Student Number: ____________

St. Catherine's School
Waverley

April 2008
HSC ASSESSMENT TASK
MID COURSE EXAMINATION

Extension 1 Mathematics

Time allowed: **2 hours**

Reading Time: **5 mins**

INSTRUCTIONS

- All questions should be attempted on the separate paper provided
- All necessary working should be shown
- Marks for each part of a question are indicated
- Start each question on a new page
- Approved scientific calculators and drawing templates may be used
- Marks may be deducted for untidy or poorly arranged work.
- Standard Integrals are provided on Page 10 of the paper

Question 1 **(12 Marks)** (Start a new page)

a) Solve the inequality $\frac{x}{2-x} \leq 4$ 2

b) Differentiate $\log_e(\sin^3 x)$ writing your answer in simplest form. 2

c) Evaluate $\int_0^{\frac{1}{2}} \frac{3dx}{\sqrt{16-9x^2}}$, correct to three decimal places 2

d) Use the substitution $u = 1-2x$ to find $\int_0^{\frac{1}{2}} 2x\sqrt{1-2x}\ dx$. 2

e) If $y = 10^x$, find $\frac{dy}{dx}$ when $x = 1$. 2

f) When the polynomial $P(x) = x^3 + ax + 1$ is divided by $(x+2)$ the remainder is 3. Find the value of a 2

Question 2 **(12 Marks)** (Start a new page) **Marks**

(a) Find $\frac{d}{dx}(x\sin^{-1}2x)$ **2**

(b) The parametric equations of a curve are given by $x=t^2, y=t^3+t$. Find the Cartesian equation of the curve with rational coefficients. **2**

(c) Evalute $\int_0^{\frac{\pi}{3}} \tan^2 x\, dx$ **2**

(d) The interval *AB* has end points *A* (5, 4) and *B* (*x*, *y*). The point *P* (−1, 3) divides *AB* internally in the ratio 2:3. Find the coordinates of *B*. **2**

(e) Evaluate $\lim_{x\to 0}\left(\frac{\sin 3x}{4x}\right)$. **2**

(f) Find, correct to the nearest degree, the ***obtuse*** angle between the lines $x+y-4=0$ and $y=2x+1$. **2**

Question 3 **(12 Marks)** (Start a new page) **Marks**

(a) (i) Divide the polynomial $P(x)=x^3+x^2+3x+4$ by $A(x)=x^2+3$ and express the result in the form **2**

$$\frac{P(x)}{A(x)}=Q(x)+\frac{R(x)}{A(x)}$$

(ii) Hence evaluate $\int_0^1 \frac{P(x)}{A(x)}dx$ **2**

(b) $P(0,1)$ and $Q(1,e)$ are points on the graph of $y=e^x$. Find to the nearest minute, the acute angle formed by the tangents to the curve at P and Q. **2**

(c) Solve $\cos\theta-\sqrt{3}\sin\theta=1$ in the domain $0\le\theta\le 2\pi$ **2**

(d) Use the factorization of a^3+b^3 to prove the following identity: **2**

$$\frac{2\sin^3 A+2\cos^3 A}{\sin A+\cos A}=2-\sin 2A \quad (\sin A+\cos A\neq 0)$$

(e) (i) Write down the expansion of $\cos(A+B)$ **1**

(ii) Hence find the exact value of $\cos\left(\frac{7\pi}{12}\right)$ in simplest surd form, with a rational denominator. **1**

Question 4 **(12 Marks)** (Start a new page) **Marks**

(a) (i) Differentiate $y = x\cos^{-1}x - \sqrt{1-x^2}$ **2**

(ii) Hence evaluate $\int_0^1 \cos^{-1}x \, dx$ **2**

(b) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. Suppose the tangents at P and Q intersect at T. Let S be the focus of the parabola.

