St Catherine's School

Year: 11 – Yearly Examination
Subject: Mathematics
Time Allowed: 2 hours plus 5 mins reading time
Date: September 2005

Student Name :

Teacher's name:

Directions to candidates:

- All questions are to be attempted.
- All questions are of equal value.
- All necessary **working** must be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Each question should be started on a **new page**.
- Approved calculators and geometry sets are required.
- Hand in your work in **1 bundle**
  Exam paper + writing booklet

TEACHER'S USE ONLY Total Marks	
Questions 1-3	/36
Questions 4-6	/36
Question 7	/12
TOTAL	/84

Question 1: **(12 marks)** Marks

(a) Simplify $\sqrt{5}+\sqrt{125}$ 1

(b) Solve $5-(3-x)=4x$ 2

(c) Solve $9^{1-2t}=\frac{1}{81}$ 2

(d) Find and test all real solutions to the equation below:

$|2x-1|=x+2$ 3

(e) Factorise the following:

(i) $9a^2-1$ 1

(ii) x^2+5x+6 1

(ii) $am-an+bn-bm$ 2

Question 2: Start a new Page (12 marks) Marks

(a) Evaluate $2\pi\sqrt{\frac{l}{g}}$ if $l = 3.1$ and $g = 9.8$.

Give your answer correct to 2 significant figures 2

(b) (i) Rationalise the denominator of $\frac{3}{3-\sqrt{2}}$ 2

(ii) Find integers a and b such that $\frac{3}{3-\sqrt{2}} = a + b\sqrt{2}$ 1

(c) The local Council increased municipal rates by $5\frac{1}{2}\%$.

The new rate for a property is \$1865. What was the previous rate for this property?
Give your answer correct to the nearest dollar. 2

(d) Find the values of x that satisfy the inequality:

$$3 - x \le \frac{x-1}{2}$$ 2

(e) (i) Solve the simultaneous equations below: 2

$2x + y = 11$

$x - 2y = -2$

(ii) Write a sentence about the geometrical significance of the solution. 1

Question 3: Start a new Page (12 marks) Marks

(a) On separate sets of axes, sketch the curves, labelling all essential features.

(i) $y = |x - 4|$ 2

(ii) $x^2 + y^2 = 4$ 2

(iii) $xy = 4$ 2

(b) What is the domain of $xy = 4$ in (a) (iii) above? 1

(c) Explain why $x^2 + y^2 = 4$ in (a) (ii) above is not a function. 1

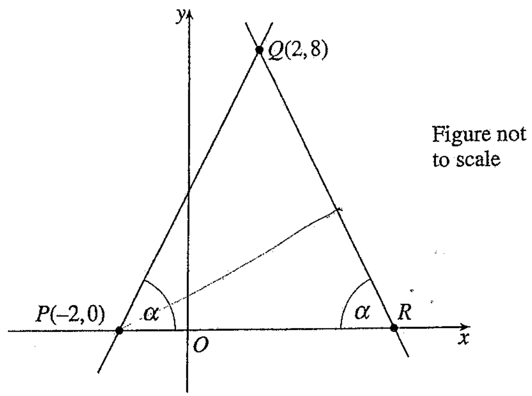
(d) What is the range of $y = x^2 - 4$? 1

(e) Is the function $f(x) = \frac{x}{x^2 - x^4}$ even, odd or neither?

Show full working for your answer. 3

Question 4: (12 marks) START A NEW PAGE **Marks**

(a)



In the diagram, P and Q have coordinates $(-2,0)$ and $(2,8)$ respectively, and $\angle QPR = \angle QRP = \alpha$.

Copy the diagram into your Writing Booklet.

