St Catherine's
School
Waverley, Sydney

Student Number:_____

Year 11 Extension I Mathematics Assessment Task 1 4 May 2010

Time allowed: 1 hour
Reading Time 5 minutes

General Instructions

- Attempt ALL questions
- Write your NAME or Student NUMBER in all the examination booklets used

Questions	Marks
Q.1	/12
Q.2	/10
Q.3	/12
Q.4	/~~12~~ 12
Total	/~~45~~ 46

Question 1 (12marks)

a) A car was sold at a discount of 10% followed by a further discount of 15%. Find the cost price to the nearest dollar, if the selling price of the car is \$16800. **2**

b) Simplify $\dfrac{x^{-1}+y^{-1}}{x+y}$ **2**

c) Find the acute angle, to the nearest degree, between the straight lines:

(i) $y=2x+5$ and $x-y+1=0$ **2**

(ii) $x=5$ and $y=\sqrt{3}x$ **2**

d) (i) Find the coordinates of the point, which divides the join of $(3,5)$ and $(5,8)$ externally in the ratio of 2:1. **2**

(ii) A point $P(1,3)$ divides the join of two points A and B internally in the ratio of 5:2. If the coordinates of the point A is $(-6,0)$, find the coordinates of the point B. **2**

Question 2 (10 Marks)

a) Solve for x: $\frac{3}{x-1} < 1$ — 3

b) Solve for x: $2x^2 < 3-5x$ — 2

c) (i) Sketch the graph of $y=|3x-1|$ and $y=x+1$ on the same set of axes, highlighting the x and y intercepts. — 2

(ii) Hence solve **graphically** the equation $|3x-1|>x+1$ — 3

Question 3 (12 marks)

a) Find the exact value of:

(i) $\tan 495^0$ — 1

(ii) $\sin 300^0$ — 1

b) Solve for x: — 2

$\tan x = \sqrt{3} \qquad -180^0 < x \leq 180^0$

c) Show that $\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$ — 3

d) Given $\cot^2\theta+2\operatorname{cosec}^2\theta=5$

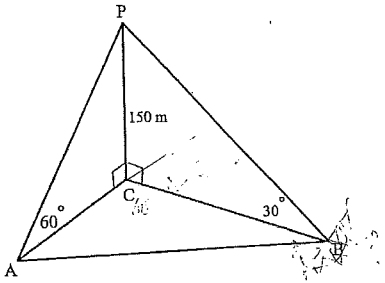
(i) show that $\cos^2\theta=\frac{1}{2}$ — 3

(ii) Hence solve for $\theta \qquad 0<\theta\leq 360^0$ — 2

12

Question 4. (12 marks)

a) PC represents a cliff, which is 150 metres high. Yacht A is to the south of the cliff and the angle of elevation from Yacht A to the top of the cliff is 60^0. Another yacht B has a bearing of 120^0 to the foot of the cliff C. The angle of elevation from B to the top of the cliff is 30^0.



(i) Calculate the exact lengths of AC and BC 2

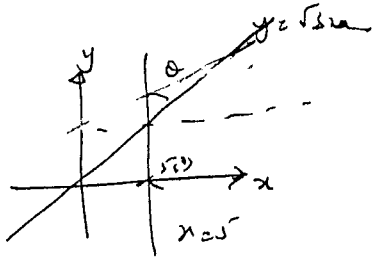
(ii) Calculate the angle ACB and hence find the distance AB between the two yachts, giving your answer to the nearest metre 3

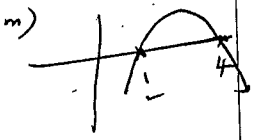
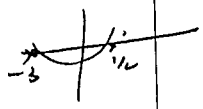
b) Sketch the region common to $y < 2x+8$ *and* $y > x^2$ 2

c) Simplify: $\dfrac{1}{2x+4}+\dfrac{1}{x^2-4}-\dfrac{1}{x-2}$ 3

d) Find the values of θ, $0 \le \theta \le 360^0$ for which $\sqrt{\sin^2\theta} = -\sin\theta$ 2