(i) Show that the equation of the tangent at P is given by $y = px - ap^2$ **2**

(ii) Find the coordinates of T. **2**

(iii) Show that $SP = a(p^2+1)$. **1**

(iv) If P and Q move such that $SP + SQ = 4a$ find the locus of T **3**

Question 5 **(12 Marks)** (Start a new page) **Marks**

(a) Determine the exact value of $\sin\left[2\cos^{-1}\left(\frac{12}{13}\right)\right]$. **2**

(b) (i) Show that the equation $x^3 + 2x - 7 = 0$ has a root $x = \alpha$ which lies between $x = 1$ and $x = 2$. **1**

(ii) By taking $x = 1\cdot 5$ as an initial approximation to the root of $x^3 + 2x - 7 = 0$, in the interval $1 < x < 2$, use one application of Newton's method to find a second approximation to this root. **2**

(c) Evaluate $\int_2^{10} \frac{x}{\sqrt{x-1}} \, dx$ using the substitution $x = t^2 + 1$ **2**

(d) Consider the function $y = \frac{1}{2}\cos^{-1}(x-1)$

(i) Find the domain and range of the function. **2**

(ii) Sketch *neatly* the graph of the function, showing clearly the coordinates of the end points. **1**

(iii) The region in the first quadrant bounded by the curve $y = \frac{1}{2}\cos^{-1}(x-1)$ and the coordinate axes is rotated through 360° about the y axis **3**

Find the volume of the solid of revolution, giving your answer in simplest *exact* form.

Question 6 **(12 Marks)** (Start a new page) **Marks**

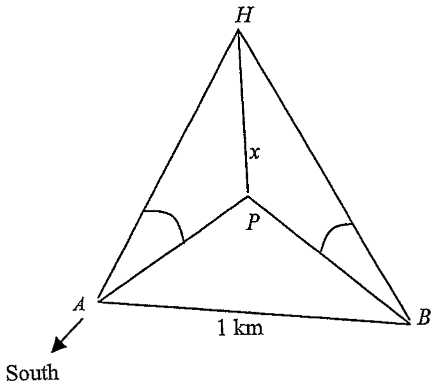
(a) Suppose $\frac{\alpha}{r}$, α and αr are the real roots of the cubic equation $2x^3-3x^2-3x+2=0$.

(i) Write down the value of the sum of all three roots. 1

(ii) Write down the value of the product of all three roots. 1

(i) Deduce that r can take on two real non-zero values and find them. 2

(b) Anna (A) is standing due south of Phillip (P) who is assisting an injured bush walker. A rescue helicopter (H) is hovering directly over P and lowering a stretcher. Anna measures the angle of elevation of the helicopter to be 60° from her position. Belinda (B) is 1 kilometre due east of A and measures the angle of elevation of the helicopter to be 30°. The height of the helicopter above P is x metres.



NOT TO SCALE

(i) Write expressions for both AP and BP in terms of x. 1

(ii) Hence or otherwise find the height of the helicopter correct to the nearest 10 m. 3

(c) Use the Principle of Mathematical Induction to show that $9^{n+2}-4^n$ is divisible by 5 for all positive integers n. 4

Question 7 **(12 Marks)** (Start a new page) **Marks**

(a) (i) State the domain and range for $f(x)=4-\sqrt{x-1}$. 2

(ii) Find the inverse function $f^{-1}(x)$ and state the domain and range for which it exists. 2

(iii) Sketch the graph of $f(x)=4-\sqrt{x-1}$ and its inverse function $f^{-1}(x)$ on the same number plane. 2

(b) Consider the function

$$f(x)=\frac{x^2-1}{x^2+1}$$

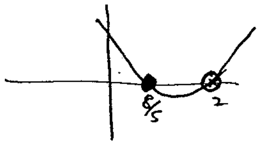
(i) Show that $f(x)=1-\frac{2}{x^2+1}$ 1

(ii) show through calculus that $f(x)$ has only one stationary point and find its coordinates and nature. 1

