Student Name:_____

St. Catherine's School
Waverley

19th March 2009
PRELIMINARY ASSESSMENT TASK 1

Weighting 15%

Extension 1 Mathematics

Time allowed: *55 minutes*
Total marks: *65*

INSTRUCTIONS

- Marks for each part of a question are indicated
- All questions should be attempted in the booklet provided
- All necessary working should be shown
- Start each question on a new page
- Approved scientific calculators and drawing templates may be used badly arranged work.
- Diagrams should be drawn using PENCIL AND RULER

Marks

QUESTION 1 (16 marks)

a) Solve the absolute equality $|x-3|=2x+1$ **2**

b) Solve the absolute inequality $|4x-3|>5$. **3**

c) Solve the quadratic inequality $6x^2-5x-4\geq 0$ **2**

d) If $\begin{cases} f(x)=3-2x^2 & x\leq -2 \\ f(x)=|2x^3-5| & -2<x<3 \\ f(x)=2x^2-1 & x\geq 3 \end{cases}$ evaluate $|f(-2)|+3f(1)+2f(3)-5$ **3**

e) Find the values of x for which $\frac{2x}{x-3}\geq 1$ **3**

f) Solve the inequality $|2x-3|+|2x+1|>6$ **3**

QUESTION 2 (13 marks)

a) State the natural domain and range for each of the following functions?

(i) $y=-\sqrt{16-x^2}$ **2**

(ii) $y=\frac{5}{x^2-3x+2}$ **2**

(iii) $y=3^{-x}+2$ **2**

b) By completing the square, or otherwise, determine the vertex of the parabola $y=x^2+4x+8$ **2**

QUESTION 2 (continued) **Marks**

c) If $f(x)=x^2-3x+1$ find the value of $f(\sqrt{2}+1)$ **3**

d) For the curves $f(x)=x^2-3x$ and $g(x)=9-x^2$ find their points of intersection. **2**

QUESTION 3 (11 marks)

a) (i) Determine whether the following function is odd, even or neither **2**

$$f(x)=\frac{x^2}{x^2-4}$$

(ii) Sketch the function showing ALL important features **3**

b) Sketch *neatly* (on the same set of axes) the following: **6**

(i) $y=\dfrac{4}{x-1}$

(ii) $x^2+y^2=25$

(iii) The region that is satisfied *simultaneously* by the inequalities
$x^2+y^2<25$ and $y\geq\dfrac{4}{x-1}$

QUESTION 4 (17 marks)

a) Find $\lim\limits_{h\to 5}\left[\dfrac{x^2-25}{x-5}\right]$ **2**

b) Find $\lim\limits_{x\to 0}\left[\dfrac{2x^2h-3xh+4h^2}{h}\right]$ **2**

c) Find $\lim\limits_{x\to\infty}\left[\dfrac{2x^2-3x-4}{3x^2-5x}\right]$ **2**

QUESTION 4 (continued) **Marks**

d) Solve $\dfrac{x+4}{x-1}\leq 6$ and graph the solution(s) on a real number line. **3**

e) Sketch neatly the following functions:

(i) $y=\dfrac{3x}{x-2}$ **2**

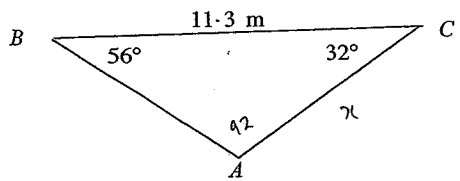
(ii) $y-1=(x+1)^2$ **2**

(iii) $y=|2x-3|+1$ **2**

(iv) $y=\left|(x-1)^3\right|$ **2**

QUESTION 5 (8 marks)

a) In the following diagram find the length of AC **3**



b) Point B is South-East ($S45°E$) of point A and a distance of 2 km from it. From point A, a point P has a bearing of 057°T and from B the bearing of point P is 348°T. **5**

