Student Name:

St. Catherine's School
Waverley

2009

ASSESSMENT TASK 3
(15%)

Mathematics
Year 11

General Instructions

- Working time: 55 minutes
- Attempt questions: 1–3
- Start each question in a new booklet using a new page for each geometry question in question 3
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work

Total marks – 40

QUESTION 1 (12 marks)

(a) Draw a number plane and on it plot $A(-4,0)$ $B(4,0)$ *and* $C(0,8)$ 1

(b) Find the gradient of AC and show that the equation of the line line AC is $2x-y+8=0$ 2

(c) If P is the point $(0,3)$ find the perpendicular distance from P to the line AC 2

(d) Find the coordinates of Q and R which are the midpoints of AC and BC respectively 1

(e) Show that QP is perpendicular to AC 2

(f) Show that the lengths of AP, BP and CP are all 5 units 2

(g) Find the equation of the circle passing through A, B and C 2

QUESTION 2 (16 marks) START A NEW BOOKLET

(a) If the roots of the equation $px^2 - x + q = 0$ are -2 and 5 find the values of p and q **2**

(b) If α and β are the solutions of $x^2 - px + 2p = 0$ find in terms of p the values of: **3**

(i) $\alpha + \beta$ (ii) $\alpha\beta$ (iii) $(\alpha - 2)(\beta - 2)$

(c) For what values of k does the equation $2x^2 - 4x + k = 0$ have no real roots? **2**

(d) Show that the graph of $y = x^2 + (p-2)x - p$ always crosses the x axis in two distinct places where p is real. **3**

(e) A parabola has its axis parallel to the y axis. Find the equation of this parabola if the focus is S(3,2) and the equation of the directrix is y=-6 **3**

(f) Solve $7x^3 = 8 - x^6$ **3**

QUESTION 3 (12 marks) START A NEW BOOKLET USING A NEW PAGE FOR EACH OF (a), (b) and (c)

(a) In the diagram below AP=PQ=QC and PQRS is a rhombus.
Copy the diagram onto your answer page

(i) If $\angle SAP = x^0$ prove that $\angle SPQ = 2x^0$ **2**

(ii) Prove that $\angle ABC = 90^0$ **2**

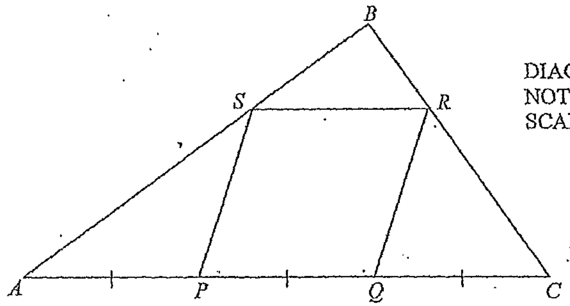
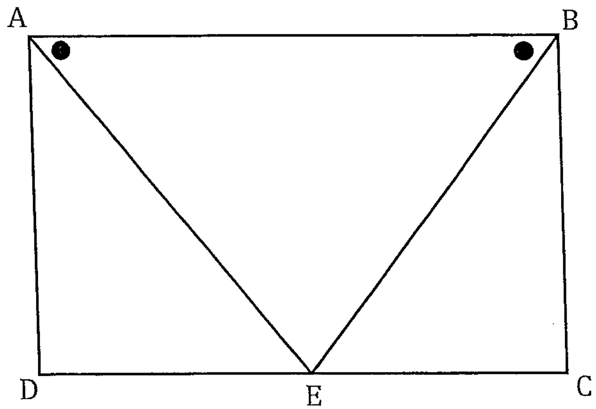


DIAGRAM NOT TO SCALE

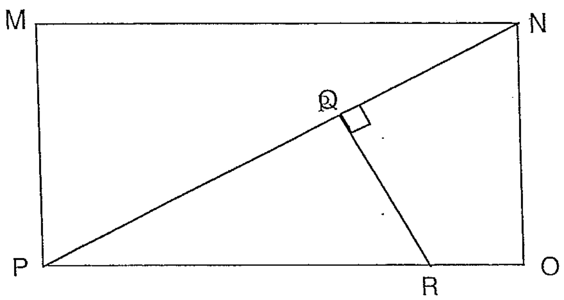
(b)