(iii) Sketch the curve $y=f(x)$ showing intercepts and asymptotes. 2

(vii) Find the exact area enclosed between the curve $y=f(x)$ and the x-axis. 2

End of Paper

Solutions	Marks	Comment
Q1 a) $\frac{x}{2-x} \le 4$ $\quad x \neq 2$		
$x(2-x) \le 4(2-x)^2$		$-\frac{1}{2}$ for $x \ge 2$
$2x - x^2 \le 16 - 16x + 4x^2$		
$\therefore 5x^2 - 18x + 16 \ge 0$ 		
$(5x-8)(x-2) \ge 0$	1	
$\therefore \quad x \le 8/5$ or $x > 2$	1	
b) $\frac{d}{dx} \log(\sin^3 x) = \frac{1}{\sin^3 x} . 3\sin^2 x . \cos x$ *	2	
$= \frac{3\cos x}{\sin x}$		
$= 3\cot x$		
c) $\int_0^{1/2} \frac{3dx}{\sqrt{16-9x^2}} = \int_0^{3/2} \frac{du}{\sqrt{4^2-u^2}}$ * $\quad u = 3x$ $du = 3dx$ $dx = \frac{du}{3}$	1	→ ½ mark for * and ½ mark f
$= \left[\sin^{-1}\frac{u}{4}\right]_0^{3/2}$ ● $\quad x=0 \quad u=0$ $x=1/2 \quad u = 3/2$		
$= \sin^{-1}\frac{3}{8} - 0$	1	
$= 0.384$ (3 d.p.)		
d) $\int_0^{1/2} 2x\sqrt{1-2x}\,dx$ $\quad u = 1-2x$ $\therefore 2x = 1-u$		1 mark fo *
$= -\frac{1}{2}\int_1^0 (1-u)\sqrt{u}\,du$ * $\quad du = -2dx$ $\frac{du}{-2} = dx$		
$= -\frac{1}{2}\int_1^0 u^{1/2} - u^{3/2}\,du$ * $\quad x=0 \quad u=1$ $x = 1/2 \quad u=0$	2	½ mark for correct integrati
$= -\frac{1}{2}\left[\frac{2u^{3/2}}{3} - \frac{2u^{5/2}}{5}\right]_1^0$		
$= -\frac{1}{2}\left[0 - \left(\frac{2}{3} - \frac{2}{5}\right)\right]$		½ mark for correct answer
$= -\frac{1}{2}\left[-\left(\frac{4}{15}\right)\right] = \frac{4}{30} = \frac{2}{15}$		

Solutions	Marks	Comments
Q1 e) $y = 10^x$		1 mark for correct solution up to *
$x = \log_{10} y$		
$x = \frac{\log_e y}{\ln 10}$	2	
$\frac{dx}{dy} = \frac{1}{y \ln 10}$ *		
$\therefore \frac{dy}{dx} = y\ln 10$		½ mark for correct $\frac{dy}{dx}$ and
$= 10^x \ln 10$		
$\therefore$ when $x=1$ $\frac{dy}{dx} = 10\ln 10$		½ mark for correct final answ
f) $P(-2) = 3$ $\quad$ now $P(-2) = -8 - 2a + 1$	1	
$\therefore -8 - 2a + 1 = 3$	½	
$-10 = 2a$	½	
$-5 = a$		
Question 2		
a) $\frac{d}{dx} x\sin^{-1}2x = \sin^{-1}2x + x . \frac{1}{\sqrt{1-4x^2}} . 2$	2	
$= \sin^{-1}2x + \frac{2x}{\sqrt{1-4x^2}}$		
b) $x = t^2$ ① $\quad y = t^3 + t$ ②		
from ① $t = \sqrt{x}$	½	
Sub in ② $y = x\sqrt{x} + \sqrt{x}$	$1\frac{1}{2}$	
$y = \sqrt{x}(x+1)$ up to here is ok		
$y^2 = x(x^2+2x+1)$		
$y^2 = x^3 + 2x^2 + x$		

Course:

Marking Scheme for Task:

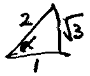
Academic Year: 2007-8

Solutions	Marks	Comments
Question 2 c) $\int_0^{\frac{\pi}{3}} \tan^2 x \, dx$		
$= \int_0^{\frac{\pi}{3}} (\sec^2 x - 1) \, dx$	½	
$= \left[\tan x - x\right]_0^{\frac{\pi}{3}}$	1	
$= \sqrt{3} - \frac{\pi}{3}$	½	
d). $\underset{(5,4)}{A}$ $\quad 2 \quad$ $\underset{(-1,3)}{P}$ $\quad 3 \quad$ $\underset{(x,y)}{B}$		
$\therefore \frac{15+2x}{5} = -1$ and $\frac{12+2y}{5} = 3$ $15+2x = -5 \qquad 12+2y = 15$ $2x = -20 \qquad 2y = 3$ $x = -10 \qquad y = \frac{3}{2}$	2	
$\therefore B(-10, \frac{3}{2})$		
e). $\lim\limits_{x \to 0} \frac{\sin 3x}{4x} = \frac{3}{4} \lim\limits_{x \to 0} \frac{\sin 3x}{3x}$	1	*1 mark for $\lim\limits_{x \to 0} \frac{\sin x}{x} =$
$= \frac{3}{4}$	1	
f). $M_2 = -1 \quad M_1 = 2$		
(acute) $\tan\theta = \left\lvert \frac{2+1}{1 [illegible] 2} \right\rvert$	½	
$\tan\theta = 3 \qquad \theta = 72°$ (nearest degree)	1	
$\therefore$ obtuse angle is $108°$ (nearest degree)	½	