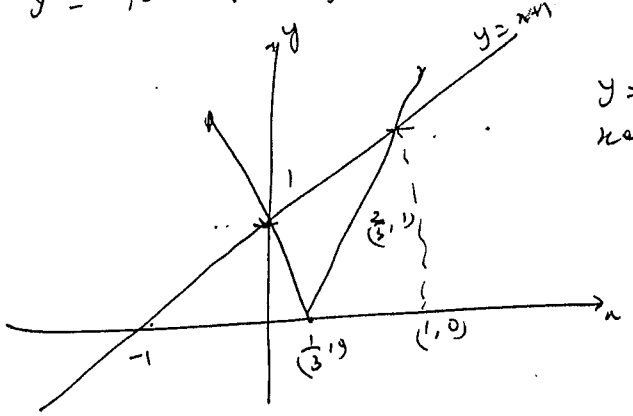
END OF PAPER

Qn	Solutions	Marks	Comments: Criteria
1a)	Let the cost price be $x. $0.9 \times 0.85x = 16800$ $x = \frac{16800}{0.9 \times 0.85} = 21960.78$ Cost price is ~~$21960.78~~ $21961	(1M) (1A)	
b)	$x^{-1} + y^{-1} = \frac{1}{x} + \frac{1}{y} = \frac{y+x}{xy}$ $\therefore \frac{x^{-1}+y^{-1}}{x+y} = \frac{\frac{x+y}{xy}}{x+y} = \frac{1}{xy}$	(1M) (1M)	
c) (i)	$y = 2x+5$; $m_1 = 2$ $x - y + 1 = 0$; $m_2 = 1$ if θ is the acute angle between the lines; $\tan\theta = \left\|\frac{2-1}{1+2}\right\| = \frac{1}{3}$ $\therefore \theta = 18^\circ$ to the nearest degree	(1M) (1A)	Correct use of formula (1M)
d (i)	 $\theta = 90^\circ - \tan^{-1}\sqrt{3}$ $= 90^\circ - 60^\circ$ $= 30^\circ$	(1M) (1A)	
d (ii)	A: (3, 5) B: (5, 8) −2 1 P(x, y) $y = \frac{-16+5}{-1} = 11$		1M – Correct use of formula. including ratio of 2:−1 or −2:1

Qn	Solutions	Marks	Comments: Criteria
(11)	A: (−6, 0) B: (x, y) 5 2 P(1, 3) $1 = \frac{5x-12}{7}$ ... $3 = \frac{5y}{7}$ $5x = 19$ $y = \frac{21}{5}$ $x = \frac{19}{5}$		
Q.2			
a)	$\frac{3}{x-1} < 1$; $x \neq 1$ $(x-1)^2 \cdot \frac{3}{x-1} < (x-1)^2$ $3(x-1) < (x-1)^2$ $(x-1)[3-(x-1)] < 0$ $(x-1)(4-x) < 0$ $x < 1$ or $x > 4$	(1m) (1m) (1m)	
b)	$2x^2 < 3 - 5x$ $2x^2 + 5x - 3 < 0$ $(2x-1)(x+3) < 0$ (1M) $-3 < x < \frac{1}{2}$. (1A)		

Qn	Solutions	Marks	Comments: Criteria

$y = |3x-1|$ & $y = x+1$



$y = |3x-1|$ key pts: $(\frac{1}{3}, 0)$, $(0, 1)$

$y = x+1$ $(0,1)$ and another pt.

$y = 3x-1$ meets $y = x+1$ at

$3x - 1 = x + 1$

$2x = 2$

$x = 1$;

$y = -3x+1$ meets $y = x+1$ at

$-3x + 1 = x + 1$

$x = 0$ or see $x=0$ from the figure

$\therefore y = |3x-1|$ meets $y = x+1$ at

$x = 0$; $x = 1$ → B. (1.5 m)

from the figure;

$|3x-1| > x+1$

for $x < 0$ or $x > 1$. → 1.5m

Qn	Solutions	Marks	Comments: Criteria

3. a) $\tan(495^\circ)$

$= \tan(135^\circ)$

$= \tan(180 - 45^\circ)$

$= -\tan 45^\circ$ — 1

$= -1$.

b) $\sin 300^\circ = \sin(360 - 60)$

$= -\sin 60^\circ$

$= -\frac{\sqrt{3}}{2}$. 1

b) $\tan x = \sqrt{3}$ (60°, 240°)

$x = 60^\circ, 240^\circ \quad (0 \le x \le 360^\circ)$

$= 60^\circ, -120^\circ \quad (-180 < x < 180)$

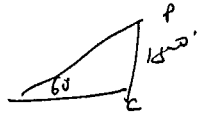
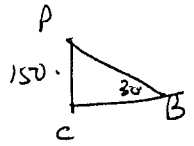
c) $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$

