Student Number:_

St. Catherine's School
Waverley

2008
ASSESSMENT TASK 3
(20%)

Mathematics Year 11

General Instructions

- Working time: 55 minutes
- Attempt questions: 1–3
- Start each question on a new page in your answer booklet.
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work

Total marks – 44

QUESTION 1 (13 marks)

P(2,-1), Q(-2,-2) and R(0, 3) are the vertices of a triangle PQR.

(i) Find the gradient of PR /1

(ii) Find the length of PR. Leave your answer as a surd. /1

(iii) Find the equation of PR. Give your answer in general form. /2

(iv) Find the perpendicular distance from Q to the line PR. Leave your answer in exact form. /2

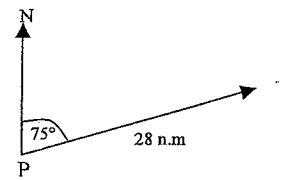
(v) Find the area of ΔPQR /2

(vi) Find the equation of the median QF (the line joining Q and F, the midpoint of PR) /3

(vii) Find the co-ordinates of D if PQRD is a parallelogram /2

QUESTION 2 (20 marks) START A NEW PAGE

(a) A ship sails from port P on a course of 075° for 28 nautical miles. It then changes its course to 130° and continues sailing for 32 nautical miles.



(i) Copy and complete the diagram into your writing booklet for the data above /1

(ii) How far is the ship from its starting point? Answer to the nearest nautical mile. /2

(iii) What is the ship's bearing from its starting point? Answer to the nearest degree. /3

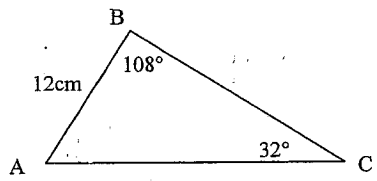
QUESTION 2 (continued)

(b) Find the exact value of

(i) $\sin 60° + \cos 30°$ /2

(ii) $\sec 60°.\ \sin 240°.\ \cos(-60°)$ /2

(c)



(i) Calculate the length of BC. Give your answer correct to 1 decimal place. /2

(ii) Calculate the area of ΔABC, giving your answer correct to two significant figures /1

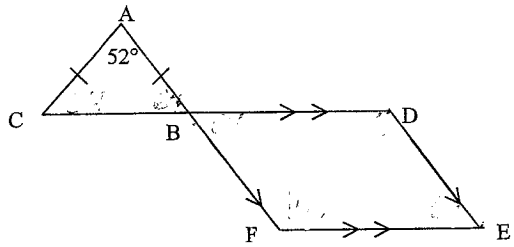
(d) Solve for θ: $2\cos\theta + 1 = 0$ *for* $0° \le \theta \le 360°$ /2

(e) Simplify $\sin^2\theta\cos^2\theta + \sin^4\theta$ /2

(f) Prove that $\dfrac{1}{\operatorname{cosec}\theta - 1} - \dfrac{1}{\operatorname{cosec}\theta + 1} = 2\tan^2\theta$ /3

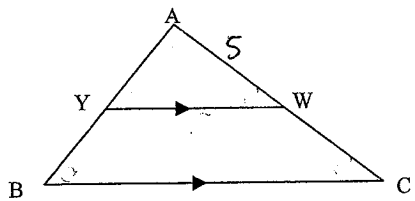
QUESTION 3 (11 marks) START A NEW PAGE

(a)



Find the size of ∠BDE giving reasons /3

(b)

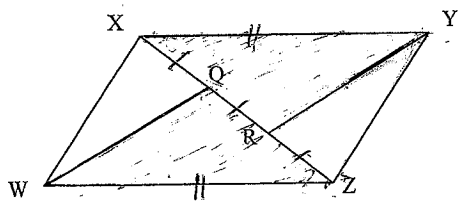


In ΔABC, YW = 8cm, BC= 12cm, AW = 5cm and YW || BC.

(i) Prove that ΔABC is similar to ΔAYW /2

(ii) Find the length of AC /2

(c)



XYZW is a parallelogram. XZ is trisected at Q and R.