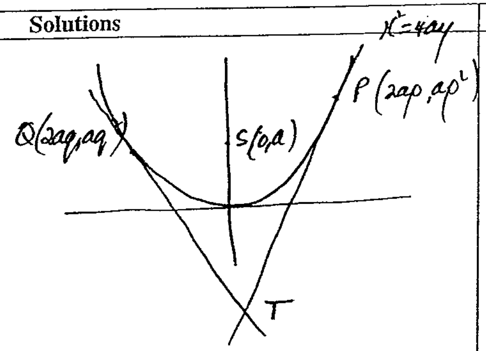
Course:

Marking Scheme for Task:

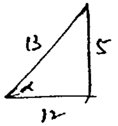
Academic Year: 2007-8

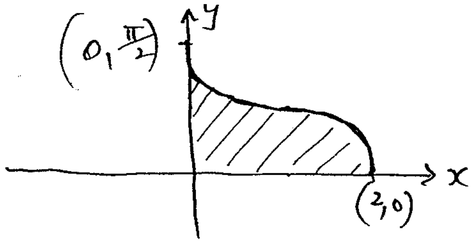
Solutions	Marks	Comments
Question 3		
a) (i) $x^3+x^2+3x+4 = (x^2+3)(x+1)+1$	1½	
$\therefore \frac{x^3+x^2+3x+4}{x^2+3} = x+1+\frac{1}{x^2+3}$	½	
$x^2+3 \,\overline{)\, x^3+x^2+3x+4}$ quotient $x+1$ $-\underline{x^3+0x^2+3x}$ x^2+4 $-\underline{x^2+3}$ 1		
(ii) $\int_0^1 \left(x+1+\frac{1}{x^2+3}\right) dx$	½	
$= \left[\frac{x^2}{2}+x+\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right]_0^1$	1	
$= \left(\frac{1}{2}+1+\frac{1}{\sqrt{3}}\cdot\frac{\pi}{6}\right)$ * up to here is ok	½	
$= \frac{3}{2}+\frac{\sqrt{3}\pi}{18} \longleftarrow$ Note: $\frac{\pi}{6\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}\pi}{18}$		
b). $y = e^x \qquad y' = e^x$	½	
at $P(0,1)$ $y' = 1$ (m_1)	½	
at $Q(1,e)$ $y' = e$ (m_2)		
$\therefore \tan\theta = \left\lvert \frac{1-e}{1+e} \right\rvert = 0.462117157$	½	
	½	
$\therefore \theta = 25°$		
c) $\cos\theta - \sqrt{3}\sin\theta = 1 \qquad \cos(\theta+\alpha) = \cos\theta\cos\alpha - \sin\theta\sin\alpha$		
$2\cos\left(\theta+\frac{\pi}{3}\right) = 1$ ✓  $\therefore \alpha = \tan^{-1}\sqrt{3}$ $\alpha = \frac{\pi}{3}$		'✓' means ½ mark
$\theta + \frac{\pi}{3} = \cos^{-1}\frac{1}{2}$ ✓		
$\theta + \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \ldots$ ✓		
$\theta = 0, \frac{4\pi}{3}, 2\pi$ ✓		

