St. Catherine's School
Waverley
PRELIMINARY ASSESSMENT TASK 4
September
2006

Mathematics

General Instructions

- Working time – 120 minutes (+ 5 min reading time)
- Write using black or blue pen
- Board-approved calculators may be used.

Total marks – 84

- Attempt all questions
- All questions are of equal value
- All necessary working should be shown

Question 1	/12
Question 2	/12
Question 3	/12
Question 4	/12
Question 5	/12
Question 6	/12
Question 7	/12

QUESTION 1 (12 Marks) *Start a new page* **Marks**

a) Evaluate $\sqrt{\dfrac{3.5\times 7.62}{\pi}}$, giving answer correct to:

(i) 3 decimal places **1**

(ii) 3 significant figures **1**

b) Simplify $\sqrt{2}+\sqrt{18}$ **1**

c) Solve $3(x-4)-(x+2)=12$ **2**

d) Factorise:

i) $am + an - bm - bn$ **2**

ii) $9x^2 - y^4$ **2**

e) Solve: $|3x+2| \le 10$ and graph the results on a number line. **3**

QUESTION 2 (12 Marks) *Start a new page* **Marks**

a) Evaluate $\frac{4}{3}\pi r^3$ to 1 decimal place given $r = 4.2$ **1**

b)

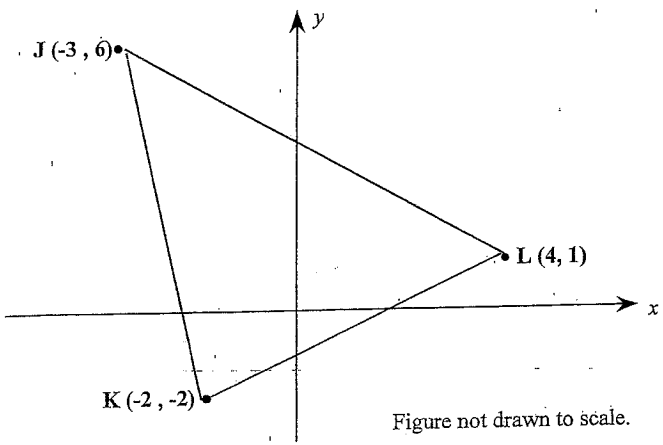


Figure not drawn to scale.

i) Find the length of KL (*leave answer in simplified surd form*) **1**

ii) Show that the gradient of KL is $\frac{1}{2}$ **1**

iii) Find the equation of the line KL in general form **2**

iv) What is the perpendicular distance of the point J from KL (*leave answer in surd form*) **2**

v) Hence or otherwise, find the area of ΔJKL **2**

c) i) Solve the simultaneous equations **2**

$5x - y = 4$
$3x + y = 12$

ii) Hence state the point of intersection of the lines **1**

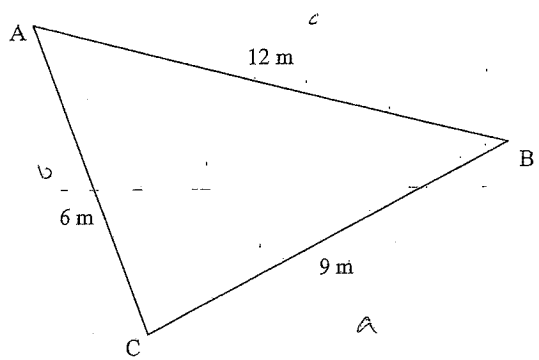
$5x - y = 4$ and $3x + y = 12$

QUESTION 3 (12 Marks) *Start a new page* **Marks**

a) i) Express $\frac{7}{4-\sqrt{2}}$ with a rational denominator **2**

ii) Hence find a and b such that $\frac{7}{4-\sqrt{2}} = a + b\sqrt{2}$ **1**

b) Use the cosine rule to find angle ABC in this triangle (to nearest minute): **3**



c) Sketch the curves showing all essential features.
Use a separate set of axes for each sketch. Each sketch should take about one quarter of a page

i) $x^2 + y^2 = 9$ **2**

ii) $y = |x + 2|$ **2**

iii) $xy = 4$ **2**

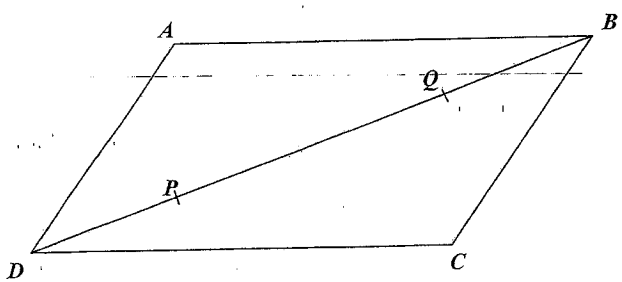
QUESTION 4 (12 Marks) *Start a new page* **Marks**

a) i) Sketch the parabola $y = -x^2 - 4x + 5$ showing its intercepts and vertex. 4

ii) What is the range for the parabola $y = -x^2 - 4x + 5$?

