St. Catherine's School

Year 11 Extension 1 Mathematics
Preliminary Task #2
May 2008

Time allowed: 45 minutes

INSTRUCTIONS

- There are 4 questions of different values
- Marks for each part of a question are indicated
- All questions should be attempted
- All necessary working should be shown
- Start each question on a new page
- Approved scientific calculators and drawing templates may be used.

Question 1 (10 marks)

a) Find the acute angle between the straight lines
$x+y=5$ *and* $2x-3y+5=0$ 3

b) Find the acute angle between the straight lines
$y=-1$ *and* $2x-y+1=0$ 2

c) Find the acute angle between the straight lines
$x=5$ *and* $y=2x+1$ 2

d) What are the two possible values of the gradients of lines that make an angle of 45^0 with the straight line $2x-3y+4=0$ 3

Question 2 (6 marks) – Start a new page

a) Find the coordinates of the point P that divides the interval between A(3,7) and B(-1,-4) internally in the ratio 5:2. 2

b) Find the coordinates of the point R which divides the interval PQ, where P is (1,8) and Q is (-2,-4) externally in the ratio 3:1. 2

c) A point P(1,4) divides the interval AB internally in the ratio 3:4. The point A is (4,-8). Find the coordinates of the point B. 2

PTO

Question 3 (9 marks) – Start a new page

Sketch the following graphs highlighting the main features including the **limits** of the functions as x tends to infinity and negative infinity and **the x and y intercepts** where relevant. State clearly the **domain** and **range**.

a) $y=\dfrac{1}{(x-2)(x+3)}$ 4

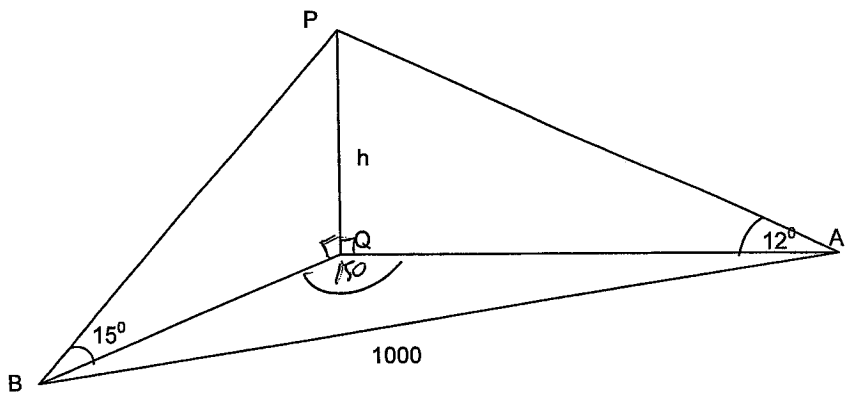
b) $f(x) = \dfrac{x-1}{x^2-9}$ 5

PTO

Question 4 (5 marks) – Start a new page

The angle of elevation of a tower PQ of height *h* from a point A due east of Q is 12^0. From another point B the bearing of the tower PQ is 060^0 and the angle of elevation is 15^0. If the distance AB is 1000 metres and AB is on the same level as Q.

5



a) Show that $\angle AQB=150^0$

b) Write AQ and BQ in terms of h.

c) Using Cosine rule on triangle AQB, find the height of the tower PQ to the nearest metre.

END OF TASK

Year 11 Ext. I Task 2. May 2008.

Question 1

a) $x+y=5$ $\quad 2x-3y+5=0$

$m_1=-1$ (½) $\quad m_2=\frac{2}{3}$ (½)

if θ is the acute angle between the lines

$\tan\theta = \left|\frac{\frac{2}{3}-(-1)}{1+\frac{2}{3}\times(-1)}\right|$ 1

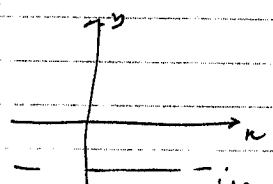
$= \left|\frac{\frac{5}{3}}{\frac{1}{3}}\right|$

$= 5$

$\theta = \tan^{-1}5$ 1

$= 78°41'$ (to near min)

b) $y=-1$ $\quad 2x-y+1=0$



$y=-1$ is a horizontal line

Gradient of $2x-y+1=0$ is 2

∴ The required angle is $\tan^{-1}2$ =

OR

$m_1=2$; $\quad m_2=0$.