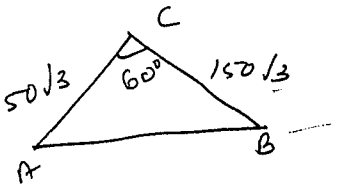
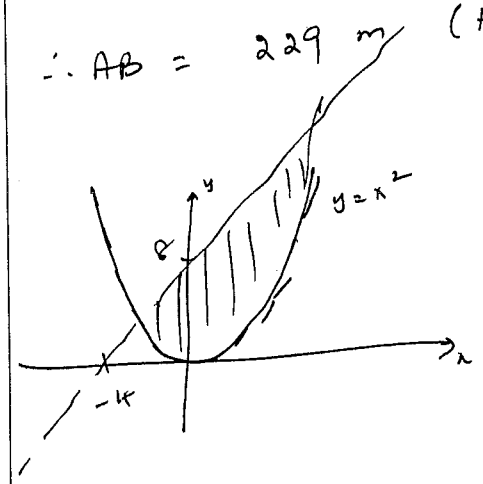
$= \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$ (1m)

$= \frac{1 + \sin^2\theta + 2\sin\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)}$ 1h

$= \frac{2 + 2\sin\theta}{\cos\theta(1+\sin\theta)}$ (1m)

$= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = \frac{2}{\cos\theta} = 2\sec\theta$ (1m) 1h

Qn	Solutions	Marks	Comments: Criteria
d)	$\cot^2\theta + 2\cos ec^2\theta = 5$		
	$\frac{\cos^2\theta}{\sin^2\theta} + \frac{2}{\sin^2\theta} = 5$	(1m)	
	$2 + \cos^2\theta = 5\sin^2\theta$	(1m)	
	$2 + \cos^2\theta = 5(1 - \cos^2\theta)$		
	$6\cos^2\theta = 3$	m.	
	$\cos^2\theta = \frac{1}{2}$		
(ii)	$\cos^2\theta = \frac{1}{2}$		$\cos\theta = \frac{1}{\sqrt{2}}$ Correctly followed through is 1m out of 2m.
	$\cos\theta = \pm\frac{1}{\sqrt{2}}$		
	$\theta = 45°, 180-45°, 180+45°, 360-45°$		
	$= 45°, 135°, 225°, 315°$		
Q.4			
	$\frac{PC}{AC} = \tan 60°$		
	$AC = \frac{150}{\sqrt{3}} = 50\sqrt{3}$		
			ignored the exact value in my working just for this task.
	$\frac{150}{BC} = \tan 30°$		
	$\therefore BC = \frac{150}{\tan 30}$		
	$= \frac{150}{1/\sqrt{3}}$	2/	
	$= 150\sqrt{3}$.		

Qn	Solutions	Marks	Comments: Criteria
	$\angle ACB = 180° - 120°$	(m)	
	$= 60°$		
			
	$AB^2 = (50\sqrt{3})^2 + (150\sqrt{3})^2 - 2\times 50\sqrt{3}\times 150\sqrt{3}\times\cos 60°$	(1m)	
	$= 3(2500 + 22500 - 7500)$		(ignored working with rounded value only for this task)
	$= 52500$		
	$\therefore AB = 229$ m (to the nearest metre)	(1m)	
b)			$\begin{cases} y = x^2 & \text{dotted} \\ y \geq 2x + 8 & \text{dotted} \end{cases}$ $-\frac{1}{2}$ only if both are not dotted ignore if one is not dotted. $(-4, 0)$ or $(0, 8)$ $[-\frac{1}{2}$ if not shown] (at this stage, I have ignored both of the above criteria.) $-\frac{1}{2}$ if pts are not labelled.
c)	$\frac{1}{2x}$		

Qn	Solutions	Marks	Comments: Criteria
	$\frac{1}{2x+4} + \frac{1}{x^2-4} - \frac{1}{x-2}$		
	$= \frac{1}{2(x+2)} + \frac{1}{(x-2)(x+2)} - \frac{1}{x-2}$	(1m)	
	$= \frac{(x-2) + 2 - (2(x+2))}{2(x-2)(x+2)}$	(1m)	
	$= \frac{x - 2 + 2 - 2x - 4}{2(x-2)(x+2)}$		
	$= \frac{-(x+4)}{2(x-2)(x+2)}$	(1m)	
d)	$\sqrt{\sin^2\theta} = \lvert\sin\theta\rvert$		
	$= -\sin\theta$ when $\sin\theta < 0$	(1m)	$\sin\theta < 0$ (1m)
	$\sin\theta < 0$ $\quad +\ -\ +$ $180^\circ < \theta < 360^\circ$.	✓✓	✓: 1m Including $180^\circ, 360^\circ$ $(-\frac{1}{2}m)$
(7)			