Student Name:______________________

St. Catherine's School

Year 11 Mathematics

Preliminary Task #2

4th May 2010

Time allowed: *90 minutes + 5 minutes reading time*

Total marks: *82 marks*

Weighting: *25%*

INSTRUCTIONS

- There are 5 questions of different values.
- Marks for each part of a question are indicated.
- Questions 1 & 2 should be attempted in one booklet.
- Question 3 should be attempted in a separate booklet.
- Questions 4 & 5 should be attempted in a separate booklet.
- All necessary working should be shown.
- Start each question on a new page.
- Approved scientific calculators and drawing templates may be used.
- Marks may be deducted for careless or badly arranged work.

QUESTION 1 (17 marks)

Marks

(a) For the function $f(x)=x^3+2$

(i) Find $f(-2)$ **1**

(ii) Find x if $f(x)=-25$ **1**

(b) State the natural domain of the following functions:

(i) $f(x)=2^x$ **1**

(ii) $f(x)=\dfrac{x^2}{x^2-4}$ **2**

(iii) $f(x)=\sqrt{x-2}$ **2**

(c) Sketch the following curves on separate diagrams, and specify their natural domain and range.

(i) $f(x)=-\dfrac{3}{x}$ **3**

(ii) $f(x)=\sqrt{100-x^2}$ **3**

(d) Is $f(x)=\dfrac{x^2+5}{x^3}$ even, odd or neither? Give reasons. **2**

(e) Graph on a number line the solution to $(2x-5)(x+3)<0$ **2**

QUESTION 2 (20 marks) START A NEW PAGE

Marks

(a) Solve for x:

(i) $3x^2 - 6x = 0$ **2**

(ii) $|4 - x| = 5$ **2**

(iii) $\frac{2x-5}{3} + 2 = \frac{3x-9}{4}$ **2**

(b) For the parabola $f(x) = 2x^2 - 4x - 6$

(i) Find the co-ordinates of the vertex. **2**

(ii) Locate the x- and y-intercepts. **2**

(iii) Hence sketch the curve. **2**

(iv) Use your graph to specify the range of $f(x)$ **1**

(v) Use your graph to specify the values of x for which the function is increasing. **1**

(c) A triangle has base $(4x + 2)\,cm$ and height $(x - 4)\,cm$, and an area of $11\ cm^2$

i) Write an equation which uses this information **1**

ii) Solve to find x using the quadratic formula. **3**

(d) The graph of $f(x) = 2x + 2$ is restricted to the domain $x > 0$. Find the range of this restricted function. **2**

QUESTION 3 (21 marks) START A NEW BOOKLET

Marks

a) Simplify $\frac{k^3 - 1000n^3}{2k^2 - 15nk - 50n^2}$ **3**

b) Find the simultaneous solution to: **2**

$$3a + 5b = 12$$

$$a - 2b = -7$$

c) Find the point(s) of intersection of the curve $y = 4x - x^2$ and the line $y = 4x - 9$ **3**

d) The function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} x & \text{for } x < 0 \\ x^3 & \text{for } x \geq 0 \end{cases}$$

(i) Find the value of $f(-5) + f(2) + 10$ **2**

(ii) Sketch $y = f(x)$ **2**

e) What are the co-ordinates of the centre of the circle $x^2 + (y - 2)^2 = 16$? What is its radius? **2**

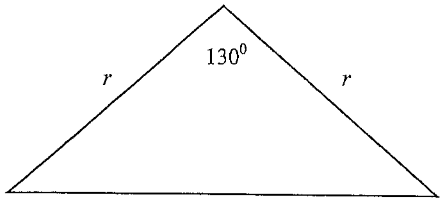
f) On the number plane, shade the region defined by $y > x^2 - 2x$ and $y < 3$ simultaneously **4**

g) Given $F(x) = x^2 + 5$, find $F(x + 2) + F(2 - x)$ **3**

QUESTION 4 (6 marks) START A NEW BOOKLET

Marks

a) 3



This isosceles triangle represents the end of a gable roof.
The base is 12 m long and the two equal sections of roof are at an angle of 130^0 to each other.

Find the length of the sloping section of roof, marked r, correct to 1 decimal place.

b) Katrina starts at point P, and walks 3 km due south and then 7 km due East to point Q.