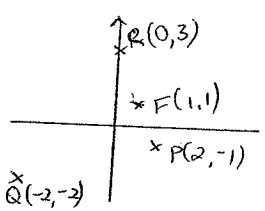
(i) Prove ΔWQZ ≡ ΔYRX /3

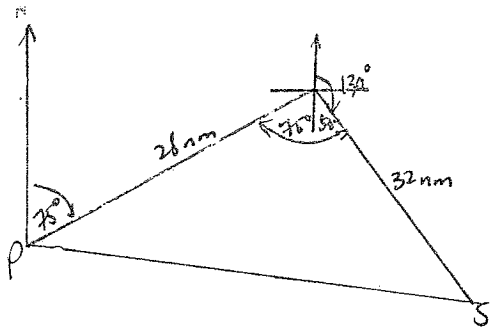
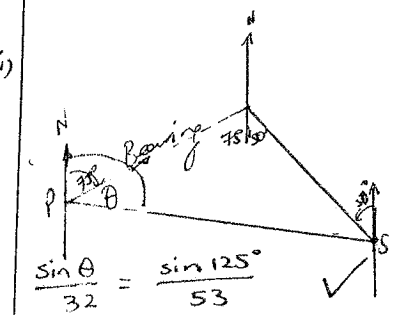
(ii) Hence, or otherwise prove that WQ=YR /1

END OF TEST

ADV 11 MATHEMATICS TASK 3 2008

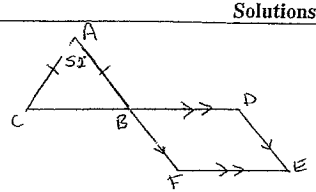
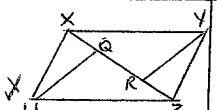
Qn	Solutions	Marks	Comments+Criteria
1(i)	$m_{PR} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3 - -1}{0 - 2}$ $= \frac{4}{-2}$ $= -2$	1	
(ii)	$d_{PR} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(0-2)^2 + (3--1)^2}$ $= \sqrt{4 + 16}$ $= \sqrt{20}$ $= 2\sqrt{5}$	1	
(iii)	$m_{PR} = -2$, $P(2,-1)$ $y - y_1 = m(x - x_1)$ $y - -1 = -2(x-2)$ $y + 1 = -2x + 4$ $2x + y - 3 = 0$	½ 2	
(iv)	$2x + y - 3 = 0$, $Q(-2,-2)$ $d = \left\lvert \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right\rvert$ $= \left\lvert \frac{2x-2 + 1x-2 + -3}{\sqrt{2^2 + 1^2}} \right\rvert$ $= \left\lvert \frac{-4-2-3}{\sqrt{5}} \right\rvert$ $= \left\lvert \frac{-9}{\sqrt{5}} \right\rvert$ $= \frac{9}{\sqrt{5}}$ $= \frac{9\sqrt{5}}{5}$	½ 1 2	
(v)	Area $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 2\sqrt{5} \times \frac{9\sqrt{5}}{5}$ $= \frac{1}{2} \times 18 \times \frac{5}{5}$ $= 9$ sq units	½ 2	

Qn	Solutions	Marks	Comments+Criteria
1(vi)	midpt$_{PR} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $= \left(\frac{2+0}{2}, \frac{-1+3}{2}\right)$ $= (1, 1)$ $\therefore F(1,1)$	1	
	eqn QF: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$ $Q(-2,-2)$, $F(1,1)$: $\frac{1--2}{1--2} = \frac{y--2}{x--2}$ $\frac{3}{3} = \frac{y+2}{x+2}$ $m=1$ $x + 2 = y + 2$ $\therefore y = x$ is eqn QF	1 1 3	
(vii)			
	Since PQRD is a parallelogram QF = FD $\therefore$ F is midpt of QD $(1,1) = \left(\frac{-2 + x_2}{2}, \frac{-2 + y_2}{2}\right)$ $1 = \frac{-2 + x_2}{2}$ $\quad 1 = \frac{-2 + y_2}{2}$ $2 = -2 + x_2$ $\quad 2 = -2 + y_2$ $x_2 = 4$ $\quad y_2 = 4$ $\therefore D$ is $(4,4)$	2	
	wrong order	½	

