# St Catherine's School Yearly Exam '04

Year: 11

Subject: Mathematics

Time Allowed: $2\frac{1}{2}$ hours

plus 5 min reading time

Date: September 2004

Tricia

Exam number: 15227508

**Directions to candidates:**

- All questions are to be attempted.
- All questions are of equal value.
- All necessary **working** must be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Each question attempted should be started on a **new page**.
- Approved calculators and geometrical instruments maybe used.
- Hand in your questions in ~~4~~ 3 separate bundles:
  - 1. Include the question paper with questions 1, 2, 3 and 4
  - 2. Questions 5, 6, 7 and 8
  - 3. Question 9 and 10
  - ~~4. Question 10~~

| TEACHER'S USE ONLY Total Marks | |
|---|---|
| Q1 | 10 /10 |
| Q2 | 10 /10 |
| Q3 | 10 /10 |
| Q4 | 10 /10 |
| Q5 | 10 /10 |
| Q6 | 10 /10 |
| Q7 | $9\frac{1}{2}$ /10 |
| Q8 | 10 /10 |
| Q9 | 9 /10 |
| Q10 | 10 /10 |
| Grand Total | $98\frac{1}{2}$ /100 |

## Question 1

(a) Find, correct to two decimal places: **2**

$$\frac{(3.24)^2}{5.74-2.85} \quad .$$

(b) Factorise and simplify: **2**

$$\frac{a^3-8}{2a-4} \quad .$$

(c) Find the values of $p$ and $q$ if $\dfrac{\sqrt{5}-2}{\sqrt{5}+2} = p + q\sqrt{5}$ . **2**

(d) Solve the following:

(i) $(x-3)(x+2)=0$ , **1**

(ii) $|x-2| \leq 5$ , graphing your solution on a number line. **3**

**Question 2** **Begin a new page.**

(a) Factorise: **1**

$$p(q+2)+q+2 \ .$$

(b) Simplify: **1**

$$12-5(3-2x) \ .$$

(c) Simplify:

(i) $2^x \times 2^{x+1}$, **1**

(ii) $3x^{-2} \times (3x)^{-2}$. **2**

(d) Solve:

(i) $(2x+1)^2 = 9$ , **2**

(ii) $v + \dfrac{1}{v} = 3$ . **3**

**Question 3** **Begin a new page.**

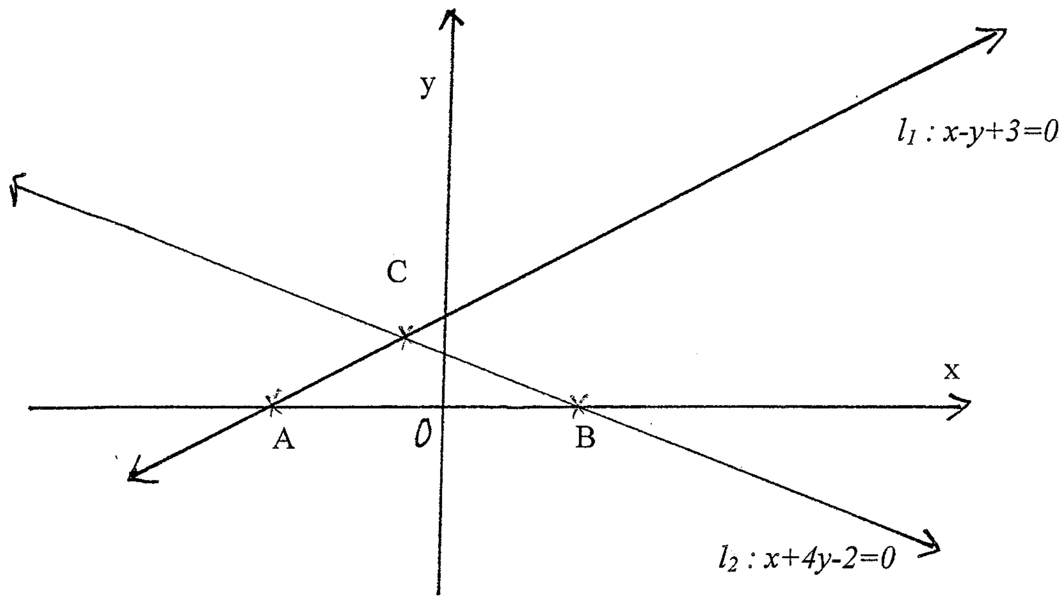



Diagram not to scale

In the diagram, $l_1$ and $l_2$ with equations $x-y+3=0$ and $x+4y-2=0$, meet at C. The lines cut the $x$-axis at A and B respectively.

