Student Number:__

St. Catherine's School
Waverley

February 2008
HSC ASSESSMENT TASK
EXAMINATION

Extension I Mathematics

Time allowed: *55 minutes*

INSTRUCTIONS

- There are 2 sections of value 17, 21
- Marks for each part of a question are indicated
- All questions should be attempted on the separate paper provided
- All necessary working should be shown
- Start each question on a new page
- Approved scientific calculators and drawing templates may be used badly arranged work.

Section A **17 Marks**

1. Find the remainder when the polynomial $P(x) = x^3 - 4x$ is divided by $(x+3)$ [2]

2. For the polynomial equation $x^3 - 2x^2 + x + 5 = 0$

(i) Show that one root of this equation lies in the interval $-2 < x < -1$. [1]

(ii) Use Newton's method once to find an approximation of this root starting with $x = -1.5$ as a first approximation. [2]

3. When the plynomial $P(x) = x^4 - 3x^3 + ax^2 + bx - 6$ is divided by $(x+1)$ [3]
the remainder is 8.
If $(x-3)$ is a factor of $P(x)$, find the values of a and b.

4. For the polynomial $P(x) = x^4 - 3x^2 - 2x$

(i) Show that $P(x)$ has a double zero at $x = -1$. [2]

(ii) Write $P(x)$ as the product of its linear factors. [2]

5. Show that $(x-1)(x-2)$ is a factor of [2]

$P(x) = x^n(2^m - 1) + x^m(1 - 2^n) + (2^n - 2^m)$

where m and n are positive integers

6. The polynomial $P(x) = 8x^3 - 20x^2 + 6x + 9$ has two equal roots. [3]
Find all the roots of $P(x) = 0$

Section B – Start a new page **21 Marks**

1. Solve the equation $2\sin^2\theta = \sin 2\theta$ for $0 \le \theta \le 360°$ [3]

2. (i) Write $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$ [2]

   (ii) Hence or otherwise solve the trigonometric equation [2]

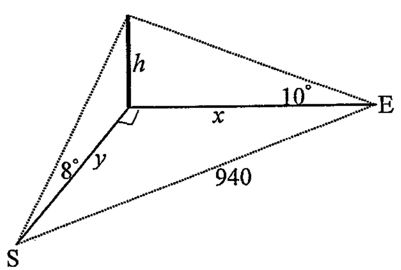
   $$\cos x - \sqrt{3}\sin x = \sqrt{3} \quad \text{for } 0° \le x \le 360°$$

3. If $\tan\dfrac{\theta}{2} = t$, prove $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = t$ [3]

4. A surveyor who is y metres south of a tower sees the top of it with an angle of elevation 8°. A second surveyor is x metres east of the tower. From his position the angle of elevation is 10° to the top of the tower.
   The two surveyors are 940m apart.

   (i) Show that $y = h\cot 8°$ [1]

   (ii) Find the height of the tower to the nearest metre. [2]



$\tan 82 = \frac{y}{h}$

$y = \tan 82 h$

$x = \tan 10 h$

$\tan 82^2 h^2 + \tan^2 80 h^2 = 940^2$

$h^2 ($

5. Prove that $\dfrac{2\tan A}{1+\tan^2 A} = \sin 2A$ [2]

6. Find the general slution of $2\cos x = \sqrt{3}$ [3]

7. Prove that $\dfrac{\tan\theta+1}{\sec\theta} - \dfrac{\cot\theta+1}{\operatorname{cos}ec\theta}$ is independent of θ [3]

End of Task

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HSC ASSESSMENT TASK
EXAMINATION

Extension 1 Mathematics

Solutions

Time allowed: *55 minutes*

INSTRUCTIONS

- There are 2 sections of value 17, 20
- Marks for each part of a question are indicated
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Marking Scheme for Task: Academic Year: 2007-8