Qn	Solutions	Marks	Comments+Criteria
2(a) (i)		1	
(ii)	$d^2 = 28^2 + 32^2 - 2\times28\times32\cos 125°$ — line 1 $d^2 = 2835.8489\ldots$ — line 2 $d = 53.2526\ldots$ — line 3 $= 53$ (nearest n.m.)	2	1 mark for line 1 ½ mark for line 2 ½ mark for line 3
(iii)	 $\frac{\sin\theta}{32} = \frac{\sin 125°}{53}$ ✓		
(b)	$53\sin\theta = 32\sin 125°$ $\therefore \sin\theta = \frac{32\sin 125°}{53}$ ✓ $\theta = 29°\,38'\,31.97''$ ✓ $\theta = 30°$ (nearest degree)	3	
	∴ Bearing of the ship from P is ... $= 30° + 75°$ ✓ $= 105°$ T		1 mark means ✓ ½ mark means ✓'

Qn	Solutions	Marks	Comments+Criteria
2(c) (i)	$\angle BAC = 180 - 108 - 32$ $= 40$ $\frac{BC}{\sin 40} = \frac{12}{\sin 32}$ 1 $BC = \frac{12}{\sin 32} \times \sin 40$ $= 14.6$ cm (1 d.p.) 1	2	
(ii)	Area $= \frac{1}{2}ab\sin C$ $= \frac{1}{2}\times12\times14.6\sin 108$ 1 $= 83$ cm² (2 s.f.)	1	Using 40° instead of 108°. (½ M)
(d)	$2\cos\theta + 1 = 0$ $\cos\theta = -\frac{1}{2}$ (cos ? = ½, ? = 60°) $\theta = 180 - 60°, 180 + 60°$ $\theta = 120°, 240°$	2	60° (½ M) 60°, 300° (1 M)
(e)	$\sin^2\theta\cos^2\theta + \sin^4\theta$ $= \sin^2\theta(\cos^2\theta + \sin^2\theta)$ 1 $= \sin^2\theta(1)$ $= \sin^2\theta$ 1	2	Dividing by $\sin^2\theta$ — (0 M) Divide by $\sin^2\theta$ to get $\cot^2\theta + \sin^2\theta = 1$ (½ M)
(f)	$\frac{1}{\operatorname{cosec}\theta - 1} - \frac{1}{\operatorname{cosec}\theta + 1} = 2\tan^2\theta$ LHS $\frac{1}{\operatorname{cosec}\theta - 1} - \frac{1}{\operatorname{cosec}\theta + 1}$ $= \frac{\operatorname{cosec}\theta + 1 - (\operatorname{cosec}\theta - 1)}{\operatorname{cosec}^2\theta - 1}$ 1 $= \frac{2}{\cot^2\theta}$ 1 $= \frac{2}{\frac{1}{\tan^2\theta}}$ $= 2\tan^2\theta$ 1 $=$ RHS	3	Just writing all the right formulae doesn't fetch marks.

√ = ½ mark (5)

Qn	Solutions	Marks	Comments+Criteria
3(a)	 $\angle ABC = \frac{180-52}{2}$ (base angle of isosceles triangle) ✓ $= 64$ ✗ $\angle DBF = 64°$ ✗ (Vertically opposite to $\angle ABC$) ✓ $\angle BDE = 180 - \angle DBF$ (cointerior angles on BF ∥ DE) ✓ $= 116°$ ✗	3	
(b)(i)	In $\triangle ABC$ and $\triangle AYW$ $\angle A$ is common ✓ $\angle ABC = \angle AYW$ (corresponding angles on BC ∥ YW) ✓ $\angle ACB = \angle AWY$ (corresponding angles on BC ∥ YW) ✓ $\therefore \triangle ABC \equiv \triangle AYW$ (equiangular) ✓	2	if no correct reason no mark
(ii)	$\frac{YW}{BC} = \frac{AW}{AC}$ $\frac{8}{12} = \frac{5}{x}$ ✓ $x = \frac{12}{8} \times 5$ $= 7.5$ cm ✓	2	
(c)(i)	In $\triangle WQZ$ and $\triangle YRX$  $WZ = YX$ (opposite sides of parallelogram) ✓ $\angle QZW = \angle YXR$ (alternate angles $XY \parallel WZ$) ✓ $XQ = QR = RZ$ (given) ✓ $\therefore QZ = RX$ $\therefore \triangle WQZ \equiv \triangle YRX$ (SAS) ✓	3	
(ii)	Since $\triangle WQZ \equiv \triangle YRX$ $WQ = YR$ (corresponding sides of congruent triangles) ✓	1	