iii) State the values of x for which $-x^2 - 4x + 5 > 0$

b) $ABCD$ is a parallelogram. Points P and Q lie on the diagonal DB such that $DP = BQ$. 4



Copy the diagram into your answer booklet. Join AQ and PC

Prove that $AQ = PC$.

c) Write in simplest form: 4

i) $\log_2 \frac{1}{8}$

ii) $\log_2 \sqrt{8}$

iii) $\log_2 9.5 + \log_2 (\frac{1}{19})$

QUESTION 5 (12 Marks)

START BOOKLET 2

a) For the Arithmetic sequence 100,94,88.....

i) Write down the first term a and common difference d 2

ii) Calculate T_{12} 2

iii) Write down the formula for S_n 1

iv) Find the smallest value of n for which $S_n < 0$ 3

b) For the Geometric Series with $a = 4$ and $r = -0.8$

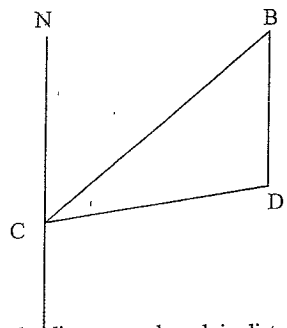
i) Find the value of T_4 2

ii) Find the sum to infinity for this series 2

QUESTION 6 (12 Marks) *Start a new page* **Marks**

a) Francine stands on point C on Coogee beach and observes a ship B 10 km out to sea on a bearing of $040°T$.

The ship is sailing due South. After 10 minutes its bearing is $085°T$ and it is at point D



NOT TO SCALE

Copy the diagram and mark in distance CB and the size of angles NCB and NCD **1**

i) Angle CBD = 40^0. Explain why. **1**

ii) Angle BCD = 45^0. Explain why. **1**

iii) Angle BDC = 95^0. Explain why. **1**

iv) Find the length of BD to 1 decimal place **3**

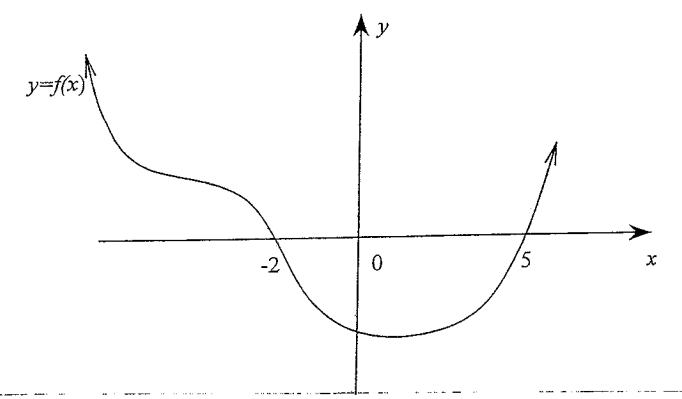
v) Find the speed of the ship in km/hr **1**

b) If $\sin\theta = -\frac{2}{3}$ and $\cos\theta > 0$, find the value of $\tan\theta$ **2**

(express your answer in surd form)

c) The graph shows the graph of $y = f(x)$

State the values of x for which $f(x) > 0$ **2**



QUESTION 7 (12 Marks) *Start a new page* **Marks**

a) Simplify $(1-\sin^2 x)\ (1+\tan^2 x)$ **3**

b) A series is given by:

$$(1+r)+(1+r)^2+(1+r)^3+\ldots+(1+r)^n+\ldots..$$

i) Show the series is Geometric. **1**

ii) For which values of r does a sum to infinity exist? **2**

c) i) Given that $PQ \| BC$ and $QR \| AB$, prove that the triangles APQ and ABC are similar. **2**

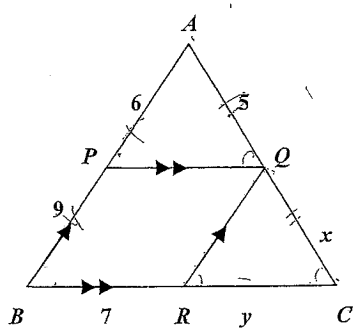


Figure not to scale

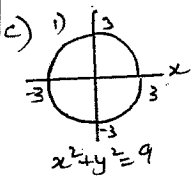
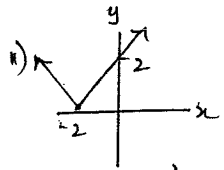
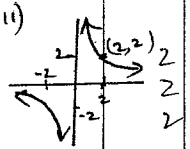
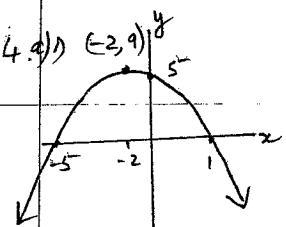
ii) Hence find the values of x and y (Give full reasons) **2**