(i) Find the gradient of PQ.
Hence, or otherwise show that $\tan\alpha = 2$ **2**

(ii) Show that the gradient of QR is -2. **1**

(iii) Show that the equation of QR is $2x + y - 12 = 0$. **2**

(iv) Find the co-ordinates of R. **1**

(v) Find the perpendicular distance from P to QR. **2**

(vi) On your diagram, shade in the region satisfying both the inequalities:

$y \le 2x + 4$ and $2x + y - 12 \ge 0$ **2**

(b) Solve $\log(w-1) + \log w = \log 2$ **3**

Question 5: Start a new Page (12 marks) **Marks**

(a) If $\sin\theta = \cos 35^\circ$, find the value of θ, if $0 \le \theta \le 90^\circ$ **1**

(b) Find the exact value of $\cot 210°$. **2**

(c) If $\cos\theta = -\frac{3}{8}$ and $\sin\theta > 0$, find the exact value of $\tan\theta$. **2**

(d) Prove that $\dfrac{\sec^2 x - 1}{\cos^2 x - 1} = -\sec^2 x$ **2**

(e) Solve the following equations, giving your answer to the nearest degree if $0^\circ \le \theta \le 360^\circ$:

(i) $\tan\theta = -\frac{2}{3}$ **2**

(ii) $2\cos 2\theta = 1$ **3**

Question 6: **Start a new Page (12 marks)** **Marks**

(a) Given the equation $2x^2 - 7x - 1 = 0$ with roots α and β, find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **2**

(b) Consider the equation $x^2 + (k-4)x + 9 = 0$. For what values of k does the equation have distinct real roots? **3**

(c) Solve the equation below for real values of x:

$$3^{2x} - 3^x - 6 = 0$$ **3**

(d) Given that $\log_5 3 = 0.683$ and $\log_5 2 = 0.431$, find the value of $\log_5 1.5$ **2**

Question 7: **Start a new Page (12 marks)** **Marks**

(a) From a lighthouse L a boat B bears 135°T and is at a distance of 10 km. From L a yacht Y bears 255°T and is at a distance of 12 km.

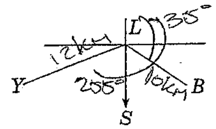


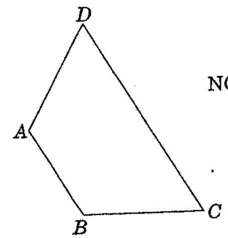
Figure not to scale

(i) Copy the diagram onto your writing booklet, marking on it the information supplied. **2**

(ii) Find the distance of boat B from yacht Y. Give your answer as a simplified surd. **3**

(iii) Find the bearing of yacht Y from boat B. Give your answer to the nearest degree. **3**

(b)



NOT TO SCALE

ABCD is a quadrilateral with $\angle CDA = \angle DCB$ and sides AD and BC equal.

(i) Copy the diagram of the figure onto your writing booklet and mark all the given information. **1**

(ii) Prove that ΔCAD is congruent to ΔDBC. **2**

(iii) Hence, or otherwise prove that $\angle DCA = \angle CDB$. **1**

END OF PAPER

Q3 (continued)

(c) $x^2+y^2=4$ is not a function because for every x value, there is more than one y-value

