Student Name:_____

St. Catherine's School
Waverley

2010
ASSESSMENT TASK 3
(15%)

Mathematics Year 11

General Instructions

- Working time: 55 minutes
- Attempt questions: 1–4
- Start each question on a new page
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work

Total marks – 50

QUESTION ONE	/17
QUESTION TWO	/9
QUESTION THREE	/18
QUESTION FOUR	/6
TOTAL MARKS	**/50**

Question One Start a new page — Marks

a. Solve $2x^2 - 7x - 15 \le 0$ — 2

b. Find the values of k for which the quadratic equation $2x^2 - 5x + 4k = 0$ has no real roots. — 2

c. If α and β are the roots of the equation $x^2 + 4x + 1 = 0$
What is the value of:

i) $\alpha + \beta$ — 1

ii) $\alpha\beta$ — 1

iii) $\alpha^2 + \beta^2$ — 1

iv) $\frac{1}{\alpha} + \frac{1}{\beta}$ — 2

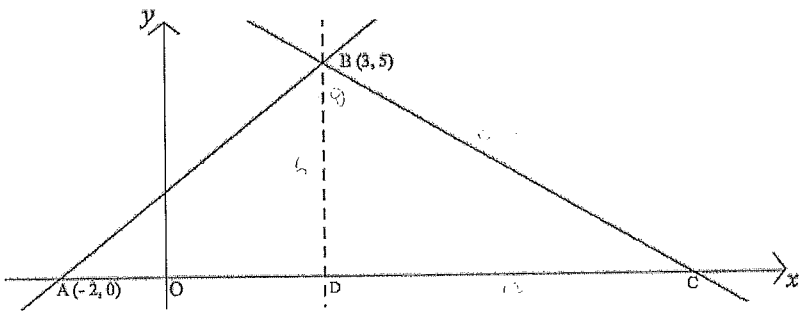
v) $\alpha - \beta$ — 2

d. Solve the equation
$9^x - 12(3)^x + 27 = 0$ — 3

e. If $x^4 + 4x^3 - x^2 - 10x + 6 \equiv a(x^2 + 2x)^2 + b(x^2 + 2x) + c$
Find the values of a, b and c. — 3

Question Two Start a new page Marks

a.



The diagram shows the points A (-2,0), B (3,5) and the point C, which lies on the x-axis. The point D also lies on the x-axis such that BD is perpendicular to AC.

i) Show that the gradient of AB is 1. **1**

ii) Find the equation of the line AB. **2**

iii) What is the size of $\angle BAC$? **1**

iv) The length of BC is 13units. Find the length of DC. **1**

v) Calculate the size of $\angle ABC$, to the nearest degree. **1**

b. Find the shortest distance between the parallel lines

$2x-y+2=0$ and $2x-y-5=0$ **3**

Question Three Start a new page Marks

a. Given that $\cos x = \frac{-4}{5}$ and $\tan x > 0$ **2**

Solve for x to the nearest degree where $0^\circ \le x \le 360^\circ$

b. Prove that $\frac{\cot\alpha}{\cos\alpha} = \operatorname{cosec}\alpha$ **2**

c. Prove that $\frac{\cot\theta\cos\theta}{\cot\theta+\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$ **3**

d. Find the exact value of

i) $\sin 60^\circ + \cos 30^\circ$ **2**

ii) $\tan 315^\circ$ **1**

d. Prove that $(\sin x+\cos x)^2+(\sin x-\cos x)^2 = 2$ **2**

e. Two cars leave a point A at the same time. One car averages 80km/hr along a straight road to B on a bearing of 025°. The other car averages 90km/hr along a straight road to C on a bearing of 135°.