Solutions	Marks	Comments
Q1 $x^3 - 4x = (x+3)(x^2 - 3x + 5) - 15$ $\therefore$ remainder is -15 * or by long division $x+3 \,\overline{)\, x^3 \quad -4x}$ quotient $x^2 - 3x + 5$ $x^3 + 3x^2$ $-3x^2 - 4x$ $-3x^2 - 9x$ $5x$ $5x + 15$ -15 * OR $P(-3) = -15$	2	
Q2 (i) for the function $f(x) = x^3 - 2x^2 + x + 5$ $f(-2) = -8 - 8 - 2 + 5 < 0$ $f(-1) = -1 - 2 - 1 + 5 > 0$	1	
$\therefore f(x) = 0$ has a root for $-2 < x < -1$ $f'(x) = 3x^2 - 4x + 1$ (ii) Newton's Method. $a_1 = a - \frac{f(a)}{f'(a)}$	1	
Let $a = -1.5$ $\quad \therefore a_1 = -1.5 - \frac{f(-1.5)}{f'(-1.5)}$ $f(-1.5) = -\frac{35}{8}$ $\quad f'(-1.5) = \frac{55}{4}$ $\therefore a_1 = -1.5 - \frac{-\frac{35}{8}}{\frac{55}{4}}$ $= -1.5 + \frac{7}{22}$ $= -1.1\dot{8}$	1	

Solutions	Marks	Comments
Q3 $P(x) = x^4 - 3x^3 + ax^2 + bx - 6$		
$P(-1) = 1 + 3 + a - b - 6 = 8$		
$\therefore a - b = 10$ —— (1)	1	
$P(3) = 81 - 81 + 9a + 3b - 6 = 0$		
$\therefore 9a + 3b = 6$ —— (2)	1	
Now (1) $\times 3 +$ (2)		
$12a = 36$		
$\therefore a = 3$	1	
and $b = -7$		
Q4 (i) $P(x) = x^4 - 3x^2 - 2x$		
$P(-1) = 1 - 3 + 2 = 0$	1	
$P'(x) = 4x^3 - 6x - 2$		
$P'(-1) = -4 + 6 - 2 = 0$	1	
$\therefore x = -1$ is a double root of $P(x)$		
(ii) $P(x) = (x+1)^2 Q(x)$		
$\therefore x^4 - 3x^2 - 2x = (x^2 + 2x + 1)(x^2 - 2x)$		
$= x(x-2)(x+1)^2$	2	
OR Clearly $P(x) = x(x^3 - 3x - 2)$		
$= x(x+1)^2(x - \alpha)$	1	
Clearly $P(x) = x(x+1)^2(x-2)$ Since product of roots of $x^3 - 3x - 2$ is 2.	1	

Solutions	Marks	Comments
Q5 $P(1) = 1^n(2^m - 1) + 1^m(1 - 2^n) + 2^n - 2^m$		
$= 2^m - 1 + 1 - 2^n + 2^n - 2^m$		
$= 0$		
$\therefore (x-1)$ is a factor.	1	
$P(2) = 2^n(2^m - 1) + 2^m(1 - 2^n) + (2^n - 2^m)$		
$= 2^{n+m} - 2^n + 2^m - 2^{m+n} + 2^n - 2^m$		
$= 0$		
$\therefore (x-2)$ is a factor		
$\therefore (x-1)(x-2)$ is a factor of $P(x)$.	1	
Q6 $P(x) = 8x^3 - 20x^2 + 6x + 9$		
Let the roots be α, α, β		
$\therefore$ sum of roots $= 2\alpha + \beta = \frac{5}{2}$ —— (1)	1	
Sum of roots 2 x $= \alpha^2 + 2\alpha\beta = \frac{3}{4}$ —— (2)		
product of roots $= \alpha^2\beta = -\frac{9}{8}$ —— (3)		
from (1) $\beta = \frac{5}{2} - 2\alpha$ —— (4)		
Sub in (2) $\alpha^2 + 2\alpha\left(\frac{5}{2} - 2\alpha\right) = \frac{3}{4}$		
$\alpha^2 + 5\alpha - 4\alpha^2 = \frac{3}{4}$		
$-3\alpha^2 + 5\alpha = \frac{3}{4}$		
$-12\alpha^2 + 20\alpha - 3 = 0$		
$12\alpha^2 - 20\alpha + 3 = 0$		
$(6\alpha - 1)(2\alpha - 3) = 0$	1	
$\therefore \alpha = \cancel{\frac{1}{6}}, \frac{3}{2}$		
now $P'\left(\frac{3}{2}\right) = 0 \therefore \frac{3}{2}$ is the double zero.		
Sub in (4) $\beta = \frac{5}{2} - 3 = -\frac{1}{2}$	1	
$\therefore$ roots are $x = \frac{3}{2}, \frac{3}{2}, -\frac{1}{2}$		

