

St Catherine's School

Year: 12 Extension 1

Subject: Mathematics

Time allowed: 55 minutes

Date: February 2006

Student number _____

Teacher's Name: _____

Directions to candidates:

- All questions are to be attempted.
- Marks may be deducted for careless or badly arranged work
- All necessary **working** must be shown
- Approved calculators may be used

Marks:

Q 1	/8
Q 2	/7
Q 3	/8
Q 4/5	/8
Total	/31

Question 1 (8 marks)

The tangent at the point P $(2ap, ap^2)$ on the parabola $x^2 = 4ay$ meets the axis of the parabola at T.

S is the focus of the parabola.

- i) Sketch the parabola showing P, T and S 1
- ii) Find the equation of the tangent at P 2
- iii) Show that the co-ordinates of T are $(0, -ap^2)$ 1
- iv) Write down the co-ordinates of S 1
- v) Show that $SP=ST$ 3

Question 2 (7 marks)

P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ are points on the parabola $x^2 = 4ay$ such that OP is perpendicular to OQ where O is the vertex of the parabola.

- i) Show that $pq = -4$ 2
- ii) Find the co-ordinates of M, the midpoint of chord PQ 1
- iii) Show that the locus of M has Cartesian equation $x^2 = 2a(y - 4a)$ and describe this curve. 3
- iv) Describe the curve. 1

Question 3 (8 marks)

- ii) When the polynomial $P(x)$ is divided by $(x-4)(x-3)$, the remainder is $2x + 3$. Find the remainder when $P(x)$ is divided by $x-4$

2

i) $Q(x) = x^3 + 6x^2 + 3x - 10$

a) Show $Q(x)$ has a factor $(x-1)$

1

b) Hence express $Q(x)$ in fully factored form.

2

- iii) A polynomial $P(x)$ of degree 3 has roots at $x = 1, 4$ and 6 , and $P(0) = 12$.

a) Find $P(x)$

2

b) Sketch $P(x)$

1

Question 4 (4 marks)

Show that the polynomial $P(x) = x^4 + x - 5$ has a zero between 1 and 2.

1

Using one application of Newton's method with a first approximation to the zero of 1.5, find a more accurate value for the zero.

3

Question 5 (4 marks)