(i) Draw a diagram showing this information. 1

(ii) Use trigonometry to find the bearing of Q from P. 2

QUESTION 5 (18 marks) START A NEW PAGE

Marks

a) i) Find x if $2^{3x} = 64$ 2

ii) Find y if $\log_3 y = 3$ 2

b) i) Express in simplest form: $\log_6 9 + \log_6 8 - \log_6 2$ 2

ii) Express $\log_4 10$ correct to 2 decimal places. 2

c) Solve for k, correct to 3 significant figures:

$$3^k = 50$$ 2

d) Express $\dfrac{\sqrt{2}}{3\sqrt{2}-1}$ in the form $a + b\sqrt{2}$ 3

e) If $g(x) = \dfrac{x}{x^2-1}$, find $g(\dfrac{1}{x})$ in simplest form. 2

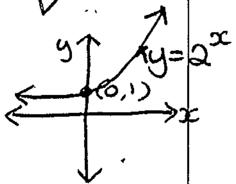
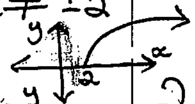
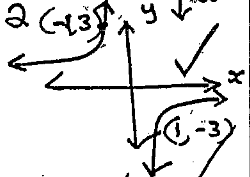
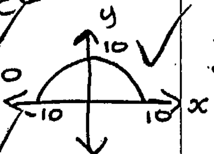
g) i) For any odd function $F(x)$, prove that $F(a) + F(-a) = 0$ 1

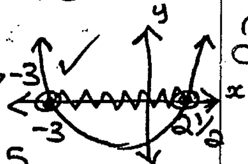
ii) Hence show that if $F(x)$ is defined at $x=0$, then $F(0) = 0$ 1

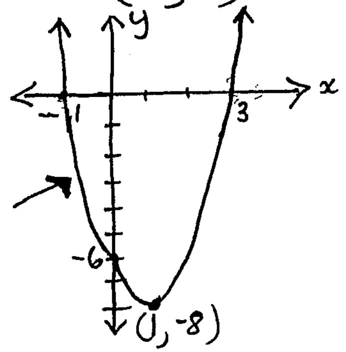
iii) Explain what this means for the graph of $y = F(x)$ 1

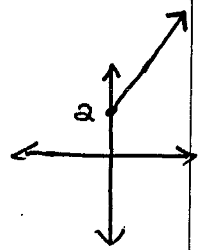
END OF PAPER

Year 11 Solutions ✓ 1 mark ✓ $\frac{1}{2}$ mark

Qn	Solutions	Marks	Comments: Criteria
1 (a)	$f(x)=x^3+2$ i) $f(-2)=(-2)^3+2$ ✓ $=-8+2$ ✓ $=-6$	1	
	ii) $f(x)=-25$ $-25=x^3+2$ ✓ $-27=x^3$ $x=\sqrt[3]{-27}$ ✓ $\therefore x=-3$	1	
(b)	i) $f(x)=2^x$  Domain: $x\in R$ ✓	1	
	ii) $f(x)=\frac{x^2}{x^2-4}$ Domain: $x\in R$ ✓ $x\neq\pm2$ ✓	2	$-\frac{1}{2}$ mark if $\pm$ not shown.
	iii) $f(x)=\sqrt{x-2}$  Domain: $x\geq2$	2	
(c)	i) $f(x)=\frac{-3}{x}$  Domain: $x\in R$ $x\neq0$ ✓ Range: $f(x)\in R$ $f(x)\neq0$ ✓	3.	
	ii) $f(x)=\sqrt{100-x^2}$  Domain: $x\in R$ $-10\leq x\leq10$ ✓ Range: $f(x)\in R$ $0\leq y\leq10$ ✓	3	
(d)	$f(x)=\frac{x^2+5}{x^3}$ $f(-x)=-f(x)$ odd function $f(x)=\frac{(x)^2+5}{(x)^3}$		