Solutions	Marks	Comment
Question3 d) $a^3+b^3=(a+b)(a^2-ab+b^2)$		
$\therefore \frac{2(\sin^3A+\cos^3A)}{\sin A+\cos A}=\frac{2\cancel{(\sin A+\cos A)}(\sin^2A-\sin A\cos A+\cos^2A)}{\cancel{(\sin A+\cos A)}}$	1	
$=2(1-\sin A\cos A)$	½	
$=2-2\sin A\cos A$	½	
$=2-\sin 2A$		
e)(i) $\cos(A+B)=\cos A\cos B-\sin A\sin B$	1	
(ii) $\cos\frac{7\pi}{12}=\cos\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)$		
$=\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$	½	
$=\cos\frac{\pi}{4}\cos\frac{\pi}{3}-\sin\frac{\pi}{4}\sin\frac{\pi}{3}$		
$=\frac{1}{\sqrt{2}}\cdot\frac{1}{2}-\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}$ up to here is ok	½	
$=\frac{1-\sqrt{3}}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$		
$=\frac{\sqrt{2}-\sqrt{6}}{4}$		
Question 4: a)(i) $y=x\cos^{-1}x-\sqrt{1-x^2}$	1	for product rule with $x\cos^{-1}x$.
$y'=\cos^{-1}x-x\frac{1}{\sqrt{1-x^2}}-\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot-2x$		
$=\cos^{-1}x-\frac{x}{\sqrt{1-x^2}}+\frac{x}{\sqrt{1-x^2}}$	1 for	$\sqrt{1-x^2}$
$=\cos^{-1}x$		
$\therefore\int_0^1\cos^{-1}x\,dx=\left[x\cos^{-1}x-\sqrt{1-x^2}\right]_0^1$	1	
$=0+1=1$	1	

Solutions	Marks	Comments
Question 4 b) 		
(i) $y=\frac{x^2}{4a}$	½	
$y'=\frac{x}{2a}$	½	
at P $y'=p$		
$\therefore$ equation of tangent is $y-ap^2=p(x-2ap)$		
$y-ap^2=px-2ap^2$	1	
$y=px-ap^2$		
(ii) $y=px-ap^2$ —— ①		
$y=qx-aq^2$ —— ②		
①-② $0=(p-q)x-ap^2+aq^2$		
$0=(p-q)x-a(p-q)(p+q)$	1	
$x=a(p+q)$		
Sub in ① $y=ap(p+q)-ap^2$	1	
$y=apq$		
$\therefore T\left(a(p+q),apq\right)$		

Solutions	Marks	Comments
Q4 b) (iii) $SP = \sqrt{(2ap-0)^2+(ap^2-a)^2}$		
$SP = \sqrt{4a^2p^2+a^2p^4-2a^2p^2+a^2}$		
$SP = \sqrt{a^2p^4+2a^2p^2+a^2}$	½✓	
$SP = \sqrt{(ap^2+a)^2}$		
$\therefore SP = ap^2+a$	½✓	
$= a(p^2+1)$		
(iv) $SP+SQ = a(p^2+1)+a(q^2+1) = 4a$		
$\therefore p^2+1+q^2+1 = 4$		
$\underline{p^2+q^2 = 2}$	1	
$T(a(p+q), apq)$		
$x = a(p+q) \qquad y = apq$		
$x^2 = a^2(p+q)^2$		
$x^2 = a^2(p^2+q^2+2pq)$		
$\therefore x^2 = a^2\left(2+\frac{y}{a}\right)$	1	
$x^2 = 2a^2+ay$		
$x^2 = a(y+2a)$		
parabola vertex $(0,-2a)$ focal length $= \frac{a}{4}$	1	

Solutions	Marks	Comments
Question 5 a) $\sin\left(2\cos^{-1}\frac{12}{13}\right)$		
$= 2\sin\left(\cos^{-1}\frac{12}{13}\right)\cos\left(\cos^{-1}\frac{12}{13}\right)$ 	1	
$= 2 \cdot \frac{5}{13} \cdot \frac{12}{13}$	1	
$= \frac{120}{169}$		
b) (i) $f(1) = -4 < 0 \qquad f'(x) = 3x^2+2 \rightarrow$	½	
$f(2) = 5 > 0 \rightarrow$	½✓	
$\therefore$ root lies between $x=1$ and $x=2$		
(ii) $x_1 = x - \frac{f(x)}{f'x} \qquad f(1.5) = -0.625$	1	½ for f(1.5
$f'(1.5) = 8.75$		½ for f'(1.5
$= 1.5 + \frac{0.625}{8.75}$	1	½ for 1.5+
$\therefore x_1 = 1.57 \ (2dp)$		½ for 1.57
c) $\int_2^{10} \frac{x}{\sqrt{x-1}}\,dx \qquad x = t^2+1$		
$t^2 = x-1$		
$= \int_1^3 \frac{t^2+1}{\cancel{t}}\,2\cancel{t}\,dt \qquad t = \sqrt{x-1}$		
$x = t^2+1$		
$= 2\int_1^3 (t^2+1)\,dt \qquad x=2 \quad t=1 \rightarrow$		1 mark for getting to the line
$x=10 \quad t=3$		
$= 2\left[\frac{t^3}{3}+t\right]_1^3 \qquad \frac{dx}{dt} = 2t$	½	
$dx = 2t\,dt$		
$= 2\left[(9+3)-\left(\frac{1}{3}+1\right)\right]$		
$= 2\left[10\frac{2}{3}\right]$	½✓	
$= 20\frac{4}{3}$ or $21\frac{1}{3}$		

