Check: $r=0$, $r=-2$		
LHS = 1 $\quad$ LHS = $\lvert -3\rvert$		
RHS = 1 $\quad$ = 3		
= LHS $\quad$ = RHS ✓		1
∴ true ✓ $\quad$ ∴ true		
∴ $r=0, r=-2$		
c, $\frac{3\times(x-1)^2}{x+1} \le 2\times(x-1)^2$ $\quad x\neq 1$ ✓		
$3(x-1)\le 2(x-1)^2$		1
$3(x-1)-2(x-1)^2\le 0$		
$(x-1)[3-2(x-1)]\le 0$		$\frac{1}{2}$
$(x-1)(3-2x+2)\le 0$	correct dir'n	
$(x-1)(5-2x)\le 0$	ineq: ½ M correct type: ½ M	$\frac{1}{2}$
		1

TASK:

SOLUTIONS	COMMENTS	M's
1d $g(-3)=4$ ✓		
$g(-2)=-2$ ✓		
$g(2)=2$ ✓		
$4-2+2=4$ ✓		2
e $f(x)=x^4+5x^2-2$		
$f(-x)=x^4+5x^2-2$		1
$=f(x)$		1
∴ even		
2 a i $f(x)=2/x+1$		
D: All real x except $x=-1$ ✓		
R: All real y except $y=0$ ✓		
	✓ correct shape ✓ each correct asymptote as per D&R. ✓ plotting a pt.	3
ii $y=\sqrt{25-x^2}$		
	½ M semi-circle ½ M radius	1
D: All real x such that $-5\le x\le 5$	½ M correct bounds & ½ M correct ineq. type for each.	1
R: All real y such that $0\le y\le 5$		1

TASK:

SOLUTIONS	COMMENTS	M's
1aiii, $y = -2x^2 + 9x + 5$		
$= -(2x^2 - 9x - 5)$ $p = -10$		
$= -(2x^2 + x - 10x - 5)$ $s = -9$		
$= -(x(2x+1) - 5(2x+1))$		
$= -(2x+1)(x-5)$		1
$(13\frac{3}{4}, 10\frac{5}{8})$		
	Shape only intercepts	½ ½
By symmetry a.o.s. $x = 5 - \frac{5}{2} \times \frac{1}{2}$		
$= 3\frac{3}{4}$		½
At $x = 3\frac{3}{4}$, $y = \frac{85}{8}$ (21)		
$= 10\frac{5}{8}$ $15\frac{1}{8}$		½
D: All real x		½
R: All real y such that $y \leq 10\frac{5}{8}$		½
bi, Proof: $\angle BAC$ is common		
$\angle ABC = \angle ADE$ (corresp. $\angle$'s equal, BC ∥ DE)		
$\therefore \angle ACB = \angle AED$ ($\angle$ sum Δ, 180°)		
$\therefore \Delta ABC \equiv \Delta ADE$ (equiangular)		3
ii) $\frac{8.4}{8.4+y} = \frac{3.1}{4.6}$ (sides in prop in similar Δ's)		
$3.1y + 26.04 = 38.64$		
$\therefore y = 4.1$ cm (to 1 d.p)		



9.2 b)

In Δs ABC and ADE

$\angle A$ is common

$\angle ABC = \angle ADE$ (corresp $\angle$s, BC ∥ DE)

$\therefore \angle ACB = \angle AED$ (Corresp $\angle$s, BC ∥ DE)

$\therefore \Delta ABC \equiv \Delta ADE$ (3M)

ii) and (iii) and $\frac{8.4}{8.4+y} = \frac{3.1}{4.6}$ (Corresp sides of similar Δs are proportional)

$(8.4+y) 3.1 = 8.4 \times 4.6$

$8.4 + y = 12.5$

$y = 4.1$ (to 1 d.p) (1½)

$\angle A = \angle A$ (common)

$\angle AFG = \angle ACB$ (corresp $\angle$s, FG ∥ BC)

$\therefore \Delta AFG \equiv \Delta ABC$

$\Delta AFG \equiv \Delta AED$

$\therefore \frac{AG}{AD} = \frac{19.6}{x}$ (Corresp sides are proportional)

$\Delta ABC \equiv \Delta ADE$

$\therefore \frac{3.1}{4.6} = \frac{19.6}{x}$

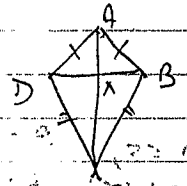
$3.1x = 90.16$

$x = 29.1$ to 1 d.p. (1½)

$\therefore AC = 29.1 - 19.6$

$= 9.5$ to 1 d.p.

c)



In $\triangle ABC$ and $\triangle ADC$

$AB = AD$ (Given)

$BC = DC$ (Given)

AC is common

$\therefore \triangle ABC \equiv \triangle ADC$ (SSS)

$\therefore \angle DAC = \angle BAC$ (corresp $\angle$s of Cong $\triangle$s) 2

Considering $\triangle ADX$ and $\triangle ABX$

$\angle DAX = \angle BAX$ (proved)

$DA = AB$ (Given)

AX is Common

$\therefore \triangle ADX \equiv \triangle ABX$ (SAS)

$\therefore \angle AXD = -\angle AXB$ (corresp $\angle$s of Cong $\triangle$s)

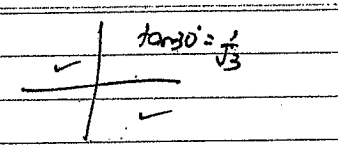
also $\angle AXD + \angle AXB = 180°$ (adjacent Supplementary $\angle$s)

$\therefore \angle AXD = \angle AXB = 90°$ 2

Thus AE and BD are at right angle to each other.

Q.3 $\cot\theta = -\sqrt{3}$

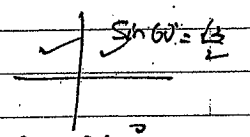
$\tan\theta = -\frac{1}{\sqrt{3}}$



$\theta = (180° - 30°), (360 - 30°)$

$= 150°, 330°$ (2M)

(ii) $\sin 2\theta = \frac{\sqrt{3}}{2}$



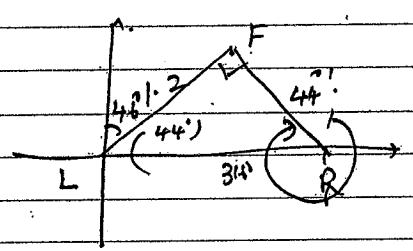
$0 \le 2\theta \le 360°$

$2\theta = 60°, 120°, 420°, 480°$

$\theta = 30°, 60°, 210°, 240°$ (2M)

c) $\frac{\sin(90-\alpha)}{\cos(180-\alpha)} = \frac{\cos\alpha}{-\cos\alpha} = -1.$

(i)



$\angle FLR = 44°$

$\angle LFR = 316 - 270°$

$= 46°$.

$\therefore \angle LFR = 180° - (44+46)$

$= 90°$.

(ii) Let $FR = x$.

$\tan 44° = \frac{x}{1.2}$

$x = 1.2 \times \tan 44°$

$= 1.16$ km (to 2 d.p)

at 2 km/h.

1.16... km is travelled in $\frac{1.16...}{2}$

$= 0.5$ hr