Student Name: ____________

St. Catherine's School

Year 11 Extension Mathematics
Preliminary Task #2
22- June -2010

Time allowed: *55 minutes*

Total marks: *35 marks*

Weighting: *25%*

INSTRUCTIONS

- There are 5 questions of different values.
- Marks for each part of a question are indicated.
- All necessary working should be shown.
- Start each question on a new page.
- Approved scientific calculators and drawing templates may be used.
- Marks may be deducted for careless or badly arranged work.

Question 1

Differentiate the following functions

a) $y = 4x^3 + 3x^2 - 5x + 6$ 2m

b) $y = \frac{x}{5}$ 1m

c) $y = (x-2)^4(2x+1)$ 2m

d) $y = \frac{5x-1}{3x+1}$ 2m

e) $y = \frac{1}{\sqrt{2x+3}}$ 2m

f) $y = \frac{5x^3 + x + 3}{x}$ 2m

Question 2

a) Find the equation of the tangent to the curve $y = x^3 - 3x + 1$ at the point where $x = 2$ 3m

b) The line $y = mx + b$ is a tangent to the curve $y = 2x^3 + 3x^2 + 5$ at the point (1,10)..

Find the value of *m* and *b*. 3m

Question 3.

Differentiate from first principles: $f(x)=\frac{1}{x}$ 3m

Question 4.

Consider the curve $y=x^4-2x^3$

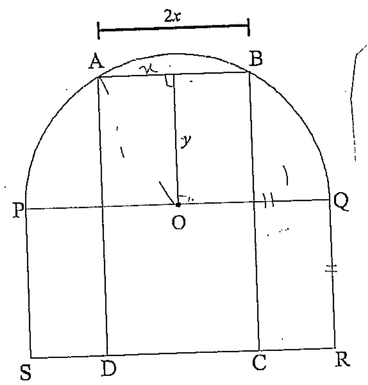
(i) Find the **stationary points** and determine their **nature** and also find any **points of inflexion.** 6m

(ii) Sketch the curve showing the above features 2m

(iii) For what values of x is the curve concave down? 1m

Question 5.

A stained glass window is in the shape of a semi circle on a rectangle PQRS.
The semi circle has a radius of 1 **unit** and OQ =QR.= 1unit, where O is the centre of the semi circle.
It requires metal strips to support along AB, AD and BC.



(i) Copy the diagram and information onto your answer page

(ii) The length of the strip AB is 2x unit and it is at a distance of y units from the centre of the semi circle.
Find an expression of y in terms of x. 1m

(iii) Show that the total length of the metal strips is given by
$L=2+2x+2\sqrt{1-x^2}$ 1m

(iv) The window will have maximum strength when the total length L of the supports is a maximum. Find the exact value of x that will allow the window to have maximum strength. (Assume that $\frac{d^2L}{dx^2}<0$ for all possible values of x) 4m

END OF PAPER

Qn	Solutions	Marks	Comments: Criteria
1.	$y = 4x^3 + 3x^2 - 5x + 6$ $y' = 12x^2 + 6x - 5$	2	
b)	$y = \frac{x}{5}$ $y' = \frac{1}{5}$	1	
c)	$y = (x-2)^4(2x+1)$ $y' = (x-2)^4(2) + (2x+1)\,4(x-2)^3$ $= 2(x-2)^3(x-2+4x+2)$ $= 2(x-2)^3(5x)$ $= 10x(x-2)^3$	2	
d)	$y = \frac{5x-1}{3x+1}$ $y' = \frac{(3x+1)(5) - (5x-1)(3)}{(3x+1)^2}$ $= \frac{8}{(3x+1)^2}$	2	
e)	$y = (2x+3)^{-\frac{1}{2}}$ $y' = -\frac{1}{2}(2x+3)^{-\frac{3}{2}}(2)$ $= \frac{-1}{\sqrt{(2x+3)^3}}$	2	
f)	$y = 5x^2 + 1 + \frac{3}{x}$ $\left(\frac{3}{x} = 3x^{-1}\right)$ $y' = 10x - \frac{3}{x^2}$	2	

Qn	Solutions	Marks	Comments: Criteria
2.	$y = x^3 - 3x + 1$ $y' = 3x^2 - 3$	$\frac{1}{2}$	
	y' at $x = 2$ $= 3 \times 2^2 - 3$ $= 9$	$\frac{1}{2}$	
	at $x = 2$; $y = 2^3 - 3 \times 2 + 1$ $= 3$	$\frac{1}{2}$	
	Eqn. of the tangent is $y - 3 = 9(x-2)$ $y = 9x - 15$	1 $\frac{1}{2}$	
b)	$y = 2x^3 + 3x^2 + 5$ $y' = 6x^2 + 6x$	$\frac{1}{2}$	
	y' at $x = 1$ $= 6 \times 1^2 + 6 \times 1$ $= 12$	$\frac{1}{2}$ 1	
	$\therefore \underline{m = 12}$ $(1, 10)$ lies on $y = mx + b$ $10 = 12 \times 1 + b$ $\therefore \underline{b = -2}$	1	

Qn	Solutions	Marks	Comments: Criteria

3.