For the polynomial

$$P(x) = 3x^3 + 12x^2 - 9x - 4$$

with roots α, β, γ , find the value of

i) $\alpha + \beta + \gamma$

1

ii) $\alpha\beta + \alpha\gamma + \beta\gamma$

1

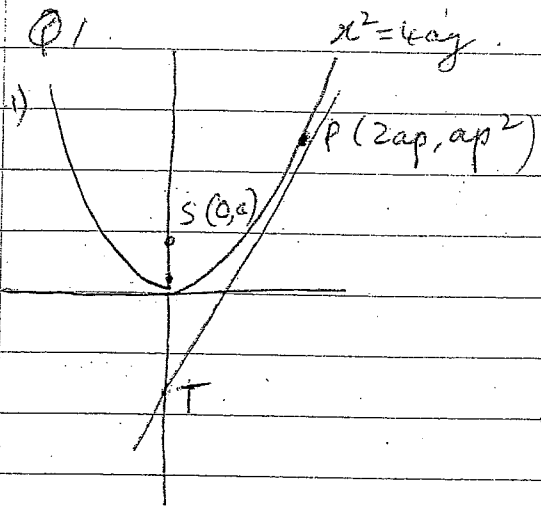
iii) $\alpha^2 + \beta^2 + \gamma^2$

2

106

Solutions JLB Ext 1 Maths

Q1



(i) $x^2 = 4ay$
 $y = \frac{x^2}{4a}$

$$\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$$

at $x = 2ap$, $\frac{dy}{dx} = \frac{2ap}{2a} = p$

∴ eq of tangent

$$y - ap^2 = p(x - 2ap)$$

$$y = px - ap^2$$

(iii) $T(0, \quad)$

at $x = 0$ on tangent

$$y = p \times 0 - ap^2$$

$$= -ap^2$$

∴ $T(0, -ap^2)$

(iv) $S(0, a)$

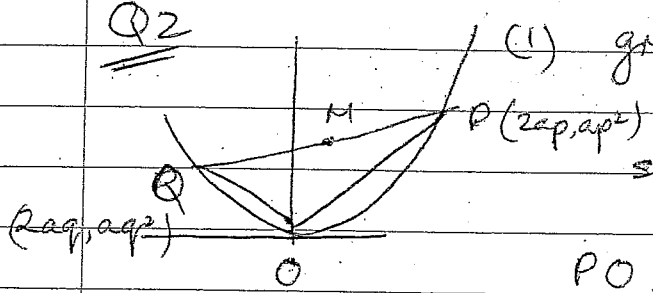
(v) $SP = \sqrt{(2ap - 0)^2 + (ap^2 - a)^2}$
 $= \sqrt{4a^2p^2 + a^2(p^2 - 1)^2}$
 $= a(p^2 + 1)$

From diagram,

$$ST = a + ap^2$$

∴ $ST = SP$

Q2



(i) $\text{grad } OP = \frac{ap^2 - 0}{2ap - 0} = \frac{p}{2}$

$\text{sum grad } OQ = \frac{q}{2}$

$PO \perp OQ \therefore \frac{p}{2} \times \frac{q}{2} = -1$

∴ $pq = -4$

(ii) $P1 = \left(\frac{2ap + 2aq}{2}, \frac{ap^2 + aq^2}{2} \right)$
 $= \left(a(p+q), \frac{a}{2}(p^2 + q^2) \right)$

(iii) $x = a(p+q)$

$y = \frac{a}{2}(p^2 + q^2) \therefore p^2 + q^2 = \frac{2y}{a}$

$(p+q) = \frac{x}{a}$

$(p^2 + q^2) = \frac{x^2}{a^2}$

$p^2 + q^2 + 2pq = \frac{x^2}{a}$

$\therefore \frac{2y}{a} + 8 = \frac{x^2}{a^2}$

$2ay - 8a^2 = x^2$

∴ $x^2 = 2a(y - 4a)$

focal length $\frac{a}{2}$

$$Q3) P(x) = (x-4)(x-3)Q(x) + 2x+3$$

$$P(4) = 0 \times 1 \times Q(x) + 8+3$$

$$\therefore P(4) = 11$$

Remainder is 11.

$$ii) a) Q(x) = x^3 + 6x^2 + 3x - 10$$

$$Q(1) = 1 + 6 + 3 - 10$$

$$= 0$$

$\therefore x-1$ is a factor.

$$b) \begin{array}{r} x^2 + 7x + 10 \\ x-1 \overline{) x^3 + 6x^2 + 3x - 10} \\ \underline{x^3 - x^2} \\ 7x^2 + 3x \\ \underline{7x^2 - 7x} \\ 10x - 10 \end{array}$$

$$P(x) = (x-1)(x^2 + 7x + 10)$$

$$= (x-1)(x+2)(x+5)$$

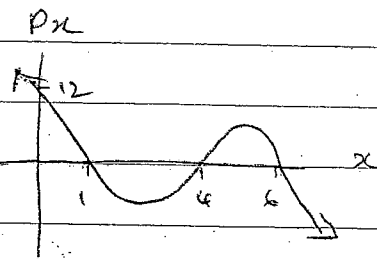
iii)

$$P(x) = k(x-1)(x-4)(x-6)$$

$$P(0) = k \times -1 \times -4 \times -6 = 12$$

$$-24k = 12 \quad \therefore k = -\frac{1}{2}$$

$$P(x) = -\frac{1}{2}(x-1)(x-4)(x-6)$$



$$Q4) P(x) = x^4 + x - 5$$

$$P(1) = 1 + 1 - 5 < 0$$

$$P(2) = 16 + 2 - 5 > 0$$

change of sign \therefore root betw 1 & 2

$$x_2 = x_1 - \frac{P(x_1)}{P'(x_1)} = 1.5 - \frac{P(1.5)}{P'(1.5)}$$

$$= 1.39$$

$$Q5) \alpha + \beta + \gamma = -\frac{b}{a} = -\frac{12}{3} = -4$$

$$\alpha\gamma + \beta\gamma + \alpha\beta = \frac{c}{a} = \frac{-9}{3} = -3$$

$$\begin{aligned} \alpha^2 + \beta^2 + \gamma^2 &= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma) \\ &= (-4)^2 - 2 \times -3 = \underline{\underline{22}} \end{aligned}$$