(i) Find the size of $\angle PAB$ and $\angle APB$

(ii) Find the distance from A to P giving your answer to 2 decimal places

End of Task

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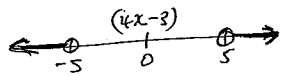
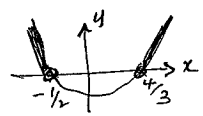
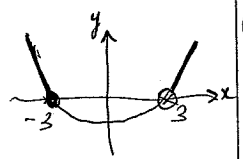
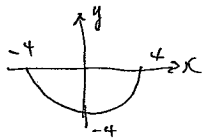
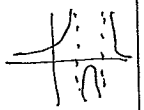
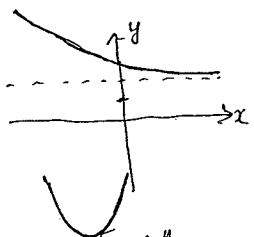
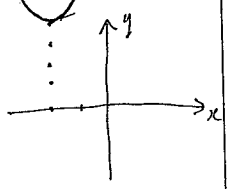
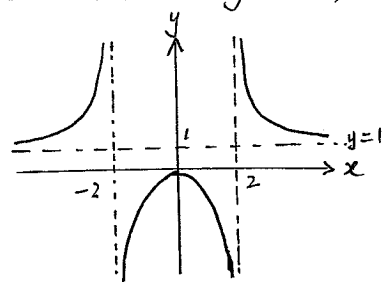
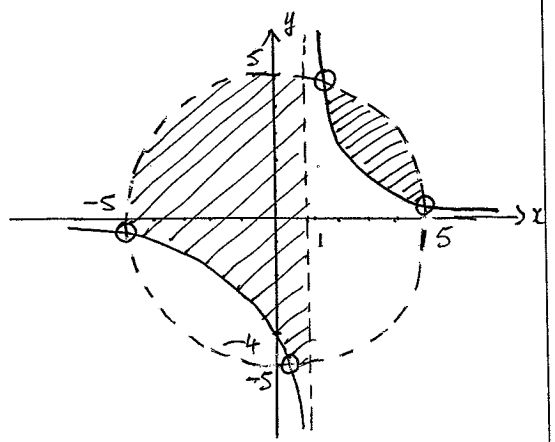
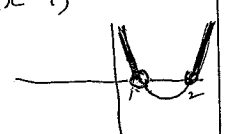
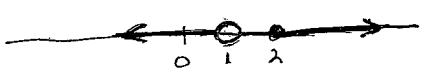
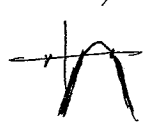
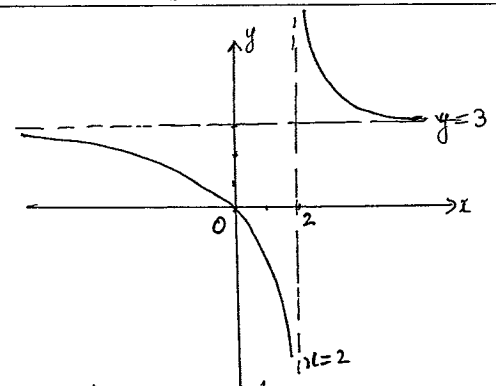
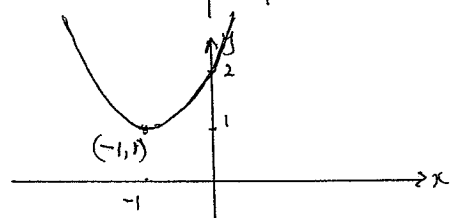
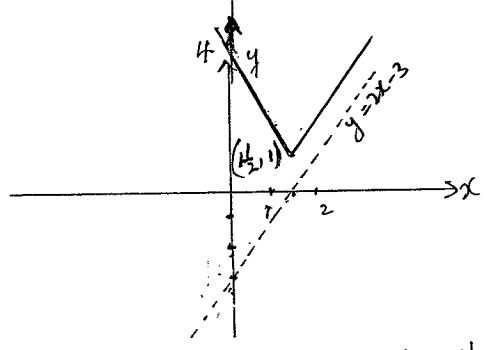
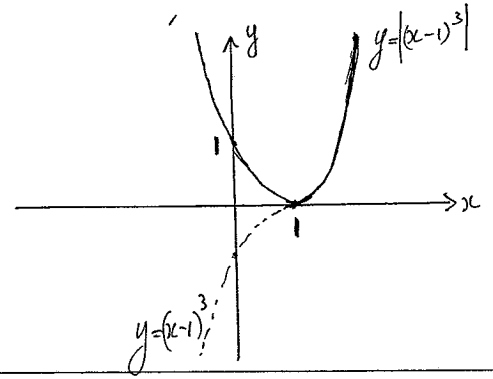
SOLUTIONS