(a) Find the coordinates of the points A and B. **2**

(b) Solve simultaneously to find the point C. **3**

(c) Find the angle of inclination of the line $l_1$ to the positive $x$-axis. **2**

(d) Find the perpendicular distance from the point (6,7) to the line $l_2$, correct to 1 decimal place. **3**

**Question 4** **Begin a new page.**

(a) Find the exact value of:

(i) $\cos 210^o$ , **1**

(ii) $\tan(-60^o)$ . **1**

(b) If $\sin\theta = \frac{3}{5}$ and $\cos\theta < 0$, **2**

find the exact value of $\tan\theta$.

(c) Prove that **2**

$$\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta \ .$$

(d) Solve for $0 \le \theta \le 360^o$

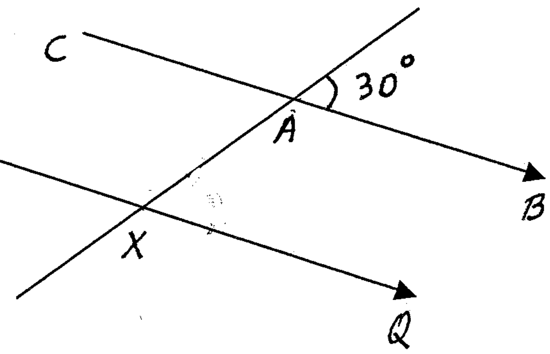
**2**

(i) $\sin\theta = -\frac{1}{2}$ , **2**

(ii) $\cos 2\theta = 0.7$ .

**Question 5** **Begin a new page.**

(a) Find the size of $\angle AXQ$, giving reasons. Diagram not to scale. **2**




(b)

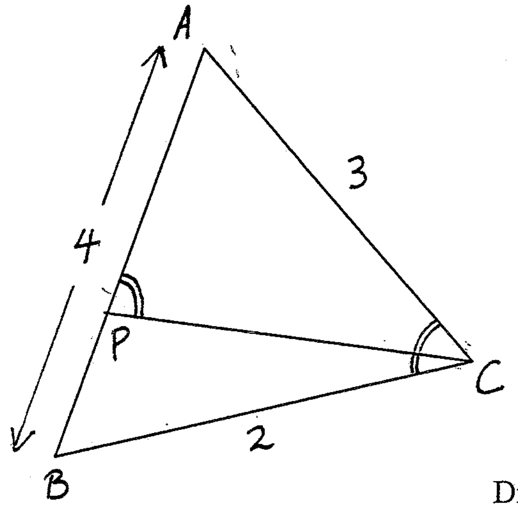



Diagram not to scale.

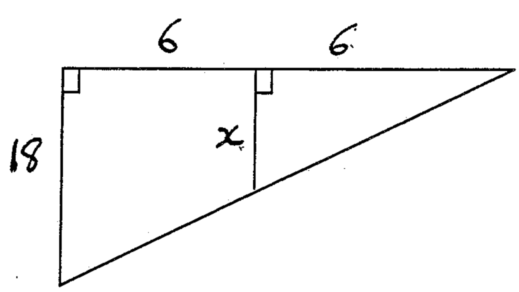
In the diagram above, $\angle APC = \angle ACB$. **3**

(i) Prove $\Delta APC \,|||\, \Delta ACB$.