OR vertical line test cutting the circle twice $\boxed{1/1}$

(d) $y=x^2-4$

R: For all real y, $y \geq -4$ $\boxed{1/1}$

(e) $f(x)=\frac{x}{x^2-x^4}$

test for even: $f(x)=f(-x)$

$f(-x)=\frac{-x}{(-x)^2-(-x)^4}$

$=\frac{-x}{x^2-x^4}$ ✓

$f(x) \neq f(-x)$ ∴ not even

test for odd: $f(-x)=-f(x)$

$f(-x)=\frac{-x}{x^2-x^4}$

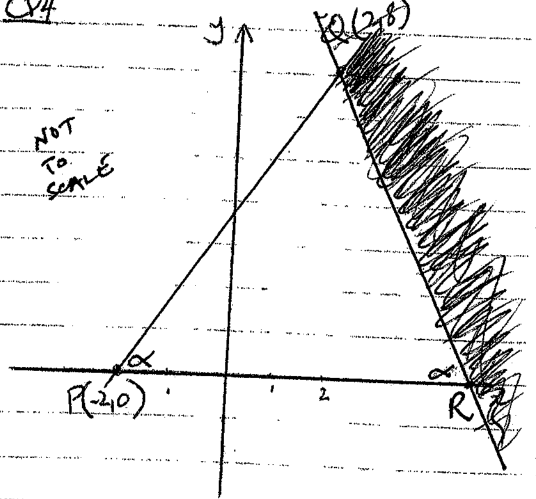
$-f(x)=-\left(\frac{x}{x^2-x^4}\right)$

$=\frac{-x}{x^2-x^4}$ ✓

$f(-x)=-f(x)$ ∴ odd ✓ $\boxed{/2}$

part units

Q4



(i) $m_{PQ}=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{8-0}{2-(-2)}$

$=\frac{8}{4}$

$=2$ ✓

using $m=\tan\alpha$

∴ $\tan\alpha=2$ ✓ $\boxed{/2}$

(ii) $\angle QRx=180°-\angle QRP=180°-\alpha$

Gradient QR $=\tan\angle QRx$

$=\tan(180°-\alpha)$ ✓

$=-\tan\alpha$

$=-2$ ✓ $\boxed{/1}$

Q4 (iii)

$m_{QR}=-2$

$Q(2,8)$

$y-y_1=m(x-x_1)$

$y-8=-2(x-2)$ ✓

$y-8=-2x+4$

$2x+y-8-4=0$ ✓

$2x+y-12=0$

(iv) R lies on line QR and the x-axis

∴ the y-coordinate is 0 ✓

i.e. when $y=0$

$2x+0-12=0$

$2x=12$

$x=6$ ✓

∴ $R(6,0)$ $\boxed{/1}$

(v) $P=\left|\frac{Ax+By+C}{\sqrt{A^2+B^2}}\right|$ ✓

$P(-2,0)$ and QR $2x+y-12=0$

$P=\left|\frac{2(-2)+0-12}{\sqrt{(2)^2+1^2}}\right|$ ✓

$=\left|\frac{-4-12}{\sqrt{4+1}}\right|$ ✓

$=\left|\frac{-16}{\sqrt5}\right|$ ✓

$=\frac{16}{\sqrt5}\times\frac{\sqrt5}{\sqrt5}=\frac{16\sqrt5}{5}$ units $\boxed{/2}$

Q4 (b)