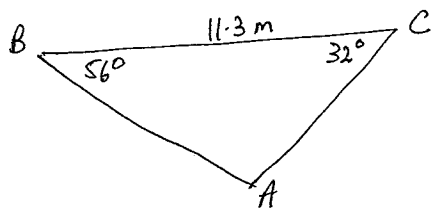
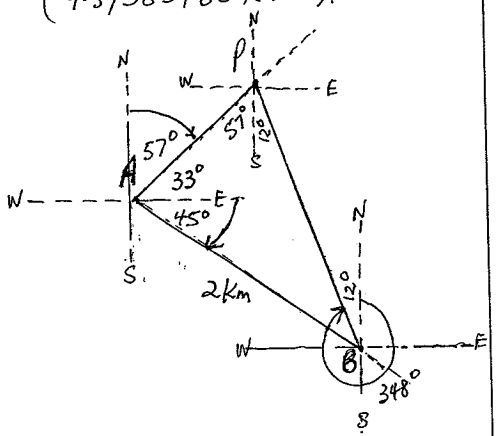
Qn	Solutions	Marks	Comments+Criteria
1a	$\lvert x-3\rvert = 2x+1$ or $x-3 = 2x+1$ $\quad x-3 = -2x-1$ $-4 = x$ $\quad 3x = 2$ $x = \frac{2}{3}$	①	
	Check $\lvert -4-3\rvert = 2(-4)+1$ $\quad \lvert \frac{2}{3}-3\rvert = 2(\frac{2}{3})+1$ False $\quad \lvert -2\frac{1}{3}\rvert = \frac{4}{3}+1$ true	①	
	$\therefore x = \frac{2}{3}$		
1b	$\lvert 4x-3\rvert > 5$ 		
	$4x-3 < -5$ or $4x-3 > 5$ $4x < -2$ $\quad 4x > 8$ $x < -\frac{1}{2}$ $\quad x > 2$	③	
1c	$6x^2-5x-4 \geq 0$  $(3x-4)(2x+1) \geq 0$ $x \leq -\frac{1}{2}$ or $x \geq \frac{4}{3}$	②	
1d	$\lvert f(-2)\rvert = \lvert -5\rvert = 5$; $f(1) = \frac{1}{3}\lvert -3\rvert$; $f(3) = 17$	①	
	$\therefore \lvert f(-2)\rvert + 3f(1) + 2f(3) - 5$	①	
	$= 5 + 9 + 34 - 5$	①	
	$= 43$		
1e	$\frac{2x}{x-3} \geq 1$ $\quad x \neq 3$		
	$2x(x-3) \geq (x-3)^2$ 	①	
	$2x^2-6x \geq x^2-6x+9$	①	
	$x^2-9 \geq 0$		
	$(x-3)(x+3) \geq 0$ $\quad \therefore x \leq -3$ or $x \geq 3$	①	$\frac{1}{2}$ off for $x \geq 3$

Qn	Solutions	Marks	Comments+Criteria
1f	$\lvert 2x-3\rvert + \lvert 2x+1\rvert > 6$		
	$2x-3+2x+1>6$ $\quad -2x+3-2x-1>6$	①	
	$4x > 8$ $\quad -4x > 4$	①	
	$x > 2$ $\quad x < -1$	①	
Q2 a)	(i) D: $-4 \le x \le 4$ 	①	
	R: $-4 \le y \le 0$	①	
	(ii) D: $x \neq 1, 2$ $\quad y = \frac{5}{(x-2)(x-1)}$ 	①	* accepting this answer actually, however, R: $y>0$ or $y \le -1/4$ discuss in class!
	$\therefore$ D: all real x; $x \neq 1, 2$	①	
	R: all real y; $y \neq 0$ *		
	(iii) D: all real x 	①	
	R: $y > 2$	①	
b)	$y = x^2+4x+8$ 		
	$y = x^2+4x+4+4$	①	
	$y = (x+2)^2+4$	①	
	Vertex $(-2, 4)$		
c)	$f(x) = x^2-3x+1$	①	
	$\therefore f(\sqrt{2}+1) = (\sqrt{2}+1)^2 - 3(\sqrt{2}+1)+1$	①	
	$= \cancel{3} + 2\sqrt{2} - 3\sqrt{2} \cancel{-3} + 1$	①	
	$= -\sqrt{2}+1$ or $1-\sqrt{2}$		