$\tan\theta = \left|\frac{2-0}{1+0}\right| = 2$

$\theta = \tan^{-1}2$.

$= 63°26'$

Question 2

a) A: (3, 7) $\quad$ B: (−1, −4)

5 : 2

P: $\left(\frac{6-5}{7}, \frac{14-20}{7}\right)$

: $\left(\frac{1}{7}, \frac{-6}{7}\right)$.

b) P: (1, 8) $\quad$ Q: (−2, −4)

−3 : 1

R: $\left(\frac{6+1}{-2}, \frac{12+8}{-2}\right)$

: $\left(-\frac{7}{2}, -10\right)$

c) A: (4, −8) $\quad$ B: (x, y)

3 : 4

D (1, 4)

$1 = \frac{16+3x}{7}$; $\quad 4 = \frac{3y-32}{7}$.

$16+3x = 7$ $\quad 3y-32 = 28$

$3x = -9$ $\quad 3y = 60$

$x = -3$ $\quad y = 20$.

B: (−3, 20)

Question 3

a)

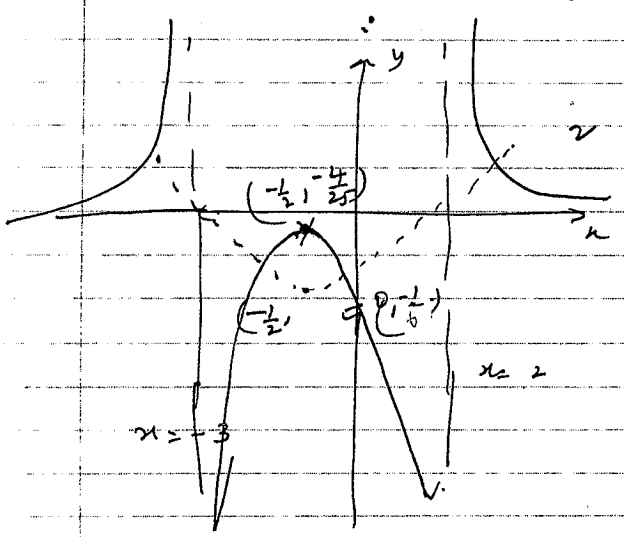
$$y = \frac{1}{(x-2)(x+3)}$$

Dom: $x \neq 2$; $x \neq -3$ ½

Vertical asymptote: $x = 2$; $x = -3$ 1

Horizontal " $y = 0$

more. as $x \to \infty$; $y \to 0$ 1



2

$x = -\frac{1}{2}$;

$y = \frac{1}{-\frac{5}{2} \times \frac{1}{2}}$

$= -\frac{4}{25}$

$x = 0$; $y = -\frac{1}{6}$

Range. $y > 0$; $y < -\frac{4}{25}$. ½

b)