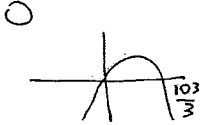
d) Simplify $\frac{4-x^2}{6-x-x^2} \times \frac{3x+9}{4x^2+16x+16}$ **2**

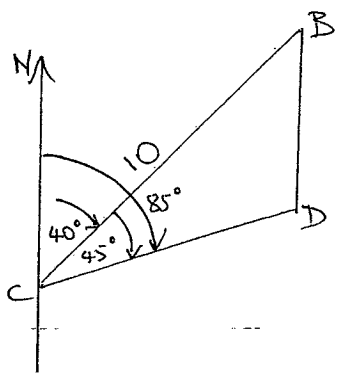
End of Paper

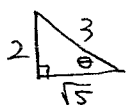
Maths Yr 11 Prelim '06 Solutions

Qn	Solutions	Marks	Comments+Criteria
1	a) i) 2.914	1	
	ii) 2.91	1	
	b) $\sqrt{2}+3\sqrt{2}=4\sqrt{2}$	1	
	c) $3x-12-x-2=12$ $2x-14=12$ $2x=26$ $x=13$	2	1 for simplifying 1 for solution
	d) i) $a(m+n)-b(m+n)$ $(a-b)(m+n)$	2	
	ii) $9x^2-y^4=(3x-y^2)(3x+y^2)$	2	
	e) $-10 \le 3x+2 \le 10$ $-12 \le 3x \le 8$ $-4 \le x \le 2\frac{2}{3}$ 	3	1 for each solution of x 1 for correct graph of solution.
2	b) i) $d=\sqrt{(4-^-2)^2+(1-^-2)^2}$ $=\sqrt{45}=3\sqrt{5}$ $\therefore KL=3\sqrt{5}$	2	Formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
	ii) $m=\frac{y_2-y_1}{x_2-x_1}=\frac{1-^-2}{4-^-2}=\frac{3}{6}=\frac{1}{2}$	1	Formula $m=\frac{y_2-y_1}{x_2-x_1}$
	iii) $y-1=\frac{1}{2}(x-4)$ $2y-2=x-4$ $x-2y-2=0$ eq. of KL	2	
	iv) $d=\left\lvert\frac{1\times-3-2\times6-2}{\sqrt{1^2+2^2}}\right\rvert=\left\lvert\frac{-17}{\sqrt{5}}\right\rvert=\frac{17}{5}$	2	Formula $d=\left\lvert\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right\rvert$
	v) $A=\frac{1}{2}b\times h=\frac{1}{2}\times3\sqrt{5}\times\frac{17}{\sqrt{5}}=25.5\,u^2$	2	
	c) $8x=16$ $\therefore x=2$ $y=6$ $\therefore$ Pt $(2,6)$	2 1	

Qn	Solutions	Marks	Comments+Criteria
3	a) i) $\frac{7}{4-\sqrt{2}}\times\frac{4+\sqrt{2}}{4+\sqrt{2}}=\frac{28+7\sqrt{2}}{16-2}=\frac{4+\sqrt{2}}{2}$	2	
	ii) $\therefore a=2$ $b=\frac{1}{2}$	1	
	b) $\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{12^2+9^2-6^2}{2\times12\times9}$ $=0.875$ $B=28^\circ27'$ (nearest min)	3	
	c) i)  $x^2+y^2=9$	2	
	ii)  $y=\lvert x+2\rvert$	2	
	iii)  $xy=4$	2	
4	a) i) 		
	ii) Range $y \le 9$ $y \in R$ ①	1	
	iii) $-5 < x < 1$ ①	1	
	← ②		
	b) In Δs ABQ, CDP $\angle ABQ=\angle CDP$ (alt $\angle$s AB $\parallel$ DC) $QB=PD$ (given) $AB=DC$ (opp side $\parallel$gram) $\therefore \Delta ABQ \equiv \Delta APC$ (SAS) $\therefore AQ=CP$ (corresp sides of cong. Δs)	2 1 1	NB Sequence is important.
	c) $\log_2\frac{1}{8}=\log_2 2^{-3}=-3$	1	
	$\log_2\sqrt{8}=\log_2 2^{3/2}=\frac{3}{2}$	1	
	$\log_2 9.5+\log_2\frac{1}{19}=\log_2\left(9.5\times\frac{1}{19}\right)$ $=\log_2\frac{1}{2}$ $=-1$.	2	

