St. Catherine's School
Waverley

2006

PRELIMINARY ASSESSMENT TASK 2
ASSESSMENT TASK

Extension Mathematics

General Instructions

- Working time – 55 minutes
- Write using black or blue pen
- Board-approved calculators may be used.
- All necessary working should be shown in every question

Total marks – 42

- Attempt Questions 1–3
- All questions are of equal value

Question 3 – 14 Marks - Start a new page

a) If $2x^2 - 5x + 1 = 0$ has roots α and β find: 6M

i. $\frac{1}{\alpha} + \frac{1}{\beta} =$

ii. $\alpha^2 + \beta^2 =$

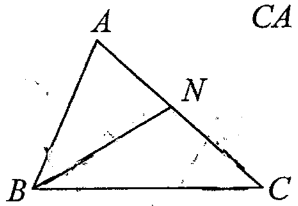
iii. $\alpha^3 + \beta^3 =$

b) The acute angle between the line $x - 2y + 3 = 0$ and the line $y = mx$ is 45°.

i. Show that $\left|\frac{2m-1}{m+2}\right| = 1$. 2M

ii. Find the possible values of m. 2M

c) ABC is a triangle and N is a point on AC. $\angle ABN = \angle CBN = \angle BCN$. $BC = 2a$, $CA = b$, $AB = c$. $BN = CN = d$.



i. Given that $\Delta ABN ||| \Delta ACB$, show that $c^2 = b^2 - 2ac$. 2M

ii. Hence show that $(a + c)^2 = a^2 + b^2$. 2M

END OF PAPER

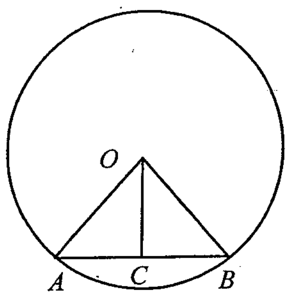
Question 1 – 14 Marks

a) Find the acute angle between the lines $2x - y = 0$ and $x + 3y = 0$, giving the answer correct to the nearest minute. 3M

b) Find the coordinates of the focus *and* the equation of the directrix of the parabola $y = x^2 + 1$.

Hint: First show $x^2 = \frac{1}{4} \times 4(y-1)$. 3M

c) O is the centre of the circle and $OC \perp AB$. Prove that $\Delta OAC \equiv \Delta OBC$. 4M



d) The circle C with equation $x^2 + y^2 - 4x + 6y - 12 = 0$ has centre $(2, -3)$ and radius 5 units.

i. Find, in terms of the constant k, the length of the perpendicular from the centre of C to the line L whose equation is $3x + 4y = k$. 2M

ii. Hence find the values of k for which L is a tangent to C. 2M

Question 2 – 14 Marks - Start a new page

a) Express $\dfrac{2x+1}{x^2+x-2}$ in the form $\dfrac{A}{x+2} + \dfrac{B}{x-1}$. 3M

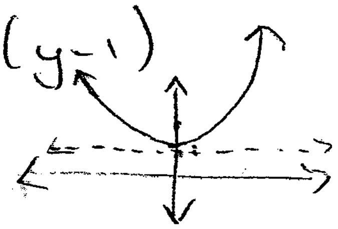
b) A is the point $(-4, 1)$ and B is the point $(2, 4)$. Q is the point which divides AB internally in the ratio $2:1$ and R is the point which divides AB externally in the ratio $2:1$. $P(x, y)$ is a variable point which moves so that $PA = 2PB$.

i. Show the co-ordinates of Q *and* R are (0, 3) and (8, 7) respectively. 4M

ii. Find the midpoint *and* length of QR. 3M

iii. Show that the locus of P is a circle with QR as diameter. 4M

Qn	Solutions	Marks	Comments+Criteria
1a	$\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$		
	$y = 2x \Rightarrow m_1 = 2$	1/2	
	$y = -\frac{x}{3} \Rightarrow m_2 = -\frac{1}{3}$	1/2	
	$\tan\theta = \left\lvert \frac{2 + \frac{1}{3}}{1 - 2/3} \right\rvert$	1	
	$= 7/3 \times 3$		1 for $\tan\theta = 1$, $\theta = 45$
	$= 7$		
	$\therefore \theta = 81°52'$	1	$-\frac{1}{2}$ for FOE
b	$y = x^2 + 1$		
	$x^2 = y - 1$		
	$\therefore x^2 = 1/4 \times 4(y-1)$	1/2	
	$\therefore a = 1/4$	1/2	
	Focus $= (0, \frac{5}{4})$	1	need $a = \frac{1}{4}$ for 1
	Directrix : $y = 3/4$	1	
c	Data: $OA = OB$ (equal radii)	1	
	OC is common	1	Has to follow proof format.
	$\angle OCA = \angle OCB = 90°$ ($OC \perp AB$)	1	
	$\therefore \triangle OAC \equiv \triangle OBC$ (RHS test)	1	
di	Centre $(2,-3)$, radius $= 5$		
	$d = \frac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2+b^2}}$ $3x+4y-k=0$		
	$= \frac{\lvert 3\times 2 + 4\times -3 - k \rvert}{\sqrt{9+16}}$	1	
	$\therefore d = \frac{\lvert -6-k \rvert}{5} = \frac{\lvert 6+k \rvert}{5}$ units	1	$-\frac{1}{2}$ for $d = \frac{\lvert 6-12-k \rvert}{5}$ $-\frac{1}{2}$ for $d = \frac{-6-k}{5}$
ii	$\frac{\lvert k+6 \rvert}{5} = 5$		
	$\lvert k+6 \rvert = 25$	1	
	$k+6 = 25$ OR $-k-6 = 25$		
	$\therefore k = 19$ OR $k = -31$	1	