$\log(w-1)+\log w=\log 2$

$\log(w-1)w=\log 2$ ✓

$w(w-1)=2$ ✓

$w^2-w=2$

$w^2-w-2=0$ ✓

$(w+1)(w-2)=0$ ✓

$w=-1, 2$

Test for $w=-1$ ∴ no solution ✓ | Test for $w=2$ ∴ only solution is $w=2$

Q5

(a) If $\sin\theta=\cos 35°$

using $\sin(90°-\theta)=\cos\theta$ ✓

∴ $\sin 90°-35°$

$=\sin 55°$

∴ $\theta=55°$ ✓ $\boxed{/1}$

(b) $\cot 210°$

$\tan 210°$ ✓

$=\tan 180°+30°$ ✓

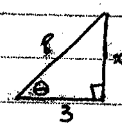
Basic angle is 30°

$\tan 30°=\frac{1}{\sqrt3}$ ✓

∴ $\cot 210°=\frac{\sqrt3}{1}=\sqrt3$ ✓

(c) $\cos\theta=\frac{-3}{8}$ and $\sin\theta>0$

∴ solution must be in the 2nd quadrant ✓



$x^2=8^2-3^2$

$=64-9$ ✓

$x^2=55$

$x=\pm\sqrt{55}$ ✓ ∴ $\tan\theta=-\frac{\sqrt{55}}{3}$

$\boxed{/3}$ 3

Year 11 - 2unit - Yearly 2005 - Solutions

7 part marks

Q1(a) $\sqrt{5} + \sqrt{125}$

$= \sqrt{5} + \sqrt{25\times 5}$

$= \sqrt{5} + 5\sqrt{5}$ ✗

$= 6\sqrt{5}$ ✗ $\boxed{/1}$

(b) $5 - (3-x) = 4x$

$5 - 3 + x = 4x$ ✓

$-4x + x = -2$

$-3x = -2$

$x = \frac{2}{3}$ ✓ $\boxed{/2}$

(c) $9^{1-2t} = \frac{1}{81}$

$9^{1-2t} = 81^{-1}$

$9^{1-2t} = (9^2)^{-1}$

$9^{1-2t} = 9^{-2}$ ✓

$1-2t = -2$

$-2t = -2-1$

$-2t = -3$

$t = \frac{3}{2}$ ✓ $\boxed{/2}$

(d) $|2x-1| = x+2$

Case 1:

$2x - 1 = x+2$

$2x - x = 2+1$

$x = 3$ ✓

case 2:

$-(2x-1) = x+2$

$-2x+1 = x+2$

$-2x - x = 2-1$

$-3x = 1$

$x = -\frac{1}{3}$ ✓

Q1(d) continued

test $x=3$

$LHS = |2x-1|$

$= |2(3)-1|$

$= |5|$

$= 5$

$RHS = x+2$

$= 3+2$

$= 5$

$LHS = RHS$ ✗

$\therefore x = 3$ is a solution

test $x = -\frac{1}{3}$

$LHS = |2x-1|$

$= |2(-\frac{1}{3})-1|$

$= |-\frac{2}{3}-1|$

$= |-1\frac{2}{3}|$

$= 1\frac{2}{3}$

$RHS = x+2$

$= -\frac{1}{3}+2$

$= 1\frac{2}{3}$

$LHS = RHS$ ✗

$\therefore x = 1\frac{2}{3}$ is a solution $\boxed{/3}$

(e)(i) $9a^2 - 1$

$= (3a)^2 - (1)^2$

$= (3a-1)(3a+1)$ ✓ $\boxed{/1}$

(ii) x^2+5x+6

$= (x+3)(x+2)$ ✓ $\boxed{/1}$

Q1(e) continued

(iii) $am - an + bn - bm$ ✓

$= a(m-n) - b(m-n)$

$= (a-b)(m-n)$ ✓ $\boxed{/2}$

Q2

(a) $2\pi\sqrt{\frac{l}{g}}$

$= 2\pi\sqrt{\frac{3.1}{9.8}}$

$= 3.53384\ldots$ ✓

$= 3.5$ (to 2 sig. fig.) ✓ $\boxed{/2}$

(b)(i) $\frac{3}{3-\sqrt{2}}$

$= \frac{3}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$ ✓

$= \frac{9+3\sqrt{2}}{9-2}$

$= \frac{9+3\sqrt{2}}{7}$ ✓ $\boxed{/2}$

(ii) From (i),

$\frac{3}{3-\sqrt{2}} = \frac{9+3\sqrt{2}}{7}$

$= \frac{9}{7} + \frac{3\sqrt{2}}{7}$

$a + b\sqrt{2}$

$\therefore a = \frac{9}{7}$ ✗ and $b = \frac{3}{7}$ ✗ $\boxed{/1}$

continued

Q2(c) Let the old rate be represented by 100%

$\therefore$ new rate $= 105\frac{1}{2}\% = \$1865$

$1\% = \$1865 \div 105.5$ ✓

$100\% = \frac{1865}{105.5} \times 100$

$= \$1767.77 = 1768$ ✓ (nearest dollar) $\boxed{/2}$

(d) $3 - x \leq \frac{x-1}{2}$

$2(3-x) \leq \frac{x-1}{2}(\times 2)$

$6 - 2x \leq x - 1$ ✓

$-2x - x \leq -1 - 6$

$-3x \leq -7$

$x \geq \frac{7}{3}$ ✓ $\boxed{/2}$

(e)(i) $2x + y = 11$ — ①

$x - 2y = -2$ — ② $\times 2$

$2x + y = 11$ — ①

$2x - 4y = -4$ — ②'

