St Catherine's School
Waverley, Sydney

Student: ....

Year 11
Assessment Task 2

Mathematics Extension I

Time allowed: 55 mins

Course weighting: 25%

General Instructions

- Attempt ALL questions
- Write your Student NUMBER at the top of this page and on the writing paper used

Question	Marks
1	
2	
3	
Total marks	

Question 1: (9 marks)

(a) What is the exact value of $\cos 225°$? **1**

(b) If $\sin A = \frac{12}{13}$ and $\cos A < 0$, find the exact value of $\sec A$ **2**

(c) Solve $4\sin^2 x° - 1 = 2$ for $0^0 \le x \le 360^0$. **3**

(d) Prove that $(1-\sin^2\theta)(1+\cot^2\theta) = \cot^2\theta$ **3**

Question 2 (13 marks)

(a) Let α and β be the roots of the equation $x^2 - 5x + 2 = 0$.

Find the values of;

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(b) For what values of 'k' will the equation **2**

$2x^2 - x - 1 + k = 0$

have real and different roots?

(c) Divide the interval AB defined by A(–1,2) and B(6,5) externally in the ratio of 4:5 **2**

(d) Find the value of k in the quadratic equation **2**

$(k-2)x^2 + (k+2)x + 2k + 1 = 0$

if the roots are reciprocals of one another.

(e) The acute angle between the lines $y = 3x + 5$ and $y = mx + 4$ is $45°$. **3**

Find the two possible values of m

QUESTION 3: (15 marks)

(a) The focus of a parabola is $S(3,4)$ and its directrix is the line $y=-2$.

(i) Sketch the parabola and indicate the coordinates of its vertex. **1**

(ii) Write down the focal length of the parabola **1**

(iii) Find the equation of the parabola **2**

(b) Determine (showing all your working) whether the line **4**

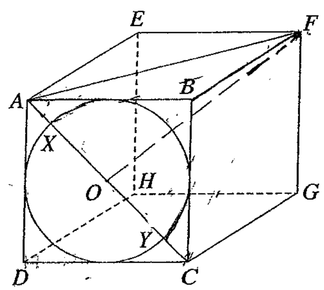
$5x - 12y + 10 = 0$ is a tangent to the circle

$x^2 + y^2 - 2x + 4y - 4 = 0$.

(c) A and B are the points (5,–4) and (–3,2) respectively. **3**
The point P(x,y) moves so that $PA^2 + PB^2 = 68$.

Determine the equation of the locus of P and describe the locus in geometrical terms.

(d) **4**

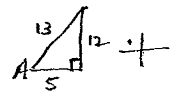
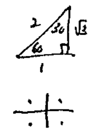


The length of each edge of the cube $ABCDEFGH$ is 2 metres. A circle is drawn on the face $ABCD$ so that it touches all four edges of the face. The centre of the circle is O and the diagonal AC meets the circle at X and Y.

(i) Explain why $\angle FAC = 60°$.

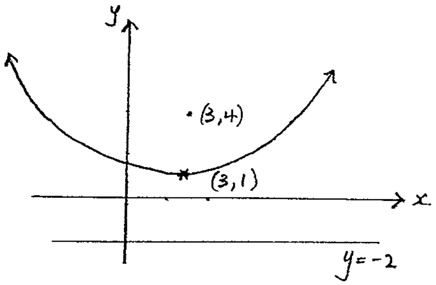
(ii) Show that $FO = \sqrt{6}$ metres.

(iii) Calculate the size of $\angle XFY$ to the nearest degree.

Qn	Solutions	Marks	Comments+Criteria
1a)	$\cos 225° = -\cos 45°$ $= -\frac{1}{\sqrt{2}}$	1	
b)	$\sin A = \frac{12}{13}$ $\therefore \sec A = \frac{-13}{5}$ 	2	
c)	$4\sin^2 x - 1 = 2$ $4\sin^2 x - 3 = 0$ $(2\sin x - \sqrt{3})(2\sin x + \sqrt{3}) = 0$ $\therefore \sin x = \pm\frac{\sqrt{3}}{2}$  $\therefore x = 60°, 120°, 240°, 300°$	2 1	
	OR $4\sin^2 x = 3$ $\sin^2 x = \frac{3}{4}$ $\sin x = \pm\frac{\sqrt{3}}{2}$ $x = 60°, 120°, 240°, 300°$	2 1	
d)	LHS $= (1-\sin^2\theta)(1+\cot^2\theta)$ $= \cos^2\theta \times \operatorname{cosec}^2\theta$ $= \cos^2\theta \times \frac{1}{\sin^2\theta}$ $= \frac{\cos^2\theta}{\sin^2\theta}$ $= \cot^2\theta$ $=$ RHS	1 1	Note: 1 mark for correct format for a proof
2a)	(i) $\alpha + \beta$ = sum of roots $= 5$	1	
	(ii) $\alpha\beta$ = product of roots $= 2$	1	
	(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ $= 25 - 4$ $= 21$	2	