Qn	Solutions	Marks	Comments+Criteria
d)	$f(x) = x^2-3x$ $\quad g(x) = 9-x^2$		
	$x^2-3x = 9-x^2$	①	
	$2x^2-3x-9 = 0$		
	$(2x+3)(x-3) = 0$		
	$\therefore x = -\frac{3}{2}, 3$		
	$y = 6\frac{3}{4}, 0$		
	$\therefore$ points of intersection are $(-\frac{3}{2}, 6\frac{3}{4}), (3, 0)$	①	
Q3a)	(i) $f(x) = \frac{x^2}{x^2-4}$ $\quad f(-x) = \frac{(-x)^2}{(-x)^2-4} = \frac{x^2}{x^2-4}$		
	$\therefore$ function is EVEN (ie symmetrical about y axis)	②	
	(ii) 	③	1 for vert asymptotes 1 for other asymptotes 1 intercepts + sketch
		⑥	1½ graph of $y = \frac{4}{x-1}$ 1½ graph of $x^2+y^2=25$ 1 dotted circle 1 inside circle ½ open circles on intersection points ½ right side of hyperbola in each section

Qn	Solutions	Marks	Comments+Criteria
Q4	a) $\lim_{x\to 5} \frac{x^2-25}{x-5} = \lim_{x\to 5} \frac{(x-5)(x+5)}{(x-5)}$ $= 10$	(2)	1 mark for correct factorising & simplifying 1 mark for correct answer
	b)* $\lim_{h\to 0} \left[\frac{2x^2h - 3xh + 4h^2}{h}\right]$ $= \lim_{h\to 0} 2x^2 - 3x + 4h$ $= 2x^2 - 3x$	(2)	* If the student had done this question with $x \to 0$ $\lim_{x\to 0}\left[\frac{2x^2h-3xh+4h^2}{h}\right] = 4h$ this answer gains full marks.
	c) $\lim_{x\to\infty} \left[\frac{2x^2 - 3x - 4}{3x^2 - 5x}\right]$ $= \lim_{x\to\infty} \left[\frac{\frac{2x^2}{x^2} - \frac{3x}{x^2} - \frac{4}{x^2}}{\frac{3x^2}{x^2} - \frac{5x}{x^2}}\right]$ $= \frac{2}{3}$	(2)	
	d) $\frac{x+4}{x-1} \leq 6 \qquad x \neq 1$ $(x-1)(x+4) \leq 6(x-1)^2$ $x^2 + 3x - 4 \leq 6x^2 - 12x + 6$ $0 \leq 5x^2 - 15x + 10$ $0 \leq x^2 - 3x + 2$ $0 \leq (x-2)(x-1)$ $\therefore x \leq 1$ or $x \geq 2$  	(1) (1) (1)	OR $(x-1)(x+4) - 6(x-1)^2 \leq 0$ $(x-1)[x+4-6(x-1)] \leq 0$ $(x-1)(-5x+10) \leq 0$  note $x \neq 1$ $-\frac{1}{2}$ mark for $x \leq 1$

Qn	Solutions	Marks	Comments+Criteria
Q4 e	(i) 	(2)	½ mark vert asymptote ½ mark horizontal asymptote ½ mark correct shape ½ mark x, y intercepts
	(ii) 	(2)	½ mark correct shape ½ mark y intercept 1 mark vertex
	(iii) 	(2)	½ mark correct shape ½ mark y intercept 1 mark correct shifting and vertex
	(iv) 	(2)	½ mark for reflection ½ mark correct shape 1 mark correct intercepts

Qn	Solutions	Marks	Comments+Criteria
Q5 a)			
	$\angle BAC = 92^\circ \quad (180^\circ - 56^\circ - 32^\circ)$.	(1)	
	$\therefore \frac{AC}{\sin 56^\circ} = \frac{11.3}{\sin 92^\circ}$	(1)	
	$AC = \frac{11.3 \times \sin 56^\circ}{\sin 92^\circ}$		
	$= 9.37\,m \ (2\,d.p.)$.	(1)	
	$(9.373834857\ldots)$.		
b)			
	(i) $\angle PAB = 33^\circ + 45^\circ$		
	$= 78^\circ$	(1)	
	$\angle APB = 57^\circ + 12^\circ$		
	$= 69^\circ$	(1)	

Qn	Solutions	Marks	Comments+Criteria
Q5	(ii) Now $\angle PBA = 180^\circ - 78^\circ - 69^\circ$		
	$= 33^\circ$	(1)	
	$\therefore \frac{AP}{\sin 33^\circ} = \frac{2}{\sin 69^\circ}$		
	$AP = \frac{2 \sin 33^\circ}{\sin 69^\circ}$	(1)	
	$= 1.17\,km. \ (2\,d.p.)$	(1)	