(ii) Find the length PC. **2**

(iii) Find the length AP. **1**

(c) Find the value of $x$, giving reasons. Diagram not to scale. **2**



**Question 6** **Begin a new page.**

(a) If $f(x)=\begin{cases} x, & x \geq 0 \\ x^2, & x<0 \end{cases}$

(i) Find $f(1)+f\left(-\frac{1}{2}\right)$. **2**

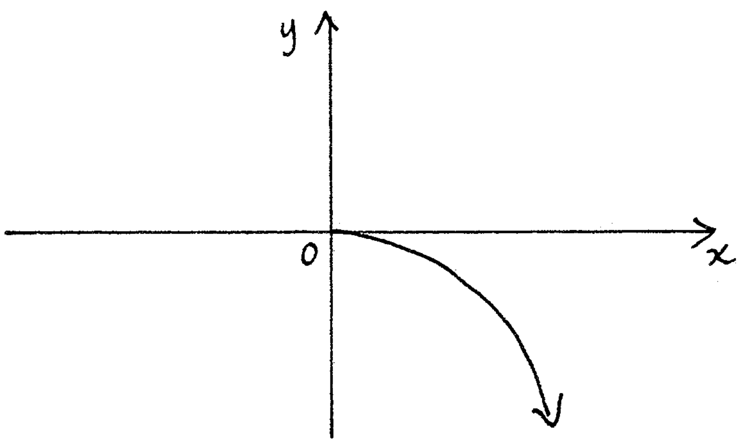
(ii) Sketch $y=f(x)$. **2**

(b) (i) Sketch $y=\frac{1}{x}$ and state the domain and range. **2**

(ii) Hence sketch (α) $y=\frac{1}{x}+2$ , **1**

(β) $y=\frac{1}{x-2}$ . **1**

(c) Consider the graph of $y=f(x)$, part of which is sketched.




(i) Copy the diagram and complete the graph if $y=f(x)$ is even. **1**

(ii) Copy and complete the graph if $y=f(x)$ is odd. **1**

## Question 7 Begin a new page.

(a) Given the equation $2x^2 - 6x + 1 = 0$ with roots α and β, find the value of:

(i) $\alpha + \beta$ , **1**

(ii) $\alpha\beta$ , **1**

(iii) $(\alpha - 1)(\beta - 1)$ . **2**

(b) Determine the value of *k* if $x^2 - 3kx + k = 0$ has real and different roots. **3**

(c) Solve $(k-1)(k+2) > 0$ . **2**

(d) Find the minimum value of the expression $3(x+4)^2 + 7$. **1**

## Question 8 Begin a new page.

(a) Find $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$ **2**

(b) Differentiate with respect to *x*:

(i) $2x^3 - 6x + 4$ , **2**

(ii) $2x(x^2 - 1)^4$, leaving your answer in factorised form. **3**

(c) Find the equation of the tangent to the curve $y = x^2 + 2x$ at the point where *x*=1. **3**

**Question 9** **Begin a new page.**

(a) Find $\lim_{n \to \infty} \dfrac{2n^2 + n - 1}{3n^2 - 2n + 1}$. **2**

(a) Consider the curve with equation $f(x) = x^2$.

(i) Find the gradient of the secant joining the points on the curve with *x*-coordinates *3* and *3+h*. **2**

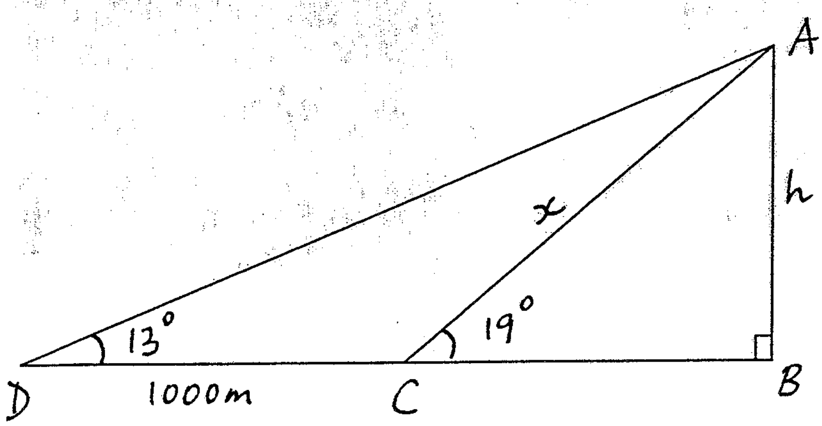
(ii) Find the equation of this secant. **2**

(iii) Find the limiting position of this secant as $h \to 0$ and describe your answer geometrically. **2**

(c) Decide whether the function $f(x) = \begin{cases} \dfrac{x^2 - 4x + 3}{x - 3}, & x \neq 3 \\ 2, & x = 3 \end{cases}$ is continuous at $x$=3. Give reasons. **2**

**Question 10** **Begin a new page.**

(a)




The angle of elevation of the top A of a mountain AB from D is 13°. After walking 1000m towards the mountain to C, the angle of elevation is 19°. Let AC=$x$ metres and AB=$h$ metres.

(i) Show that $x = \dfrac{1000\sin 13^\circ}{\sin 6^\circ}$. **2**

**2**

(ii) Hence show that $h = \dfrac{1000\sin 13^\circ \sin 19^\circ}{\sin 6^\circ}$ and evaluate $h$ correct to 3 significant figures.