Solutions	Marks	Comment
Q5 d) $y = \frac{1}{2}\cos^{-1}(x-1)$		
(i) D: $-1 \le x-1 \le 1$		
$0 \le x \le 2$	1	
R: $0 \le y \le \frac{\pi}{2}$	1	
(ii) 	1	
(iii) $V = \pi\int_a^b x^2\,dy$ $\quad x-1 = \cos 2y$		
$x = \cos 2y + 1$		
$= \pi\int_0^{\frac{\pi}{2}}(\cos 2y + 1)^2\,dy$	½	
$= \pi\int_0^{\frac{\pi}{2}}(\cos^2 2y + 2\cos 2y + 1)\,dy$		
$= \pi\int_0^{\frac{\pi}{2}}\left[\frac{1}{2}(1+\cos 4y) + 2\cos 2y + 1\right]dy$	½	
$= \frac{\pi}{2}\int_0^{\frac{\pi}{2}}(1 + \cos 4y + 4\cos 2y + 2)\,dy$		
$= \frac{\pi}{2}\left[3y + \frac{1}{4}\sin 4y + 2\sin 2y\right]_0^{\frac{\pi}{2}}$	½	
$= \frac{\pi}{2}\left[\left(\frac{3\pi}{2} + 0 + 0\right) - (0+0+0)\right]$		
$= \frac{3\pi^2}{4}\,U^3$	½	

Solutions	Marks	Comments
Question 6: a) (i) Sum of roots $= \frac{3}{2}$	1	
(ii) Product of roots $= -1$	1	
(iii) product of roots $= \frac{\alpha}{r} \times \alpha \times \alpha r = \alpha^3$		
$\therefore \alpha^3 = -1$		
$\therefore \alpha = -1$	1	
$\therefore$ roots are $-\frac{1}{r}, -1, -r$		
Now sum of roots $= -\frac{1}{r} - 1 - r = \frac{3}{2}$		
$\therefore -1 - r - r^2 = \frac{3r}{2}$		
$\therefore -2 - 2r - 2r^2 = 3r$		
$\therefore 2r^2 + 5r + 2 = 0$		
$(2r+1)(r+2) = 0$		
$\therefore r = -\frac{1}{2}, -2$	1	
b) (i) $AP = \frac{x}{\tan 60°} = x\cot 60° = \frac{x}{\sqrt{3}}$		$\frac{1}{2}$ each
$BP = \frac{x}{\tan 30°} = x\cot 30° = x\sqrt{3}$	1	
(ii) Now in ΔAPB		
$AP^2 + AB^2 = BP^2$	1	
$\frac{x^2}{3} + 1 = 3x^2$		
$x^2 + 3 = 9x^2$	1	
$8x^2 = 3$		
$x = \sqrt{\frac{3}{8}}$ Km		
$= 610$ m (nearest 10m)	1	

Marking Scheme for Task: Academic Year: 2007-8

Solutions	Marks	Comments
Question 6: c) ① true for $n=1$ $9^3 - 4^1 = 729 - 4$ $= 725$ divisible by 5! $\therefore$ true for $n=1$ ② assume true for $n=k$ $\therefore$ assume $9^{k+2} - 4^k = 5N$	1	
③ aim to prove true for $n=k+1$ i.e. aim to prove $9^{k+3} - 4^{k+1} = 5M$ now: LHS $= 9.9^{k+2} - 4.4^k$ $= 9[5N + 4^k] - 4.4^k$ $= 45N + 9.4^k - 4.4^k$ $= 45N + 5.4^k$ $= 5[9N + 4^k]$ $\therefore$ divisible by 5	2	
④ Since true for $n=k+1$ if true for $n=k$ then since true for $n=1$ by Mathematical Induction it is true for $n = 2, 3, 4. \ldots$ $\therefore$ true for all $n \geq 1$	1	$\frac{1}{2}$ off for misunderst of final Statement