Qn	Solutions	Marks	Comments+Criteria
2a	$\dfrac{2x+1}{x^2+x-2} \equiv \dfrac{A}{x+2} + \dfrac{B}{x-1}$		
	$2x+1 \equiv A(x-1) + B(x+2)$		
	c: $1 = -A + 2B$ (1)		
	x: $2 = A + B$ (2)		
	(1)+(2) $3 = 3B$		
	$B = 1$	$1\frac{1}{2}$	For solving simultaneously
	Subst $B=1$ into (2)		
	$2 = A + 1$		
	$A = 1$ $\therefore \dfrac{2x+1}{x^2+x-2} = \dfrac{1}{x+2} + \dfrac{1}{x-1}$	$1/2$	For stating the equivalent sum
bi	$x = \dfrac{mx_2 + nx_1}{m+n}$ $y = \dfrac{my_2 + ny_1}{m+n}$		
	where $m:n$ $A(-4,1)$, $B(2,4)$		
	$2:1$ $Q = \left(\dfrac{2\times 2 + 1\times -4}{2+1}, \dfrac{2\times 4 + 1\times 1}{2+1}\right)$		
	$= (0,3)$	1	
	$-2:1$ $R = \left(\dfrac{-2\times 2 - 4}{-2+1}, \dfrac{-2\times 4 + 1}{-1}\right)$		
	$= \left(\dfrac{-8}{-1}, \dfrac{-7}{-1}\right)$		
	$= (8,7)$	1	
ii	$\text{Mdpt} = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$	½	
	$= \left(\dfrac{0+8}{2}, \dfrac{3+7}{2}\right)$		
	$= (4,5)$	1	
	$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$	$1/2$	
	$= \sqrt{8^2 + (7-3)^2}$		
	$= \sqrt{64+16}$		
	$= \sqrt{80}$		
	$\therefore d = 4\sqrt{5}$ units	1	

Qn	Solutions	Marks	Comments+Criteria
(iii)	$PA^2 = 4PB^2$		
	$(x+4)^2 + (y-1)^2 = 4[(x-2)^2 + (y-4)^2]$	1	
	$x^2+8x+16+y^2-2y+1 = 4(x^2-4x+4+y^2-8y+16)$		
	$x^2+8x+y^2-2y+17 = 4x^2-16x+4y^2-32y+80$		
	$3x^2-24x+3y^2-30y+63=0$		
	$x^2-8x+y^2-10y+21=0$	1	
	$(x-4)^2+(y-5)^2 = -21+16+25$		
	$\therefore (x-4)^2+(y-5)^2 = 20$		
	$\therefore$ Centre $(4,5)$, radius $= 2\sqrt{5}$ units	1	
	$\therefore$ Diameter $= 4\sqrt{5}$ units & the centre is the midpt of QR		
	$\therefore$ Locus of P is a circle on QR as diameter.	1	
3a	$2x^2-5x+1=0$		
	$\alpha+\beta = -\frac{b}{a} = \frac{5}{2}$	$\frac{1}{2}$	
	$\alpha\beta = \frac{c}{a} = \frac{1}{2}$	$\frac{1}{2}$	
i	$\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$		
	$= \frac{5}{2}\times 2$		
	$=5$	1	
ii	$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	1	
	$= \left(\frac{5}{2}\right)^2 - 2\times\frac{1}{2}$		
	$= \frac{25}{4} - 1$		
	$= \frac{21}{4}$	1	
iii	$\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$	$\frac{1}{2}$	
	$= (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$		
	$= \frac{5}{2}\times\left(\frac{21}{4}-\frac{1}{2}\right)$	$\frac{1}{2}$	
	$= \frac{5}{2}\times\frac{19}{4}$		
	$= \frac{95}{8}$	1	

Qn	Solutions	Marks	Comments+Criteria
3bi	$\tan\theta = \left\lvert \dfrac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$		
	$x - 2y + 3 = 0$		$\frac{1}{2}$ for
	$y = \frac{x}{2} + \frac{3}{2} \Rightarrow m_1 = \frac{1}{2}$		$\tan 45 = 1$
	$y = mx \Rightarrow m_2 = m$		$\therefore 1 = \left\lvert \dfrac{2m-1}{m+2} \right\rvert$
	$\tan 45^\circ = \left\lvert \dfrac{\frac{1}{2} - m}{1 + \frac{m}{2}} \right\rvert$	1	
	$1 = \left\lvert \dfrac{1-2m}{2+m} \right\rvert$		
	$\therefore \left\lvert \dfrac{2m-1}{m+2} \right\rvert = 1$	1	
ii	$\dfrac{2m-1}{m+2} = 1$ OR $\dfrac{1-2m}{m+2} = 1$		
	$2m - 1 = m + 2$ $1 - 2m = m + 2$		
	$m = 3$ OR $-1 = 3m$		
	$\therefore m = 3$ or $m = -\frac{1}{3}$	1+1	
c.i	$\dfrac{AB}{AC} = \dfrac{AN}{AB} = \dfrac{BN}{CB}$ ($\triangle ABN \parallel\!\mid \triangle ACB$ & corresp. sides in ratio)		
	$\dfrac{c}{b} = \dfrac{b-d}{c} = \dfrac{d}{2a}$	1	
	$\therefore c^2 = b^2 - bd$ ① & $bd = 2ac$ ②		
	subst ② into ①		
	$c^2 = b^2 - 2ac$	1	
c.ii	$c^2 = b^2 - 2ac$		
	$c^2 + 2ac = b^2$	1	
	$c^2 + 2ac + a^2 = a^2 + b^2$		
	$\therefore (a+c)^2 = a^2 + b^2$	1	