i) Draw a diagram in your booklets

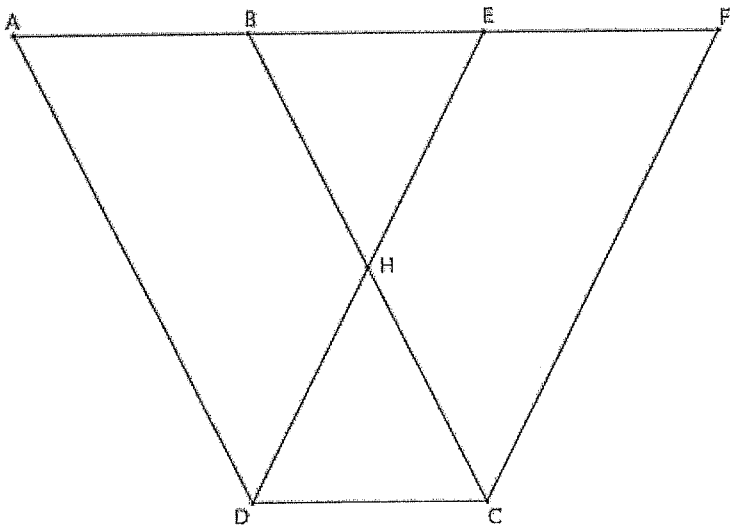
ii) How far apart are the cars after 3 hours?(correct to the nearest whole number) **3**

iii) Find $\angle BCA$ to the nearest degree **2**

iv) Find the bearing of B from C. **2**

Question Four Start a new page

Marks



A, B ,E and F are collinear points. ABCD and EFCD are parallelograms. BC and ED intersect at H such that H is the midpoint of BC.
Copy or trace the diagram into your worksheet.

i) Prove that $\Delta BHE \equiv \Delta CHD$ 3

ii) Show that $DC = BE$ 1

iii) Hence or otherwise, show that $AF = 3DC$ 2

Qn	Solutions	Marks	Comments+Criteria

Year 11 – 2unit – Solutions

Question (1)

a) $2x^2 - 7x - 15 \leq 0$
$(2x+3)(x-5) \leq 0$
$-\frac{3}{2} \leq x \leq 5.$ — 2 marks

b) no roots $b^2 - 4ac < 0$
$2x^2 - 5x + 4k = 0$
$a = 2 \quad b = -5 \quad c = 4$
$b^2 - 4ac < 0$ — 2 marks
$(-5)^2 - 4 \times 2 \times 4k < 0$
$25 - 32k < 0$ ✓1
$-32k < -25$
$k > \frac{25}{32}$ ✓1

$25 < 32k$
$\frac{25}{32} < k$
or
$k > \frac{25}{32}$

c) $x^2 + 4x + 1 = 0 \quad a = 1, \ b = 4, \ c = 1$

i) $\alpha + \beta = \frac{-b}{a} = \frac{-4}{1} = -4$ — 1 mark

ii) $\alpha\beta = \frac{c}{a} = \frac{1}{1} = 1$ — 1 mark

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= (-4)^2 - 2.1$
$= 16 - 2$
$= 14$ — 1 mark

iv) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-4}{1} = -4$ — 2 marks

v) $\alpha - \beta = \sqrt{(\alpha-\beta)^2}$
$(\alpha-\beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$
$= (14) - 2.1$
$\alpha - \beta = \pm\sqrt{12}.$ — 2 marks

Qn	Solutions	Marks	Comments+Criteria

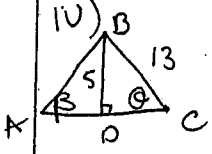
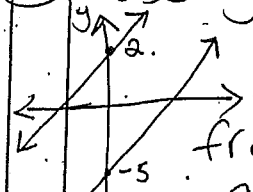
d) $9^x - 12(3^x) + 27 = 0$
let $v = 3^x$ ½ $9^x = (3^x)^2$ — 3 marks.
$v^2 - 12v + 27 = 0$ ½
$(v-9)(v-3) = 0$
$v = 9$ or $v = 3$
$3^x = 9$ | $3^x = 3$ |
$\therefore x = 2 \quad x = 1$