$$y = \frac{x-1}{x^2-9}$$

$$= \frac{x-1}{(x-3)(x+3)}$$

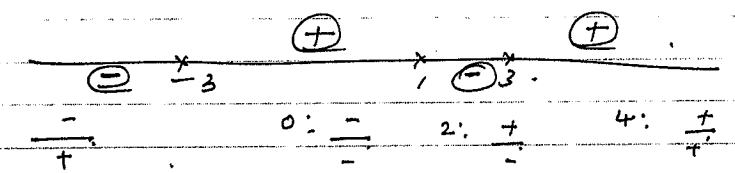
$x = 1$; $y = 0$

Dom: $x \neq 3$; $x \neq -3$. ½

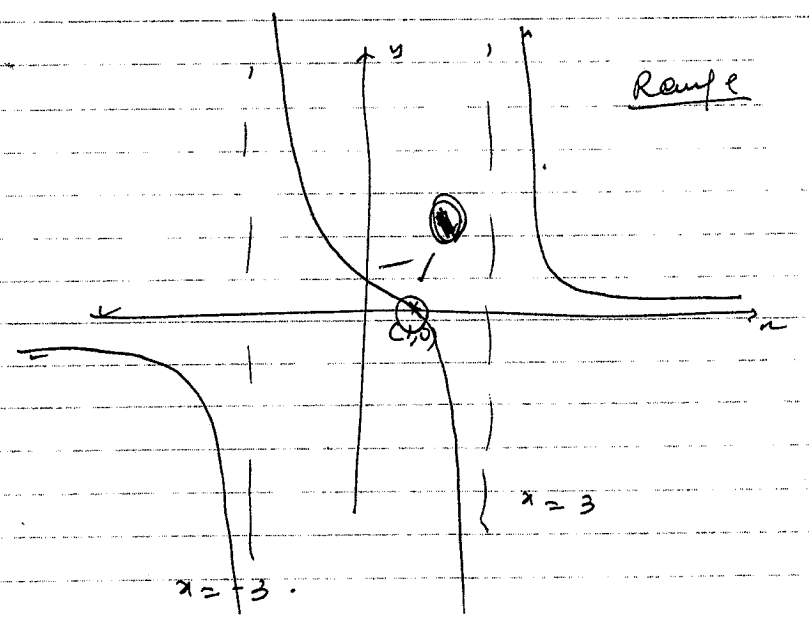
Vertical asymptote $x = -3$; $x = 3$. 1 } in for all 5

As $x \to \infty$; $y \to 0$. 1

Sign of y:

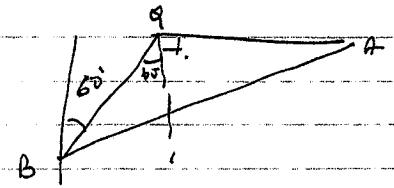


1 in graph if found



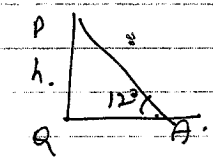
Range all y. ½

Question 4



$\angle BQA = 60° + 90°$

$= 150°$. |



$\frac{PQ}{AQ} = \tan 12$

$\therefore AQ = h \cos 12°$ ½

Similarly BQ is $h \cos 15°$ ½

c) In $\triangle AQB$

$1000^2 = h^2 \cos^2 12 + h^2 \cos^2 15 - 2h \cos 12 h \cos 15 \cos 150°$ |

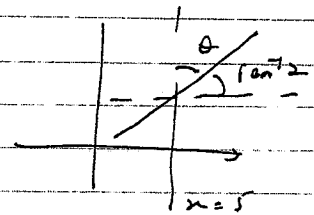
$1000^2 = h^2 (\cos^2 12 + \cos^2 15 - 2 \cos 12° \cos 15° \cos 150°)$ |

$\therefore h = \frac{1000}{\sqrt{\cos^2 12 + \cos^2 15 - 2 \cos 12 \cos 15 \cos 150°}}$ |

$= 123$ m (to near m)

Question 1

c)



$x = 5$ is a vertical line with grad. ∞.

grad. of $y = 2x + 1$ is 2

$\theta = 90° - \tan^{-1} 2$

$= 26°34'$

d) Let m be the gradient of any line that make $45°$ to $2x - 3y + 4 = 0$.

grad. $2x - 3y + 4 = 0$ is $\frac{2}{3}$.

$\tan 45° = \left| \frac{\frac{2}{3} - m}{1 + \frac{2m}{3}} \right|$ ½

$\therefore 1 = \left| \frac{2 - 3m}{3 + 2m} \right|$ #

$\frac{2-3m}{3+2m} = 1$

$2 - 3m = 3 + 2m$

$-1 = 5m$

$m = -\frac{1}{5}$

$\frac{2-3m}{3+2m} = -1$

$2 - 3m = -3 - 2m$

$5 = m$

1½