St Catherine's School

Year: 12
Subject: Extension II Mathematics
Time Allowed: 55 mins
Assessment Task 1 - 2006

Student number: 16361275

Directions to candidates:

- All questions are to be attempted.
- All questions are of equal value.
- All necessary **working** must be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Approved calculators and geometrical may be used.
- Place the question paper inside your exam booklet.
- State the student Number in your question paper.

TEACHER'S USE ONLY **Total Marks**
A
B
TOTAL

Extension II Mathematics Assessment task 1

27/2/06

$-\sqrt{2}+\sqrt{2}i$

Q.1 If $z = \cancel{-\sqrt{2}+i}$ and w = $\sqrt{3}-i$, find the modulus and argument of the following :

(a) (i) z

(ii) w

(iii) z^2w^3

2/(4m)

Q.2. Sketch the locus of z in each of the following:

(i) $|z-2| = |z-4i|$ (2m) 1/2

(ii) $\text{Arg}\,(z-1) = \frac{\pi}{4}$ (1m) 0

(iii) $\text{Arg}\,(1-z) = \frac{\pi}{4}$ (1m) 0

(iv) $\frac{z+i}{z}$ is a real number, where $z = x+iy$ (2m) 1/2

Q.3 If α, β *and* γ are the roots of the equation $x^3+5x-1=0$, find the polynomial equation, whose roots are

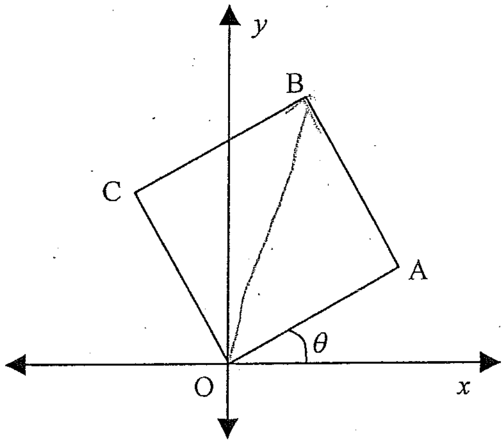
(i) α^2, β^2 *and* γ^2 (2m)

(ii) $2\alpha+3$, $2\beta+3$ *and* $2\gamma+3$ (Do not simplify) (2m)

3/4

Q.4. OABC is a square in the complex plane and the point A represents the complex number $z = \cos\theta + i\sin\theta$

State in modulus argument form (in terms of θ) the complex numbers represented by the points B and C. 0/ (4m)



Q.5 (i) Given that $1+i$ is a root of the polynomial equation $x^4 + 3x^2 - 6x + 10 = 0$, solve the equation over the Complex Number System $1\frac{1}{2}$/(3m)

Q.6 Show that $G(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{\cancel{3\times2}\;6}$ cannot have a double root 3/(3m)

Q.~~4~~ 7 (i) Use De Moivre's theorem and the expansion of $(\cos\theta + i\sin\theta)^5$ to express $\cos 5\theta$ in terms of $\cos\theta$ *and* $\sin\theta$ and $\sin 5\theta$ in terms of $\cos\theta$ *and* $\sin\theta$.

(Note: $(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$)

(3m)

(ii) Hence show that $\tan 5\theta = \dfrac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$ (1m)

(iii) Show that $\tan\frac{\pi}{5}, \tan\frac{2\pi}{5}, \tan\frac{3\pi}{5}$ *and* $\tan\frac{4\pi}{5}$ are the roots of the equation $x^4 - 10x^2 + 5 = 0$ (3m) $\frac{1}{2}$ $= 2$

(iv) Solve $x^4 - 10x^2 + 5 = 0$ and hence find the exact values of $\tan\frac{\pi}{5}$ *and* $\tan\frac{2\pi}{5}$ (3m) $\frac{1}{2}$

6

$\frac{6}{10}$

$2 + 3 + 1$

$= 6$

END OF PAPER