Qn	Solutions	Marks	Comments+Criteria
5(a)	100, 94, 88, ...		
	(i) $a = 100$	✓	
	$d = -6$	✓	
	(ii) $T_{12} = a + 11d$	✓	
	$= 100 - 66$		
	$= 34$	✓	
	(iii) $S_n = \frac{n}{2}(2a + (n-1)d)$	✓	accept either formula
	$\left[= \frac{n}{2}(200 + (n-1)\times -6)\right.$		$-\frac{1}{2}$ for both with one incorrect.
	$= \frac{200n}{2} - 3n(n-1)$		
	$\left.= 103n - 3n^2\right]$		
	(iv) $S_n < 0$ means		
	$103n - 3n^2 < 0$		
	$n(103 - 3n) < 0$	✓	
	$n > \frac{103}{3}$ 	✓	−1 for dividing by x without explanation that $n>0$
	~~$n < 0$~~ as $n > 0$		
	$\therefore n > 34\frac{1}{3}$	✓	$-\frac{1}{2}$ for not stating $n = 35$
	ie $n = 35$ $\left[S_{35} = -70\right]$		
(b)	$a = 4 \quad r = -0.8$		
	(i) $T_4 = ar^3$	✓	
	$= 4 \cdot (-0.8)^3$ $\left[= \frac{-256}{125}\right]$	✓	
	$= -2.048$ $\left[= -2\frac{6}{125}\right]$	✓	

Qn	Solutions	Marks	Comments+Criteria
5(b)	(ii) $S_\infty = \frac{a}{1-r}$		
	$= \frac{4}{1+0.8}$	✓	
	$= \frac{20}{9} = 2.\dot{2}$	✓	
6(a)		✓	
	(i) $\angle CBD = 40°$ (alt ∠s on // lines BD, NC)	✓	accept (alt ∠s) only 1
	(ii) $\angle BCD = 45°$ (Adj angle to $\angle NCB$ and $\angle NCD = 85°$)	✓	accept subtraction of ∠s if explained
	(iii) $\angle BDC = 95°$ (∠ sum △ BCD)	✓	accept (∠ sum △)
	(iv) $\frac{BD}{\sin 45°} = \frac{10}{\sin 95°}$	✓	1 for $\sin 45 = \frac{BD}{10}$ if correct RO
	$\therefore BD = \frac{10 \sin 45°}{\sin 95°}$		
	$= 7.0980 \ldots \doteq 7.1$ (1dp)	✓(✓)(RO)	

Qn	Solutions	Marks	Comments+Criteria
6(a)(v)	Speed $= \frac{D}{T}$ $= \frac{7.0980\ldots}{\frac{1}{6}}$ $= 42.5884\ldots$ km/hr	✓	Ignore RDE
(b)	$\sin\theta = -\frac{2}{3}$ $\cos\theta > 0$ $\therefore +ve$	✓	
	 $\therefore \tan\theta = -\frac{2}{\sqrt{5}}$ $= -\frac{2\sqrt{5}}{5}$	✓	
(c)	$f(x) > 0$ for $x < -2$ $x > 5$	✓ ✓	−1 for $f(x) < -2$ $f(x) > 5$
			$-\frac{1}{2}$ for $-2 > x > 5$
			−1 for $x \leq -2$ $x \geq 5$

Qn	Solutions	Marks	Comments+Criteria
7.	a) $(1-\sin^2 x)(1+\tan^2 x)$		
	$= \cos^2 x \times \sec^2 x$	2	
	$= \cos^2 x \times \frac{1}{\cos^2 x} = 1$	1	
	b) $\frac{T_n}{T_{n-1}} = \frac{T_3}{T_2} = \frac{T_2}{T_1} = 1+r$	1	
	S_∞ exists when $\lvert 1+r \rvert < 1$		
	$\therefore -1 < 1+r < 1$		
	$-2 < r < 0$	2	
	c) In $\triangle$s APQ, ABC		
	$\angle A$ is common		
	$\angle APQ = \angle ABC$ (corresp $\angle$, PQ ∥ BC)		
	$(\angle AQP = \angle ACB$, " ")	2	
	$\therefore \triangle APQ \,\|\|\| \triangle ABC$		
	$\therefore \frac{6}{15} = \frac{5}{5+x}$ corresp sides of sim $\triangle$s	1	
	$x = 7.5$		
	also $PQ = 7$ (opp sides of p'gram)		
	$\frac{6}{7} = \frac{15}{7+y}$ (same)		
	$42+6y = 105$	1	
	$6y = 63$ $y = 10.5$		
	d) $\frac{4-x^2}{6-x-x^2} \times \frac{3x+9}{4x^2+16x+16}$		
	$= \frac{(2-x)(2+x)}{(2-x)(3+x)} \times \frac{3(x+3)}{4(x+2)(x+2)}$	2	
	$= \frac{3}{4(x+2)}$		