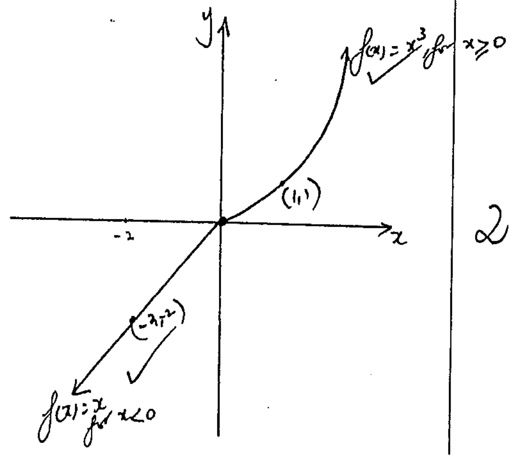
Qn	Solutions	Marks	Comments: Criteria
	$f(-x)=\frac{x^2+5}{-x^3}$ ✓ $-f(x)=\frac{x^2+5}{-x^3}$ ✓ $\therefore$ the function is odd.	2.	$=\frac{x^2+5}{}$ $=-\frac{(x^2+5)}{x^3}$
(e)	$(2x-5)(x+3)<0$ $x<\frac{5}{2}$ ✓ $x>-3$ ✓  $\therefore x: -3<x<2.5$	2.	-1 if $2x-5<0$ $x+3<0$ Q1 /17
2 (a)	i) $3x^2-6x=0$ ✓ $3x(x-2)=0$ ✓ $3x=0$ $x-2=0$ $x=0$ ✓ $x=2$ ✓	2	1 factorising 1 solution.
	ii) $\lvert 4-x\rvert=5$ case one $4-x>0$ case 2: $4-x<0$ $4-x=5$ or $4-x=-5$ $-1=x$ $9=x$ $\therefore x=-1$ ✓ $\therefore x=9$ ✓	2	1 for each case.
	iii) $\frac{2x-5}{3}+2=\frac{3x-9}{4}$ ✓ $\frac{2x-5+6}{3}=\frac{3x-9}{4}$ ✓ $\frac{(2x+1)}{3}=\frac{(3x-9)}{4}$ ✓ $4(2x+1)=3(3x-9)$ ✓ $8x+4=9x-27$ $31=x$ $\therefore x=31$ ✓	2.	$\frac{1}{2}$ mark for common denominator
(b)	$f(x)=2x^2-4x-6$ $x=\frac{-b}{2a}$ ✓ $a=2$ $b=-4$ $=\frac{--4}{2\times2}\Rightarrow1$ ✓ $c=-6$		

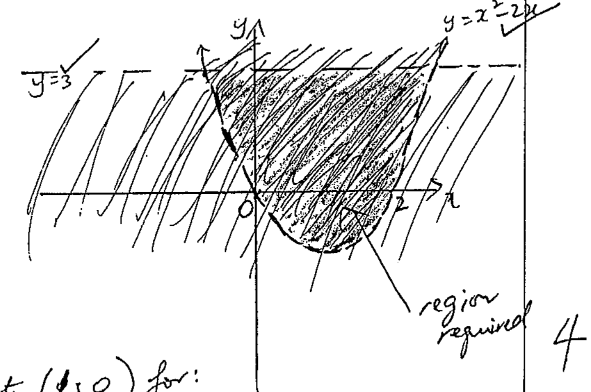
Qn	Solutions	Marks	Comments: Criteria
	when $x=1$ $f(x)=?$ $f(x) = 2(1)^2 - 4\times 1 - 6$ $= 2 - 4 - 6$ $= -8$ ✓ $\therefore$ vertex is $(1,-8)$ ✓	2	
	ii) y intercept is when $x=0$ $f(0) = 2(0)^2 - 4\times 0 - 6$ $= -6$ ✓ $\therefore$ y intercept is $(0,-6)$ x intercept is when $y=0$ or $f(x)=0$ $0 = 2x^2 - 4x - 6$ $0 = x^2 - 2x - 3$ ✓ $0 = (x-3)(x+1)$ $\therefore x = 3$ and $x = -1$ $\therefore$ x intercepts are $(3,0)$ ✓ and $(-1,0)$ ✓	2	
	iii)  $f(x) = 2x^2 - 4x - 6$	2	$\frac{1}{2}$ for x int $\frac{1}{2}$ for y int $\frac{1}{2}$ for vertex $\frac{1}{2}$ concaved up.
	iv) range: $f(x) \in R$, $f(x) \geq -8$ ✓	1	
	v) the function is increasing when $x > 1$ ✓	1	