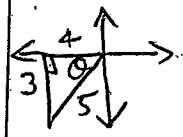
e) $x^4 + 4x^3 - x^2 - 10x + 6$
$\equiv a(x^2+2x)^2 + b(x^2+2x) + c$
$\equiv a(x^4 + 4x^3 + 4x^2) + bx^2 + 2bx + c$ — 3 marks
$\equiv ax^4 + 4ax^3 + 4ax^2 + bx^2 + 2bx + c$ |
$\therefore a = 1$ ½ $4a + b = -1$
$4 + b = -1$ |
$c = 6$ ½ $b = -5$
$x^4 + 4x^3 - x^2 - 10x + 6 \equiv (x^2+2x)^2 - 5(x^2+2x) + 6$

Question Two.

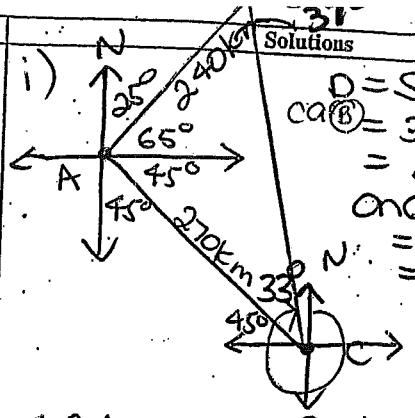
a) i) grad AB $= \frac{y_2 - y_1}{x_2 - x_1}$ — 1 mark.
$m = \frac{5-0}{3+2} = 1$

ii) equation AB
$y - y_1 = m(x - x_1)$
$y - 0 = 1(x+2)$
$y = x + 2$ — 2 marks.
$\therefore x - y + 2 = 0$ is the equation of the line AB.

Qn	Solutions	Marks	Comments+Criteria
iii)	$< BAC \Rightarrow m = 1$ $m = \tan\beta$ $1 = \tan\beta$ $\therefore \beta = 45°$ $\therefore < BAC = 45°$		1mark.
iv)	 In ΔDBC $OC^2 = BC^2 + BD^2$ $= 169 - 25$ $OC = \sqrt{144}$ $OC = 12$ units $\therefore$ C has coords $(15, 0)$		1mark.
v)	$< BCD \Rightarrow \sin\theta = \frac{5}{13}$ $\therefore \theta = 22°37'$ $< ABC = 180° - (45° + 22°37')$ $= 180° - 68°$ $\therefore < ABC = 112°$ (to the nearest deg.)		1mark
b)	Shortest distance is perpendicular distance. $2x - y + 2 = 0 \quad y = 2x + 2$ $2x - y - 5 = 0 \quad y = 2x - 5$  $d = \left\| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right\|$ (1) from $(0,2)$ to line (1) $\therefore a = 2 \quad b = -1 \quad c = -5$ $d = \left\| \frac{0 \times 2 + -1 \times 2 - 5}{\sqrt{2^2 + (-1)^2}} \right\|$ $= \left\| \frac{-2-5}{\sqrt{5}} \right\| = \frac{7}{\sqrt{5}}$ $\therefore d = \frac{7}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{7\sqrt{5}}{5}$ units		3marks.

Qn	Solutions	Marks	Comments+Criteria
	Question Three		
a)	$\cos x = -\frac{4}{5} \quad \tan x > 0$ $\therefore x$ must lie in the 3rd quadrant.	1/2	2marks.
	 $\cos x = \frac{4}{5}$ $x = 37°$	1/2	
	In the 3rd quadrant $x = 180 + \theta$ $= 180 + 37°$ $\therefore x = 217°$	1	
b)	$\frac{\cot\alpha}{\cos\alpha} = \operatorname{cosec}\alpha$ LHS $= \frac{1}{\tan\alpha} \times \frac{1}{\cos\alpha}$ 1/2 $= \frac{\cos\alpha}{\sin\alpha} \times \frac{1}{\cos\alpha}$ 1/2 $= \frac{1}{\sin\alpha}$ 1/2 $= \operatorname{cosec}\alpha$ $=$ RHS. 1/2		2marks.
c)	$\frac{\cot\theta\cos\theta}{\cot\theta + \cos\theta} = \frac{\cos\theta}{1 + \sin\theta}$ $\text{LHS} = \frac{\cot\theta\cos\theta}{\cot\theta + \cos\theta}$ $= \frac{\frac{\cos\theta}{\sin\theta} \cdot \cos\theta}{\frac{\cos\theta}{\sin\theta} + \cos\theta}$	1	