$5y = 15$

$y = 3$ ✓

when $y = 3$, $x = ?$

$x - 2y = -2$

$x - 6 = -2$

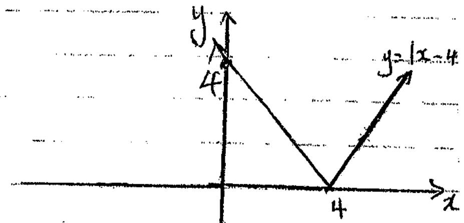
$x = -2+6$ ✓

$x = 4$ $\boxed{/2}$

7 part marks

(ii) The geometrical significance of $(4,3)$ is that $(4,3)$ is the pt of intersection of the 2 lines ✓ $\boxed{/1}$

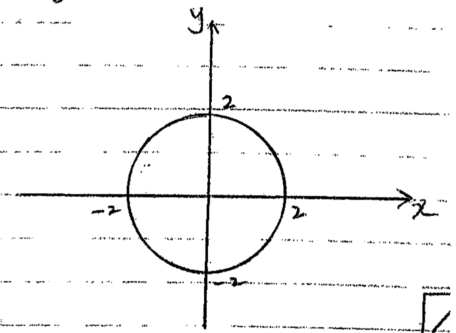
Q3(a)(i) $y = |x-4|$



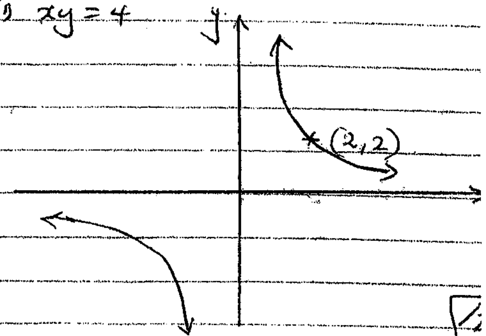
1 for sh

1 for feat

(ii) $x^2+y^2=4$



(iii) $xy = 4$



(b) D: For all real x, $x \neq 0$

Q5 (continued)

(d) LHS $= \dfrac{\sec^2 x - 1}{\cos^2 x - 1}$

$= \dfrac{\tan^2 x}{-\sin^2 x}$

$= \dfrac{\sin^2 x}{\cos^2 x} \div -\sin^2 x$

$= \dfrac{\sin^2 x}{\cos^2 x} \times \dfrac{-1}{\sin^2 x}$

$= -\dfrac{1}{\cos^2 x}$

$= -\sec^2 x = $ RHS

/2

(e) (i) $\tan\theta = -\dfrac{2}{3}$

Basic $\theta = 33°41'$

$= 34°$ (nearest degree)

S | A
T | C

tan ratio is −ve in 2nd + 4th quad.

$\therefore \theta = 180° - 34°, 360° - 34°$

$= 146°, 326°$

/2

(ii) $2\cos 2\theta = 1$

$\cos 2\theta = \dfrac{1}{2}$

$2\theta = 60°$ however $0° \le \theta \le 360°$ (×2)

i.e. $0° \le 2\theta \le 720°$

cos ratio is +ve in 1st + 4th quad.