$$f(x) = \frac{1}{x}$$

$$f(x+h) = \frac{1}{x+h}$$

$$f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x}$$

$$= \frac{x - (x+h)}{x(x+h)}$$

$$= \frac{-h}{x(x+h)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{-1}{x(x+h)} \qquad (\tfrac{1}{2})$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad (\tfrac{1}{2} m)$$

$$= \lim_{h \to 0} -\frac{1}{x(x+h)} \qquad (\tfrac{1}{2} m)$$

$$= -\frac{1}{x^2}. \qquad (\tfrac{1}{2} m)$$

Qn	Solutions	Marks	Comments: Criteria

4.

$$y = x^4 - 2x^3$$

$$y' = 4x^3 - 6x^2 = 2x^2(2x-3)$$

$$y'' = 12x^2 - 12x = 12x(x-1)$$

At stationary pts ; $y' = 0$

$$2x^2(2x-3) = 0$$

$$x = 0 \quad ; \quad x = 3/2 \qquad (1M)$$

$$y = 0 \quad ; \quad y = \left(\tfrac{3}{2}\right)^4 - 2\left(\tfrac{3}{2}\right)^3$$

$$= -\frac{27}{16} \qquad (1M)$$

<u>Nature</u> $(0,0)$ and $\left(\frac{3}{2}, -\frac{27}{16}\right)$ are stat. pts.

at $x = 0$

$y'' = 0$; $\quad y'' = 12x^2 - 12x$

x	-0.1	0	0.1
y''	>0	0	<0

There is change in concavity

$\therefore$ <u>$(0,0)$</u> is a <u>horizontal pt. of inflexion</u>. (2M)

$y''_{\text{at } x = 3/2} \quad 12\left(\frac{3}{2}\right)\left(\frac{3}{2} - 1\right) > 0.$

$\therefore$ <u>$\left(\frac{3}{2}, -\frac{27}{16}\right)$</u>. is a <u>minimum turning pt</u> (1M)

Comments: Criteria:

Establishing $(0,0)$ is a horiz. pt of inflexion (in part (i) or part (ii)) is <u>2 marks</u>

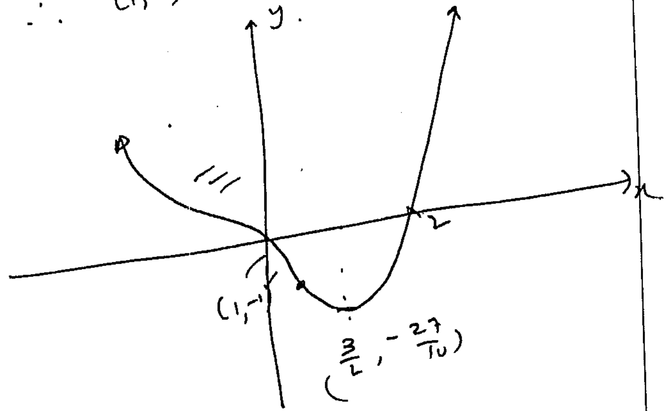
$\left(\frac{3}{2}, -\frac{27}{16}\right)$; min t. pt. is <u>1 m</u>

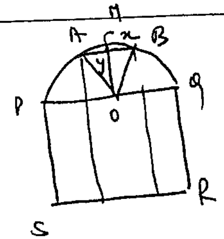
$\left(\frac{1}{2}, -1\right)$ is a pt of inflexion <u>1 mark</u>

Qn	Solutions	Marks	Comments: Criteria
(ii)	At pts of inflexion $y''=0$		
	$12x(x-1)=0$		
	$x=0$; $x=1$		
	$y=0$; $y=1^4-2(1)^3$ $=-1$		
	$(0,0)$ is a H. pt. of inflexion		

$12x(x-1)$

x	0.9	+1	+1.1
y''	<0	0	>0

Qn	Solutions	Marks	Comments: Criteria
	There is a change in concavity	(1M)	
	$\therefore$ $(1,1)$ is a pt. of inflexion		
			1 mark for the general sketch 1 mark for the correct concavity around $x=0$, $x=1$
	Concave down ; $y''<0$		
	$12x(x-1)<0$		
	$0<x<1$.	(1m)	

Qn	Solutions	Marks	Comments: Criteria
			
	In ΔOAB ; $OA=OB$		
	$OM \perp AB$; $AM=MB=x$		
	$OB^2=OM^2+MB^2$		
	$1^2=y^2+x^2$		
	$\therefore y=\sqrt{1-x^2}$	(1m)	
	$L=AB+BC+AD.$		
	$=2x+(y+1)+(y+1)$		
	$=2x+2+2y$		
	$=2x+2+2\sqrt{1-x^2}.$	(1m)	
(iv)	Max/min L ; $\frac{dL}{dx}=0$		$\frac{dL}{dx}=2-\frac{2x}{\sqrt{1-x^2}}$ (2m)
	$2+2\cdot\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)=0$		
	$2-\frac{2x}{\sqrt{1-x^2}}=0.$		
	$2x=2\sqrt{1-x^2}$		
	$x^2=1-x^2$		
	$2x^2=1$		
	$x=\frac{1}{\sqrt{2}}$ $(x>0)$	($\frac{1}{2}$m)	2M
	$\frac{d^2L}{dx^2}<0$ (given)		
	$\therefore L$ is maximum, when $x=\frac{1}{\sqrt{2}}$	($\frac{1}{2}$m)	