(b) Differetiate from first principles $f(x) = \dfrac{2}{x}$. **3**

(c) Consider the circle $x^2 + y^2 = 25$ and the line 3x+4y=7. **3**

Determine whether the line meets the circle in one point, two points or not at all.

# YEAR 11 MATHEMATICS EXAM YEARLY 2004 ST. CATHERINE'S SOLUTIONS 100

1 a) $\dfrac{(3.24)^2}{5.74-2.85} = 3.63238$

$= 3.63$ (2dp) (2)

b) $\dfrac{a^3-8}{2a-4} = \dfrac{(a-2)(a^2+2a+4)}{2(a-2)}$

(2) $= \dfrac{a^2+2a+4}{2}$

c) $p+q\sqrt{5} = \dfrac{\sqrt{5}-2}{\sqrt{5}+2} \times \dfrac{\sqrt{5}-2}{\sqrt{5}-2}$

$= \dfrac{(\sqrt{5})^2 - 2\sqrt{5}.2 + 2^2}{(\sqrt{5})^2 - 2^2}$

(2) $= \dfrac{5-4\sqrt{5}+4}{5-4}$

$= \dfrac{9-4\sqrt{5}}{1}$

$= 9-4\sqrt{5}$

$\therefore p = 9,\ q = -4$

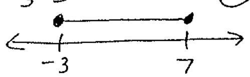
d) i) $(x-3)(x+2)=0$

(1) $x = 3$ or $x = -2$

ii) $|x-2| \le 5$

$-5 \le x-2 \le 5$

$-3 \le x \le 7$ (3)




2. a) $p(q+2)+q+2$

$= p(q+2)+1(q+2)$

$= (p+1)(q+2)$ (1)

b) $12-5(3-2x)$

$= 12-15+10x$ (1)

$= -3+10x$ or

$10x-3$

2 cont'd

c) i) $2^x \times 2^{x+1} = 2^{2x+1}$ (1)

ii) $3x^{-2} \times (3x)^{-2}$

$= \dfrac{3}{x^2} \times \dfrac{1}{3\,9x^2}$ (2)

$= \dfrac{1}{3x^4}$

d) i) $(2x+1)^2 = 9$

$2x+1 = 3$ or $-3$

$2x = 2$ or $-4$

(2) $x = 1$ or $-2$

ii) $v + \dfrac{1}{v} = 3$

$v^2+1 = 3v$ (3)

$v^2-3v+1=0$

$v = \dfrac{3 \pm \sqrt{(-3)^2-4(1)(1)}}{2(1)}$

$v = \dfrac{3 \pm \sqrt{5}}{2}$

or 2.62, 0.38 (2dp)

3. a) A : sub $y=0$ :

$x - 0 + 3 = 0$

$\therefore x = -3 \ \therefore A(-3,0)$

B : sub $y = 0$ (2)

$x + 0 - 2 = 0$

$x = 2 \ \therefore B(2,0)$

b) $x - y + 3 = 0$ ①

$x + 4y - 2 = 0$ ②

① − ② $-5y+5=0$

$\therefore 5y = 5$

$y = 1$

3 b) Cont'd sub $y=1$ into ①

$x - 1 + 3 = 0$ (3)

$\therefore x + 2 = 0$

$x = -2 \ \therefore C(-2,1)$

c) $x - y + 3 = 0 \Rightarrow y = x+3$

$\therefore m = 1 \quad \because \tan\theta = m$

$\therefore \tan\theta = 1$ (2)

$\therefore \theta = 45°$

$\therefore$ angle of inclination is $45°$.

d) $d = \left|\dfrac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

$= \left|\dfrac{6+4(7)+-2}{\sqrt{1^2+4^2}}\right|$

$= \left|\dfrac{32}{\sqrt{17}}\right|$ (3)

$= 7.8$ (1 dp)

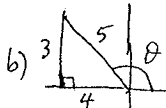
4. a) i) $\cos 210° = -\cos 30°$

$= -\dfrac{\sqrt{3}}{2}$ (1)

ii) $\tan(-60°) = -\tan 60°$

$= -\sqrt{3}$ (1)

b)




$\tan\theta < 0$

(2) $\therefore \tan\theta = \dfrac{-3}{4}$.

c) $LHS = \dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta}$

(2) $= \dfrac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta}$

$= \dfrac{2}{\cos^2\theta}$

$= 2\sec^2\theta = RHS$ QED.