Qn	Solutions	Marks	Comments+Criteria
	$\frac{\cos^2\theta}{\sin\theta}$		
	$= \frac{\cos\theta}{\frac{\cos\theta + \cos\theta\sin\theta}{\sin\theta}}$	½	
	$= \frac{\cos\theta}{\frac{\cos^2\theta}{\sin\theta}} \times \frac{\sin\theta}{\cos\theta(1+\sin\theta)}$	½	3 marks total
	$= \frac{\cos\theta}{1+\sin\theta}$	1	
	= RHS.		
d)	i) $\sin 60^\circ + \cos 30^\circ$		
	$\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{2}$	1½	2 marks
	$= \sqrt{3}$.	½	
	ii) $\tan 315^\circ =$		1 mark.
	$= -\tan(360^\circ - 45^\circ)$		
	$= -\tan 45^\circ$		
	$= -1$		
e	$(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$.		2 marks
	LHS =		
	$= \sin^2 x + 2\sin x\cos x + \cos^2 x + \sin^2 x + \cos^2 x - 2\sin x\cos x$	1	
	$= 2\sin^2 x + 2\cos^2 x$	½	
	$= 2(\sin^2 x + \cos^2 x)$		
	$= 2 \times 1$ as $\sin^2 x + \cos^2 x = 1$		
	= RHS.	½	

Qn	Solutions	Marks	Comments+Criteria
e)	i) 		
	D = S × T		
	car (B) = 3hr × 80km/L		
	= 240 km		
	and car (C)		
	= 3hrs × 90km/L		
	= 270km.		
	$\angle BAC = 110^\circ$ b = 270 km	½	
	c = 240 km.		
	ii) Distance BC is side 'a'		3 marks.
	$a^2 = b^2 + c^2 - 2bc\cos 110^\circ$	1	
	$= 270^2 + 240^2 - 2 \times 270 \times 240 \times \cos 110^\circ$		
	$a^2 = 174825.8106$		
	$a = 418.12$	1	
	= 418 km		
	iii) $\angle BCA$? use sine rule		2 marks.
	$\frac{\sin BCA}{240} = \frac{\sin 110^\circ}{418}$		
	$\sin BCA = \frac{240\sin 110^\circ}{418}$		
	$\angle BCA = 33^\circ$ (to the nearest degree)		
	iv) The bearing of B from A is		1 marks
	$= 270^\circ + 45^\circ + \angle BCA$ (33°)		
	$= 348^\circ$		
	= 348T.		

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Qn	Solutions	Marks	Comments+Criteria
	Question Four		
	AB ∥ DC (opposites sides of a parallelogram AB, CD are parallel)		
	∴ AF ∥ DC ABEF are collinear.		
i)	In ΔBHE and ΔCHD		3marks
	∠EBH = ∠DCH (alternate angles BE ∥ DC) ✓		
	∠BHE = ∠CHD (vertically opp angles) ✓		
	BH = HC (H is the midpoint given) ✓		-½ if not stating the test
	∴ ΔBHE ≡ ΔCHD (ASA)		
ii)	DC = BE (corresponding sides in congruent Δ's proven) ✓		1mark
iii)	BE = DC ✓		-½ if reason not stated correctly
	AB = DC (opposite sides in a parallelogram AB ∥ CD) ✓		
	EF = DC		
	∴ AF = AB + BE + EF ✓		2mark
	AF = 3DC		
	∴ shown.		
			total = 6