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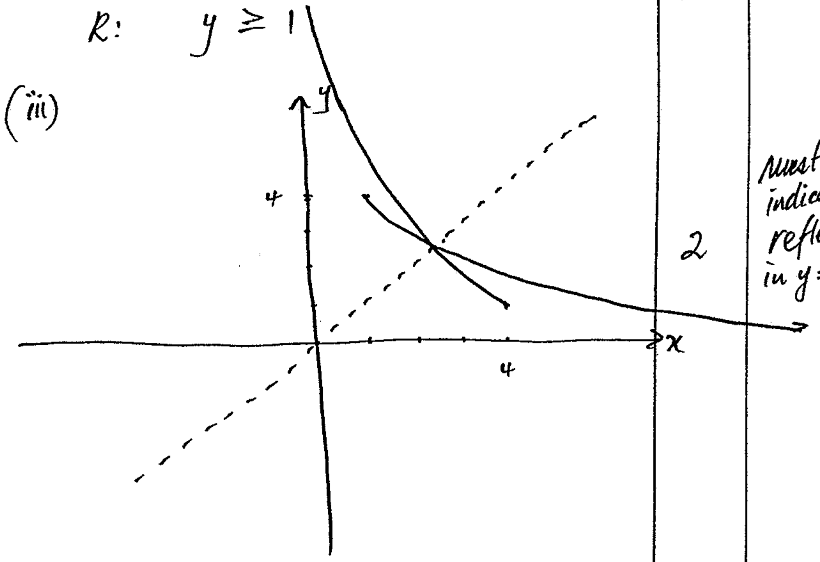
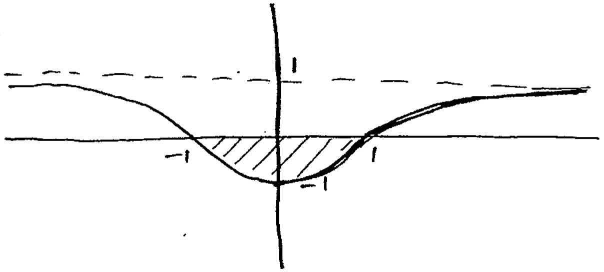
Marking Scheme for Task: Academic Year: 2007-8

Solutions	Marks	Comments
Question 7: a)(i) $f(x) = 4 - \sqrt{x-1}$ D: $x - 1 \geq 0 \therefore x \geq 1$	1	
R: $y \leq 4$	1	
(ii) Now $y = 4 - \sqrt{x-1}$ $\therefore$ Inverse: $x = 4 - \sqrt{y-1}$ $\sqrt{y-1} = 4 - x$ $y - 1 = (4-x)^2$ $y = 1 + (4-x)^2$	1	
D: $x \leq 4$ R: $y \geq 1$	1	$\frac{1}{2}$ each
(iii) 	2	Must indicate reflection in $y=x$

Solutions	Marks	Comment
Question 7. b) $f(x) = \frac{x^2-1}{x^2+1}$		
(i) $f(x) = \frac{x^2+1-2}{x^2+1}$		
$= 1 - \frac{2}{x^2+1} \left(= 1-2(x^2+1)^{-1}\right)$	1	
(ii) $f'(x) = 2(x^2+1)^{-2}. 2x$		
$= \frac{4x}{(x^2+1)^2} = 0$ for $x=0$ only	1	
$\therefore$ only one st pt at $(0,-1)$		
$x = 0-\varepsilon$ $f(x) < 0$	1	
$x = 0+\varepsilon$ $f'(x) > 0$ $\therefore$ min turning pt		
(iii) 	1	
(iv) $A = 2\left\|\int_0^1 \left(1 - \frac{2}{x^2+1}\right) dx\right\|$	1	
$= 2\left\|\left[x - 2\tan^{-1}x\right]_0^1\right\|$		
$= 2\left\|\left(1-\frac{\pi}{2}\right) - 0\right\|$ Note $\left\|1-\frac{\pi}{2}\right\|$		
$= 2\left(\frac{\pi}{2}-1\right)$ $= \frac{\pi}{2}-1$		
$= \pi - 2$ Since $\frac{\pi}{2} > 1$	1	