# YEAR 11 MATHS EXAM YEARLY 2004 ST. CATH'S SOL'NS cont'd

Question 4 cont'd

4 d) i) $\sin\theta = -\frac{1}{2}$ (2)

basic $\theta = 30°$ in Q 3 or 4

$\therefore \theta = 210°, 330°$

ii) $\cos 2\theta = 0.7$

If $0° \le \theta \le 360°$

then $0 \le 2\theta \le 720°$ (2)

basic $2\theta \doteqdot 45.6$ in Q 1 & 4

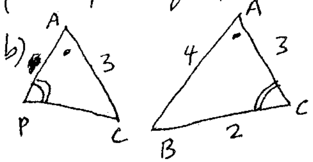
$\therefore 2\theta \doteqdot 45.6, 314.4, 405.6, 674.4$

$\therefore \theta \doteqdot 23°, 157°, 203°, 337°$

Q5. a) $\angle AXQ = 30°$ (2)

(Corresp. ∠s equal, CB ∥ PQ)

b)




i) Proof : In ΔAPC, ΔACB

(3) 1. ∠PAC = ∠BAC (Common ∠)

2. ∠APC = ∠ACB (given)

∴ ΔAPC ||| ΔACB (angle angle test)

ii) $\dfrac{PC}{CB} = \dfrac{AC}{AB}$ (sides of sim Δs in prop'n)

$\therefore \dfrac{PC}{2} = \dfrac{3}{4}$ (2)

$\therefore PC = \dfrac{3}{2}$.

iii) $\therefore \dfrac{AP}{AC} = \dfrac{AC}{AB}$ (prop'n sides of sim Δ)

(1) $\therefore \dfrac{AP}{3} = \dfrac{3}{4}$

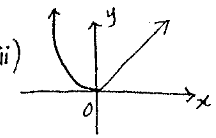
$\therefore AP = \dfrac{9}{4}$

5 c) $x = 9$ (line through midpt of one side ∥ to other side is half its length) (2)

6. a) i) $f(1) + f(-\frac{1}{2}) = 1 + (\frac{1}{2})^2$

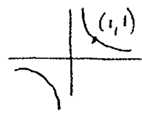
$= 1\frac{1}{4}$ (2)

ii)




(2)

b) i)

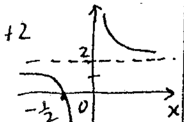



D: {all real $x \ne 0$}

(2) R: {all real $y \ne 0$}

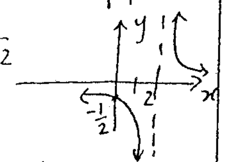
ii) α) $y = \frac{1}{x}+2$

(1)

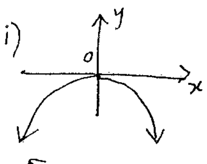



β) $y = \dfrac{1}{x-2}$

(1)

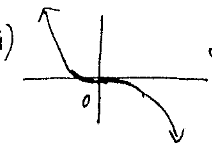



c) i)




even (1)

ii)




odd (1)

7. a) i) $\alpha+\beta = \dfrac{-b}{a}$

$= \dfrac{6}{2}$ (1)

$= 3$

ii) $\alpha\beta = \dfrac{c}{a}$ (1)

$= \dfrac{1}{2}$

iii) $(\alpha-1)(\beta-1) = \alpha\beta - (\alpha+\beta) + 1$

(2) $= \frac{1}{2} - 3 + 1$

$= -1\frac{1}{2}$

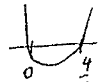
7 cont'd

b) $\Delta > 0 \ \therefore b^2-4ac > 0$

$\therefore (-3k)^2 - 4(1)(k) > 0$

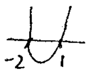
$9k^2 - 4k > 0$

$k(9k-4) > 0$




$\therefore k < 0$ or $k > \frac{4}{9}$ (3)

c) $(k-1)(k+2) > 0$




$k < -2$ or $k > 1$ (2)

d) Min value $= 7$ since

$3(x+4)^2 \ge 0$ (1)

$\therefore 3(x+4)^2 + 7 \ge 7$.