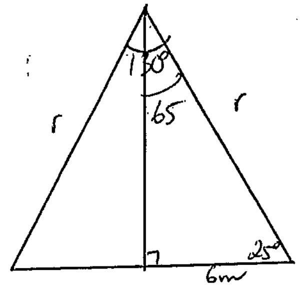
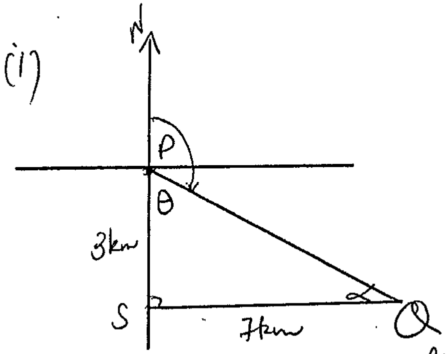
Qn	Solutions	Marks	Comments: Criteria
c	$(4x+2)$, $(x-4)$ $A = \frac{1}{2}bh$ Area $= 11cm^2$ i) $\therefore 11 = \frac{1}{2}(4x+2)(x-4)$ ✓	1	
	ii) $11 = \frac{1}{\cancel{2}}\cancel{2}(2x+1)(x-4)$ $11 = (2x+1)(x-4)$ $11 = 2x^2 - 8x + x - 4$ $0 = 2x^2 - 7x - 15$ ✓ $0 = (2x+3)(x-5)$ ✓ $\therefore x = 5$ ✓ $2x = -3$ ✓ $x = -\frac{3}{2}$ $\therefore x = 5$ is the only solution as $x = -\frac{3}{2}$ cannot be a solution. ✓ (length cannot be -ve.)	3	
d	$f(x) = 2x+2$ $x > 0$ $f(0) = 2\times 0 + 2$ $= 2$. $f(1) = 2\times 1 + 2$ $= 4$ $\therefore$ range $y : y > 2$ ✓✓ 	2	

✓ means 1 mark

Qn	Solutions	Marks	Comments: Criteria
3	(a) $\dfrac{k^3 - 1000n^3}{2k^2 - 15nk - 50n^2}$ $= \dfrac{k^3 - (10n)^3}{(k-10n)(2k+5n)}$ ✓ $= \dfrac{\cancel{(k-10n)}(k^2 + 10kn + 100n^2)}{\cancel{(k-10n)}(2k+5n)}$ ✓ ✓ $= \dfrac{k^2 + 10kn + 100n^2}{2k+5n}$ ✓	3	
	(b) $3a + 5b = 12$ —(1) $a - 2b = -7$ —(2) $\times 3$ ✓ $3a + 5b = 12$ —(1) $-\ 3a - 6b = -21$ —(2)' ✓ $11b = 33$ $b = 3$ ✓ sub. $b=3$ into (2) to find the value of a $a - 2b = -7$ $a - 6 = -7$ $a = -1$ ✓ $\therefore a = -1$ & $b = 3$	2	

Qn	Solutions	Marks	Comments: Criteria
3	(c) $y = 4x - x^2$ and $y = 4x - 9$ $\therefore 4x - x^2 = 4x - 9$ ✓ $x^2 = 9$ $\sqrt{x^2} = \sqrt{9}$ ✓ $x = \pm 3$ ✓ when $x = 3$ ✓: $y = 4(3) - 9$, $y = 3$ ✓ when $x = -3$: $y = 4(-3) - 9$, $y = -21$ ✓ $\therefore$ the points of intersection are $(3, 3)$ and $(-3, -21)$	3	
	(d) (i) $f(-5) = -5$ ✓ $f(2) = 2^3 = 8$ ✓ $\therefore f(-5) + f(2) + 10$ $= -5 + 8 + 10$ $= 13$ ✓	2	
	(ii) 	2	-0.5 mark in total for not labelling both graphs.

Qn	Solutions	Marks	Comments: Criteria
3 (e)	$x^2+(y-2)^2=16$ $(x-a)^2+(y-b)^2=r^2$ where (a,b) is the centre and r = radius ∴ Centre $(0,2)$ ✓ Radius = 4 units ✓	2	
(f)	 Test $(1,0)$ for: $y > x^2-2x$ $0 > 1-2$ $0 > -1$ (true) & $y<3$ $0<3$ (true)	4	1 mark for each sketch 0.5 mark for each curve of shading 1 mark for where the two regions ~~simultaneously~~ would ~~overlap~~ (minus 0.5 if line of each graph is not broken)
(g)	$F(x)=x^2+5$ $F(x+2)+F(2-x)$ $=(x+2)^2+5+(2-x)^2+5$ ✓ $=x^2+\cancel{4x}+4+5+4\cancel{-4x}+x^2+5$ $=2x^2+18$ ✓	3	−0.5 if students solve for x.