In the diagram above ABCD is a rectangle and E is a point on DC **4**
Copy this diagram onto your answer page

If $\angle EAB = \angle EBA$ prove that E is the midpoint of DC

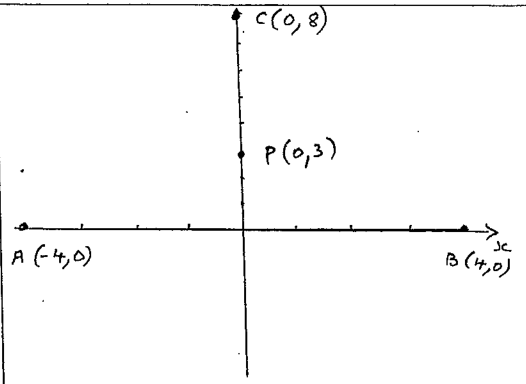
(c)



PN is a diagonal of the rectangle MNOP. R is a point on PO and $\angle PQR = 90^0$

(i) Prove that ΔPQR is similar to ΔNMP 2

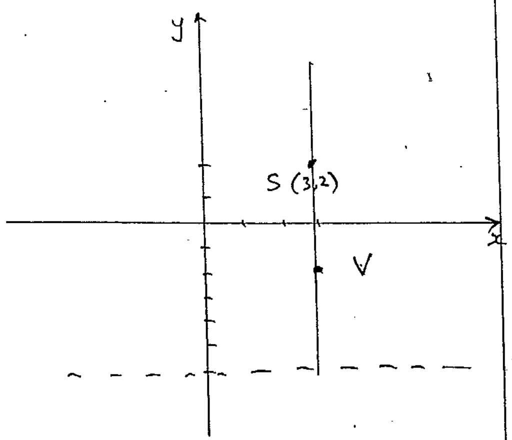
(ii) Given MP=5cm, MN=10cm and QR=2cm find the length of PQ 2

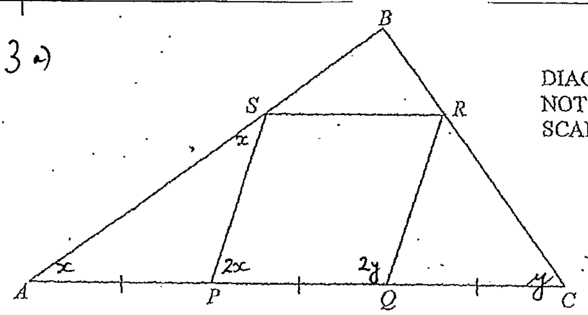
Qn	Solutions	Marks	Comments+Criteria
1 a)		1	
b)	Gradient AC = $\frac{y_2-y_1}{x_2-x_1} = \frac{8-0}{0-^-4} = 2$ line AC $y-y_1 = m(x-x_1)$ OR $y = mx + b$ $\Rightarrow y = 2x+8$ $\Rightarrow 2x-y+8=0$ as required	1 1	
c)	$d = \left\lvert \frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}} \right\rvert$ P(0,3) $= \left\lvert \frac{0 \; -3+8}{\sqrt{5}} \right\rvert = \frac{5}{\sqrt{5}} = \sqrt{5}$ units	2	$1\frac{1}{2}/2$ if not simplified to $\sqrt{5}$
d)	Q midpt AC = $\left(\frac{-4+0}{2}, \frac{0+8}{2}\right)$ $= (-2, 4)$ R midpt BC = $\left(\frac{4+0}{2}, \frac{0+8}{2}\right)$ $= (2, 4)$	$\frac{1}{2}$ $\frac{1}{2}$	