$\therefore$ Solution is

$\theta = 60°, 360° - 60°, 60° + 360°, 300° + 360°$

$30°, 150°, 210°, 330°$

/3

Q6

(a) $2x^2 - 7x - 1 = 0$

$a = 2$
$b = -7$
$c = -1$

(i) $\alpha + \beta = -b/a$

$= -\dfrac{-7}{2}$

$= 7/2$

/1

(ii) $\alpha\beta = c/a$

$= \dfrac{(-1)}{2}$

$= -\dfrac{1}{2}$

/1

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

$= \dfrac{\beta + \alpha}{\alpha\beta}$

$= \dfrac{7/2}{-1/2}$

$= \dfrac{7}{2} \times -\dfrac{2}{1}$

$= -7$

/2

(b) $x^2 + (k-4)x + 9 = 0$

$\Delta > 0$ (distinct real roots)

$\Delta = b^2 - 4ac$

$= (k-4)^2 - 4 \times 1 \times 9$

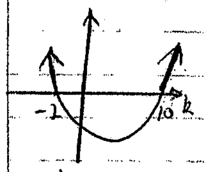
$= k^2 - 8k + 4^2 - 36$

$= k^2 - 8k - 20$

Q6 (b) continued

$k^2 - 8k - 20 > 0$

$(k-10)(k+2) > 0$



$k < -2$ or $k > 10$

/3

(c) $3^{2x} - 3^x - 6 = 0$

let $m = 3^x$

$m^2 - m - 6 = 0$

$(m-3)(m+2) = 0$

$m = 3, -2$

However $3^x = m$

i.e. $3^x = 3$

$\therefore x = 1$

and $3^x = -2$

$\therefore$ no solution

/3

$\therefore x = 1$ is the only solution

(d) $\log_5 1.5$

$\log_5 \dfrac{3}{2}$

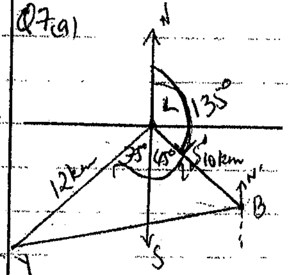
$= \log_5 3 - \log_5 2$

$= 0.683 - 0.431$

$= 0.252$

/2

Q7 (a)



(b) $\angle SLB = 45°$ (from bearing 150° for L to B)

$\angle SLY = 75°$ (from bearing 255° for L to Y)

$\therefore \angle BLY = 45° + 75°$

$= 120°$ 1

using the cosine rule to find YB

$YB^2 = YL^2 + LB^2 - 2YL.LB \cos \angle YLB$

$= 12^2 + 10^2 - 2(12)(10)\cos 120°$ 1

$YB = \sqrt{244 - 240\cos 120°}$

$= \sqrt{364}$

$= \sqrt{91 \times 4}$ $\frac{1}{2}$

$= 2\sqrt{91}$ km $\frac{1}{2}$

(c)(i) $\dfrac{\sin \angle LBY}{12} = \dfrac{\sin 120°}{2\sqrt{91}}$ 1

$2\sqrt{91} \sin \angle LBY = 12 \sin 120°$

$\sin \angle LBY = \dfrac{12 \sin 120°}{2\sqrt{91}}$

$= 0.5447\ldots$

$\angle LBY = 33.00\ldots°$

$= 33°$ (to the nearest degree) 1

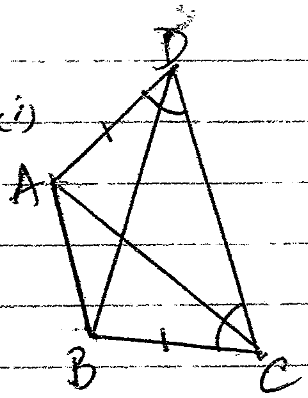
$\angle LBN' = 45°$ (alternate $\angle$ to $\angle SLB$)

$\angle N'BY = 45° + 33° = 78°$

$\therefore$ bearing from B to Y $= 360° - 78°$

$= 282°$T (to the nearest degree) 1

Q7(b)(i)



1

(ii) In Δs DAC and CBD,

AD = BC (given)

$\angle ADC = \angle BCD$ (given)

DC is common

$\therefore \Delta DAC \equiv \Delta CBD$ (SAS)

½

(iii) Since Δs DAC and CBD are congruent, all corresponding sides and angles are equal

$\therefore \angle ACD = \angle BDC$ 1