Qn	Solutions	Marks	Comments+Criteria
2b)	$2x^2 - x - (1-k) = 0$		
	$a = 2 \quad b = -1 \quad c = -(1-k)$		
	for real different roots $\Delta > 0$		
	$\therefore \quad b^2 - 4ac > 0$	1	
	$1 + 4(2)(1-k) > 0$		
	$1 + 8 - 8k > 0$		
	$9 - 8k > 0$		
	$9 > 8k$		
	$\frac{9}{8} > k \quad$ or $k < \frac{9}{8}$	1	
c)	A $(-1,2)$ $\quad (-4:5 \quad$ B $(6,5)$	1	
	$= \left(\frac{-5-24}{1}, \frac{10-20}{1}\right)$		
	$= (-29, -10)$	1	
d)	$(k-2)x^2 + (k+2)x + (2k+1) = 0$		
	if roots are reciprocals product $= 1$	1	
	$\therefore \quad \frac{2k+1}{k-2} = 1$		
	$\therefore \quad 2k+1 = k-2$		
	$\therefore \quad k = -3$	1	
e)	$m_1 = 3 \quad m_2 = m$	1	
	$\tan\theta = \left\lvert\frac{m_1 - m_2}{1 + m_1 m_2}\right\rvert \qquad \tan 45^\circ = 1$		
	$\therefore \quad \left\lvert\frac{3-m}{1+3m}\right\rvert = 1$		
	$\frac{3-m}{1+3m} = \pm 1$	1	
	$3 - m = 1 + 3m \quad$ or $\quad 3 - m = -1 - 3m$		
	$2 = 4m \qquad 2m = -4$		
	$\frac{1}{2} = m \qquad m = -2$	1	

Qn	Solutions	Marks	Comments+Criteria
3c)	$A(5,-4) \qquad B(-3,2) \qquad P(x,y)$		
	$PA^2 = (x-5)^2 + (y+4)^2 \quad PB^2 = (x+3)^2 + (y-2)^2$		
	$\therefore (x-5)^2 + (y+4)^2 + (x+3)^2 + (y-2)^2 = 68$		
	$\therefore x^2 - 10x + 25 + y^2 + 8y + 16 + x^2 + 6x + 9 + y^2 - 4y + 4 = 68$	1	
	$\therefore 2x^2 + 2y^2 - 4x + 4y = 14$		
	$\therefore x^2 - 2x + 1 + y^2 + 2y + 1 = 7 + 1 + 1$		
	$(x-1)^2 + (y+1)^2 = 9$	1	
	locus is a circle centre $(1,-1)$ radius 3	1	
d)	(i) $AC = AF = FC$ (equal diagonals of a cube)		
	$\therefore \Delta AFC$ is equilateral		
	$\therefore \angle FAC = 60^\circ$	1	
	(ii) $AD = DC = 2$ (given - sides of cube)		
	$\therefore AC = 2\sqrt{2}$ (by pythagoras)		
	$\therefore AO = \sqrt{2}$ (O midpoint of AC)		
	Now $AF = AC = 2\sqrt{2}$ (equal diagonals)		
	$FO^2 = AF^2 - AO^2$ (pythagoras)	$1\frac{1}{2}$	
	$= 8 - 2$		
	$= 6$		
	$\therefore FO = \sqrt{6}$		
	(iii) Diameter of circle $= 2$ (given)		
	$\therefore XO = 1$		
	$FO = \sqrt{6}$ (part i)		
	$\therefore \tan\angle XFO = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$	$1\frac{1}{2}$	
	$\therefore \angle XFO = \tan^{-1}\frac{\sqrt{6}}{6}$		
	$\therefore \angle XFY = 2\tan^{-1}\frac{\sqrt{6}}{6}$		
	$= 44^\circ$ (to nearest degree)		

Qn	Solutions	Marks	Comments+Criteria
3a)	(i) 	1	
	(ii) focal length: $a = 3$	1	
	(iii) $(x-3)^2 = 12(y-1)$ $x^2-6x+9 = 12y-12$ $x^2-6x-12y+21=0$	2	
b)	$5x-12y+10=0$ $x^2+y^2-2x+4y-4=0$ $x^2-2x+1+y^2+4y+4 = 9$ $(x-1)^2+(y+2)^2 = 9$ Centre $(1,-2)$ radius $= 3$.	2	
	perpendicular distance from $5x-12y+10=0$ to the centre $(1,-2)$ $d = \left\lvert \frac{5+24+10}{\sqrt{25+144}} \right\rvert$ $= \left\lvert \frac{39}{13} \right\rvert$ $= 3$	1	
	this distance equals the radius $\therefore$ $5x-12y+10=0$ is a tangent.	1	