Qn	Solutions	Marks	Comments+Criteria
1 e)	Q (-2, 4) P (0, 3) gradient QP = $\frac{y_2-y_1}{x_2-x_1} = \frac{3-4}{0-^-2} = \frac{-1}{2}$ since gradient AC = 2 from b) Then QP ⊥ AC since $-\frac{1}{2} \times 2 = -1$	2	
f)	A(-4,0) P(0,3) B(4,0) C(0,8) AP $= \sqrt{(0-^-4)^2+(3-0)^2} = 5$ BP $= \sqrt{(4-0)^2+(0-3)^2} = 5$ CP = 5 as P and C are both on y axis and are 5 units apart	2	
g)	Since P is equidistant from A, B and C a circle can be drawn with centre P and radius 5 which will pass through each of A, B, C. Equation is of form $(x-h)^2 + (y-k)^2 = r^2$ $\Rightarrow x^2 + (y-3)^2 = 25$	2	$\frac{1}{2}$ MARK FOR WRITING THIS IF WRONG MIDPOINT AND/OR RADIUS STATED BUT THEN USED CORRECTLY

Qn	Solutions	Marks	Comments+Criteria
2 a)	$px^2 - x + q = 0$ Sum solutions $= -\frac{b}{a} = \frac{1}{p}$ since solutions are -2 and 5 then $-2+5 = \frac{1}{p}$ $3 = \frac{1}{p}$ $\therefore p = \frac{1}{3}$ Product solutions $= \frac{c}{a}$ $\therefore \frac{q}{p} = -10$ $\therefore \frac{q}{\frac{1}{3}} = -10$ $q = \frac{1}{3} \times -10 = -3\frac{1}{3}$ OR since -2 and 5 are solutions they can be substituted into the equation $x=-2$ $\quad p \times 4 - {-2} + q = 0$ $\Rightarrow 4p + q = -2$ ——— (1) $x = 5$ $\quad 25p - 5 + q = 0$ $\Rightarrow 25p + q = 5$ ——— (2) Solving by subtracting (1) from (2) $21p = 7$ $p = \frac{7}{21} = \frac{1}{3}$ $\therefore$ using (1) $4 \times \frac{1}{3} + q = -2$ $q = -2 - \frac{4}{3} = -3\frac{1}{3}$	2	

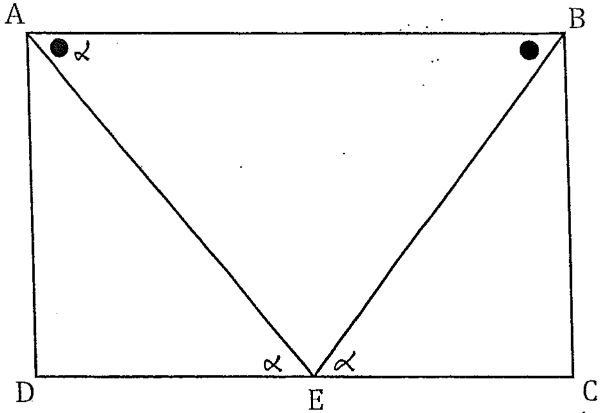
Qn	Solutions	Marks	Comments+Criteria
2 b)	$x^2 - px + 2p = 0$		
	i) $\alpha + \beta = -\frac{b}{a} = p$	1	
	ii) $\alpha\beta = \frac{c}{a} = 2p$	1	
	iii) $(\alpha-2)(\beta-2)$ $= \alpha\beta - 2\alpha - 2\beta + 4$ $= \alpha\beta - 2[\alpha+\beta] + 4$ $= 2p - 2 \times p + 4$ $= 4$	1	FULL MARKS FOR (iii) IF USING INCORRECT ANSWERS FROM i) OR ii) BUT CORRECT LOGIC
c)	$2x^2 - 4x + k = 0$ If no real roots $\Delta < 0$ $\therefore b^2 - 4ac < 0$ $\therefore 16 - 4 \times 2 \times k < 0$ $16 - 8k < 0$ $-8k < -16$ $k > 2$	2	$\frac{1}{2}$ MARK OFF IF THIS NOT MENTIONED
d)	$y = x^2 + (p-2)x - p$. For the graph to cross x axis twice $\Delta > 0$ $\Delta = b^2 - 4ac = (p-2)^2 - 4 \times 1 \times -p$ $= p^2 - 4p + 4 + 4p$ $= p^2 + 4$ Now $p^2 + 4 > 0$ for all real p $\therefore \Delta > 0$	2	$\frac{1}{2}$ MARK OFF IF GRAPH NOT MENTIONED