Qn	Solutions	Marks	Comments: Criteria
4 (a)	 $\sin 65° = \frac{6}{r}$ ✓ ∴ $r\sin 65° = 6$ $r=\frac{6}{\sin 65°}$ ✓ $=6.620267\ldots$ ✓ $=6.6$ (to 1 d.p.) ✓	3	1 mark for halving the Δ to find ∠65° and side 6 m since Δ given is an isosceles Δ.
(b) (i)		1	
(ii)	Let ∠QPS = θ $\tan\theta=\frac{7}{3}$ ✓ ∴ $\theta = 66°48'5.07''$ ✓ $\theta = 67°$ (nearest degree) The bearing of Q from P is $180°-67°=113°T$ ✓	2	or find α $\tan\alpha=\frac{3}{7}$ ∴ $\alpha = 23°$ ∴ bearing $= 90°+23° = 113°$

Qn	Solutions	Marks	Comments: Criteria
5 (a) (i)	$2^{3x} = 64$		
	$2^{3x} = 2^6$ ✓	2	
	$\therefore 3x = 6$ ✓		
	$x = 2$ ✓		
(ii)	By definition, ✓		
	if $\log_3 y = 3$, then $y = 3^3$ ✓	2	
	$= 27$ ✓		
(b) (i)	$[\log_6 9 + \log_6 8] - \log_6 2$		
	$= \log_6 9 \times 8 - \log_6 2$		
	$= \log_6 72 - \log_6 2$		
	$= \log_6 \frac{72}{2}$ ✓		
	$= \log_6 36$	2	
	$= \log_6 6^2$ ✓		
	$= 2 \log_6 6$ ✓		
	$= 2 \times 1$		
	$= 2$ ✓		
(ii)	$\log_4 10$		
	Change the base to 10	2	
	$\frac{\log_{10} 10}{\log_{10} 4}$ ✓ $= \frac{1}{\log_{10} 4} = 1.660964\ldots$ ✓		
	$= 1.66$ (to 2 d.p.) ✓		

Qn	Solutions	Marks	Comments: Criteria
5 (c)	$3^k = 50$		
	$\log 3^k = \log 50$ ✓		
	$k \log 3 = \log 50$ ✓		
	$\therefore k = \frac{\log 50}{\log 3}$ ✓	2	
	$= 3.560876\ldots$		
	$= 3.56$ (to 3 sig. fig.) ✓		
(d)	$\frac{\sqrt{2}}{3\sqrt{2} - 1}$		
	$= \frac{\sqrt{2}}{3\sqrt{2} - 1} \times \frac{3\sqrt{2} + 1}{3\sqrt{2} + 1}$ ✓		
	$= \frac{6 + \sqrt{2}}{(3\sqrt{2})^2 - 1^2}$ ✓	3	
	$= \frac{6 + \sqrt{2}}{18 - 1}$		
	$= \frac{6 + \sqrt{2}}{17}$ ✓		
	$= \frac{6}{17} + \frac{\sqrt{2}}{17}$ ✓		
	$\therefore a = \frac{6}{17}$ and $b = \frac{1}{17}$		

5

Qn	Solutions	Marks	Comments: Criteria
e)	$g(x) = \frac{x}{x^2-1}$		
	$g\left(\frac{1}{x}\right) = \frac{\frac{1}{x}}{\left(\frac{1}{x}\right)^2 - 1}$		
	$= \frac{\frac{1}{x}}{\frac{1}{x^2} - 1}$ ✓		
	$= \frac{\frac{1}{x}}{\frac{1-x^2}{x^2}}$ ✓	2	
	$= \frac{1}{x} \div \frac{1-x^2}{x^2}$		
	$= \frac{1}{\cancel{x}} \times \frac{\cancel{x^2}}{1-x^2}$ ✓		
	$= \frac{x}{1-x^2}$ ✓ or $\frac{x}{(1-x)(1+x)}$		
g)	(i) For any odd function, $F(-a) = -F(a)$ *	1	For part (i) • if students are giving an example of a correct odd function (0.5/1)
	$\therefore F(a) + F(-a)$		
	$= F(a) + [-F(a)]$ from *		
	$= 0$		
	(ii) If $F(x)$ is defined at $x = 0$ then $F(0) = 0$ since zero is unsigned i.e. not +ve or −ve	1	or • stating the condition of an odd function
	(iii) This means that $F(x)$ has point symmetry at $(0,0)$	1	