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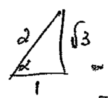
Solutions	Marks	Comments
Section B		
Q1 $\quad 2\sin^2\theta = \sin 2\theta$		
$2\sin^2\theta - \sin 2\theta = 0$		
$2\sin^2\theta - 2\sin\theta\cos\theta = 0$	1	
$2\sin\theta(\sin\theta - \cos\theta) = 0$		
$\therefore \sin\theta = 0$ or $\sin\theta - \cos\theta = 0$		
$\sin\theta = \cos\theta$		
$\theta = 0°, 180°, 360°$ $\quad \tan\theta = 1 \ (\cos\theta \neq 0)$	1	
or $0, \pi, 2\pi$ $\quad \theta = 45°, 225°$		
or $\frac{\pi}{4}, \frac{5\pi}{4}$		
$\therefore \theta = 0°, 45°, 180°, 225°, 360°$	1	
Q2 (i) $\cos x - \sqrt{3}\sin x$		$r=2$ 1 mark
$(\cos x\cos\alpha - \sin x\sin\alpha)$ 		$\alpha=60°$ 1 mark
$\tan\alpha = \sqrt{3}$, $\alpha = 60°$		
$\therefore \cos x - \sqrt{3}\sin x = 2\cos(x+60°)$	2	
(ii) $\cos x - \sqrt{3}\sin x = \sqrt{3}$		
$\therefore 2\cos(x+60°) = \sqrt{3}$		
$\cos(x+60°) = \frac{\sqrt{3}}{2}$		
$x+60° = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$		
$x+60° = 30°, 330°, 390°, \ldots$	1	
$\therefore x = \cancel{-30°}, 270°, 330°, \ldots$		
$\therefore x = 270°, 330°$	1	

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Solutions	Marks	Comments
Q3 $\quad \dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = \dfrac{1+\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$	1	
$= \dfrac{1+t^2+2t-1+t^2}{1+t^2+2t+1-t^2}$	1	
$= \dfrac{2t^2+2t}{2+2t}$		
$= \dfrac{\cancel{2}t\cancel{(t+1)}}{\cancel{2(t+1)}}$	1	
$= t$		
Q4 (i) $\tan 8° = \frac{h}{y}$		
$\therefore y = \frac{h}{\tan 8°}$		
$= h \cdot \frac{1}{\tan 8°}$	1	
$= h\cot 8°$		
(ii) now similarly $x = h\cot 10°$		
and by pythagoras' theorem	1	
$(h\cot 8°)^2 + (h\cot 10°)^2 = 940^2$		
$h^2\cot^2 8° + h^2\cot^2 10° = 940$		
$\therefore h^2 = \dfrac{940^2}{\cot^2 8° + \cot^2 10°}$		
$h^2 = 10672.60126\ldots$		
$\therefore h = 103\text{ m}$ (nearest m)	1	

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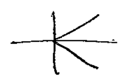
Solutions	Marks	Comments
Q5 $\dfrac{2\tan A}{1+\tan^2 A} = \dfrac{2\cdot\frac{\sin A}{\cos A}}{\sec^2 A}$		
$= \dfrac{\frac{2\sin A}{\cos A}}{\frac{1}{\cos^2 A}}$	1	
$= \dfrac{2\sin A\cos^2 A}{\cos A}$		
$= 2\sin A\cos A$		
$= \sin 2A$	1	
Q6 $2\cos x = \sqrt{3}$ $\cos x = \frac{\sqrt{3}}{2}$  $x = 30^\circ$		
$\therefore x = 0^\circ+30^\circ,\ 360^\circ-30^\circ,\ 360^\circ+30^\circ,\ 720^\circ+30^\circ,\ 720^\circ-30^\circ, \ldots$ $x = 0\times180+30^\circ,\ 2\times180-30^\circ,\ 2\times180+30^\circ,\ 4\times180^\circ+$ $\therefore x = 2n\times180^\circ \pm 30^\circ$ $n = 0, 1, 2, 3 \ldots$ or n an integer	3	
Q7 $\dfrac{\tan\theta+1}{\sec\theta} - \dfrac{\cot\theta+1}{\operatorname{cosec}\theta}$		
$= \dfrac{\frac{\sin\theta}{\cos\theta}+1}{\frac{1}{\cos\theta}} - \dfrac{\frac{\cos\theta}{\sin\theta}+1}{\frac{1}{\sin\theta}}$		
$= \sin\theta+\cos\theta - \cos\theta-\sin\theta$		
$= 0$	3	