Qn	Solutions	Marks	Comments+Criteria
2 e)	 Vertex must be halfway between focus and directrix and lie on axis of parabola. Hence vertex is $(3,-2)$ Focal length $= 4$ Hence required equation is $(x-3)^2 = 16(y+2)$	 1 1 1	
f)	$7x^3 = 8 - x^6$ $\therefore\ x^6 + 7x^3 - 8 = 0$ Let $a = x^3$ $a^2 + 7a - 8 = 0$ $(a+8)(a-1) = 0$ $a = -8, 1$ $\therefore\ x^3 = -8$ or 1 Hence $x = -2$ or 1.	 1 1 1	

Qn	Solutions	Marks	Comments+Criteria
3 a)	 DIAGRAM NOT TO SCALE		
	i) R.T.P. If $\angle SAP = x$ Prove $\angle SPQ = 2x°$ Now PQRS is a rhombus (given) $\therefore\ PQ = SP$ But $PQ = AP$ (given) $\therefore\ SP = AP$ making $\triangle APS$ isosceles Hence $\angle PAS = \angle PSA = x$ In $\triangle SPA$ $\angle SPQ = \angle PAS + \angle PSA$ (ext $\angle$ = 2 int opp) $\therefore\ \angle SPQ = x + x = 2x°$	2	
	ii) Let $\angle QCR = y°$ Using similar methods as in i) $\angle RQP = 2y$ From rhombus PQRS $2x + 2y = 180°$ (co-interior) $\therefore\ x + y = 90°$ In $\triangle BAC$ $\angle ABC = 180° - (x+y)$ $= 180° - 90°$ $= 90°$	2	

Qn	Solutions	Marks	Comments+Criteria

3b)



R.T.P. E is midpoint DC

Proof: To prove DE = CE we show

$\triangle ADE \equiv \triangle BCE$

Let $\angle EAB = \alpha$

$\therefore \angle AED = \alpha$ [alternate angles]

Since $\angle EBA = \alpha$

then $\angle BEC = \alpha$ [alternate angles]

In $\triangle$ ADE and BCE

AD = BC (opp sides rectangle)

$\angle ADE = \angle BCE$ (90° angles in rectangle)

$\angle AED = \angle BEC$ (Proved)

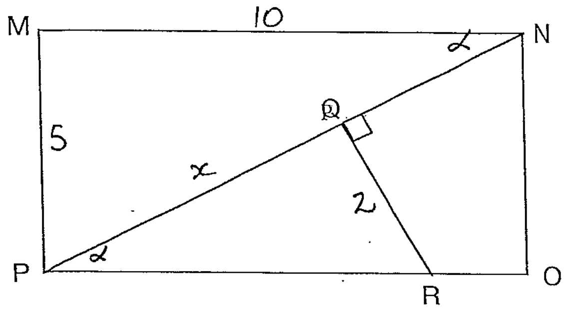
$\therefore \triangle ADE \equiv \triangle BCE$ (AAS)

$\therefore$ DE = CE (corresponding sides congruent $\triangle$s)

Comments+Criteria: $\frac{1}{2}$ MARK FOR ANSWER, $\frac{1}{2}$ MARK FOR REASONING IN EACH OF 4 REASONS

Qn	Solutions	Marks	Comments+Criteria

3c)



i) RTP $\triangle PQR \parallel\!\mid \triangle NMP$

$\angle PQR = 90^\circ$ [supp to $\angle PQN$]

$\angle PMN = 90^\circ$

$\therefore \angle PQR = \angle PMN$

Now MN ∥ PO

$\therefore \angle MNP = \angle QPR$ [alternate]

$\therefore \triangle PQR \parallel\!\mid \triangle NMP$ (equiangular) — 2

Comments+Criteria: Lose $\frac{1}{2}$ MARK if no mention of parallel

ii) From similar triangles PQR and NMP

$\frac{RQ}{PM} = \frac{PQ}{MN}$ [corresponding sides of similar $\triangle$s]

$\frac{2}{5} = \frac{x}{10}$

$\therefore 5x = 20$

$\Rightarrow x = 4$ — 2

Comments+Criteria: Lose $\frac{1}{2}$ MARK if no reason mentioned