8. a) $\lim\limits_{x\to 3} \dfrac{x^2-2x-3}{x-3}$ (2)

$= \lim\limits_{x\to 3} \dfrac{(x-3)(x+1)}{x-3}$

$= 3+1$

$= 4$

b) i) $\dfrac{d}{dx}(2x^3-6x+4)$

$= 6x^2 - 6$ (2)

ii) $\dfrac{d}{dx}\left(2x(x^2-1)^4\right)$

$= u'v + v'u$

let $u = 2x$

$u' = 2$

(3) $v = (x^2-1)^4$

$v' = 4(x^2-1)^3.2x$

$= 8x(x^2-1)^3$

$\therefore = 2(x^2-1)^4 + 8x(x^2-1)^3.2x$

$= 2(x^2-1)^3[x^2-1+8x^2]$

$= 2(x^2-1)^3(9x^2-1)$

Q8 Cont'd

c) $y = x^2 + 2x$ at $x = 1$ $y = (1)^2 + 2(1) = 3$

$y' = 2x + 2$ at $x = 1$ $y' = 2(1) + 2 = 4$

$\therefore$ Eq'n of tangent $y - y_1 = m(x - x_1)$

$y - 3 = 4(x - 1)$ (3)

$y - 3 = 4x - 4$

$\therefore 4x - y - 1 = 0$ is eq'n of tangent.

9. a) $\lim_{n\to\infty} \frac{2n^2+n-1}{3n^2-2n+1} = \lim_{n\to\infty} \frac{\frac{2n^2}{n^2} + \frac{n}{n^2} - \frac{1}{n^2}}{\frac{3n^2}{n^2} - \frac{2n}{n^2} + \frac{1}{n^2}}$

$= \lim_{n\to\infty} \frac{2 + \frac{1}{n} - \frac{1}{n^2}}{3 - \frac{2}{n} + \frac{1}{n^2}}$ (2)

$= \frac{2}{3}$

b) $f(x) = x^2$. $x_1 = 3$ $x_2 = 3 + h$

$y_1 = 9$ $y_2 = (3+h)^2$

i) $m_{secant} = \frac{y_2 - y_1}{x_2 - x_1}$ (2)

$= \frac{(3+h)^2 - 9}{(3+h) - 3}$

$= \frac{9 + 6h + h^2 - 9}{h}$

$= \frac{h(6+h)}{h}$

$= 6 + h$

ii) Eq'n : $y - 9 = (6+h)(x-3)$ (2)

$y - 9 = 6x - 18 + hx - 3h$

$\therefore 6x + hx - 3h - y - 9 = 0$

iii) $\lim_{h\to 0}(6x + hx - 3h - y - 9) = \lim_{h\to 0} 0$ (2)

$\therefore 6x - y - 9 = 0$ is eq'n of tangent.

c) $\lim_{x\to 3} \frac{x^2-4x+3}{x-3} = \lim_{x\to 3} \frac{(x-3)(x-1)}{x-3}$ (2)

$= \lim_{x\to 3} x - 1$

$= 2$

$= f(3)$. Yes it is cts as limit $= f(x)$.

10. a) i) $\angle DAC = 19 - 13$ (ext. $\angle$ of $\triangle ADC$)

$= 6°$

$\therefore$ In $\triangle ADC$ : $\frac{x}{\sin 13°} = \frac{1000}{\sin 6°}$ (2)

$\therefore x = \frac{1000 \sin 13°}{\sin 6°}$ as required.

ii) In $\triangle ACB$, $\sin 19° = \frac{h}{x}$

$\therefore h = x \sin 19°$

(2) $h = \frac{1000 \sin 13° \sin 19°}{\sin 6°}$ as required.

$\therefore h = 701$ (3 sig. fig.)

b) $f(x) = \frac{2}{x}$. $f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$

$= \lim_{x\to c} \frac{\frac{2}{x} - \frac{2}{c}}{x - c}$

$= \lim_{x\to c} \frac{2c - 2x}{xc} \times \frac{1}{x - c}$ (3)

$= \lim_{x\to c} \frac{-2(x-c)}{xc} \cdot \frac{1}{x-c}$

$= \frac{-2}{c^2}$ $\therefore f'(x) = \frac{-2}{x^2}$.

c) $x^2 + y^2 = 25$ Centre $= (0,0)$ $r = 5$.

$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$ $3x + 4y - 7 = 0$

$= \left| \frac{3(0) + 4(0) + -7}{\sqrt{3^2 + 4^2}} \right|$

$= \frac{7}{5}$

$= 1\frac{2}{5}$ (3)

$<$ radius of 5

$\therefore$ line cuts circle in 2 points since $d < r$.