St Catherine's School
Waverley, Sydney

Student Number: ______________

Year 12
Assessment Task 2
Half Yearly

Mathematics Extension II

Time allowed: 2 hours

Reading time: 5 minutes

Course weighting: 30%

General Instructions

- Attempt ALL questions
- Write your Student NUMBER at the top of this page and on the writing paper used

Sections	Marks
Total marks	

Question 1.

a) Consider the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$

(i) Sketch this ellipse clearly showing the coordinates of its foci and the equations of its directrices (4m)

(ii) Find the equation of the tangent at $P\ (x_1,y_1)$ to this ellipse (2m)

b) Consider the rectangular hyperbola $x^2-y^2=8$.

(i) Show that the eccentricity of this hyperbola is $\sqrt{2}$ (1m)

(ii) Sketch this Hyperbola clearly showing the coordinates of its vertices and foci and the equations of its directrices and asymptotes. (4m)

This Hyperbola is rotated by 45^0 to form the hyperbola $xy=4$

(iii) Find the coordinates of its vertices. (1m)

(iv) Find the coordinates of its foci. Show working (2m)

(v) Find the equations of the directrices. Show working (2m)

Question 2

a) (i) Show that i is a root of the quadratic equation

$$(2-i)z^2-(1-i)z+3=0$$ (1m)

(ii) Find the other root in terms of $a+ib$ (2m)

b) (i) Show that $cis\,2\theta-1=2\sin\theta(-\sin\theta+i\cos\theta)$ (2m)

(ii) Given that $\dfrac{z+1}{z}=cis\dfrac{2\pi}{7}$, show that $z=-\dfrac{1}{2}(1+i\cot\dfrac{\pi}{7})$ (3m)

c) P, Q and R are complex numbers z_1, z_2 and z_3 respectively. In addition if $z_2-z_1=i(z_3-z_1)$ what can you conclude about triangle PQR? Give reasons. (3m)

d) (i) Find, in term of k, the coordinates and the nature of the stationary points on the curve $y=2x^3-3x^2-12x+6k$, where k is real. (3m)

(ii) Find the values of k for which $2x^3-3x^2-12x+6k=0$ has three real and different roots. (2m)

Question 3

a) $P(cp,\dfrac{c}{p})$ and $Q(cq,\dfrac{c}{q})$ are two points on a rectangular hyperbola $xy=c^2$

(i) Show that the equation of the chord PQ is $x+pqy=c(p+q)$ (2m)

(ii) P and Q move such that PQ passes through the point A $(0,1)$.
Show that $pq=c(p+q)$ (1m)

(iii) Show that the equation of the tangent at P is $x+p^2y=2cp$ (2m)

(iv) Find the coordinates of R, the point of intersection of the tangents at P and Q. (2m)

(v) Find the locus of R. (2m)

(b) (i) Write down $\cos 2\theta$ and $\tan 2\theta$ in terms of $\tan\theta$ (1m)

(ii) Show that $\cos 4\theta=\dfrac{1-6\tan^2\theta+\tan^4\theta}{1+2\tan^2\theta+\tan^4\theta}$ (2m)

(iii) Hence find the roots of $x^4-6x^2+1=0$ (2m)

(iv) Hence deduce that $\tan^2\dfrac{\pi}{8}+\tan^2\dfrac{3\pi}{8}=6$ (2m)

Question 4

a) If ω is a complex root of $z^3 = 1$

(i) Show that ω^2 is also a root (1m)

(ii) Show that $1+\omega+\omega^2 = 0$ (1m)

(iii) Form a quadratic equation whose roots are given by $2+\omega$ and $2+\omega^2$ (2m)

(iv) Show that $\dfrac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = \omega^2$. (2m)

b) Consider the function $y = \sin^{-1}(\cos x)$

(i) State the domain and range (2m)

(ii) Show that $\dfrac{dy}{dx} = \pm 1$ (2m)

(iii) Sketch the function $y = \sin^{-1}(\cos x)$ (2m)

c) It is known that $\sin^{-1} x$ *and* $\cos^{-1} x$ are acute.

(i) Show that $\sin(\sin^{-1} x - \cos^{-1} x) = 2x^2 - 1$ (2m)

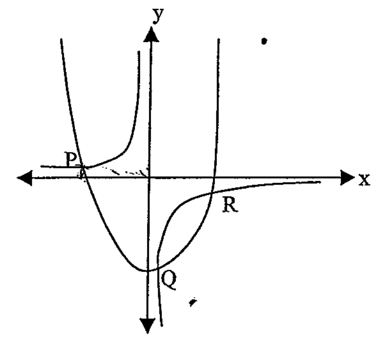
(ii) Hence solve the equation $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(3x-2)$ (2m)

Question 5

a) (i) If $y = mx + c$ is a tangent to the Hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, show that $c^2 = a^2m^2 - b^2$ (3m)

(ii) Show that the tangents drawn from the point $(\sqrt{3}, 2)$ to the hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$ are at right angles. (2m)

b)



P (x_1,y_1), Q(x_2,y_2) and R(x_3,y_3) are the points of intersection of the curves $xy = -1$ *and* $y = x^2 - 9$.

(i) Show that x_1, x_2 and x_3 are the roots of the cubic equation $x^3 - 9x + 1 = 0$ (1m)

(ii) Write an expression for OP^2 in terms of x_1. (1m)

(iii) Write a polynomial whose roots are x_1^2, x_2^2 *and* x_3^2 (2m)

(iv) Write a polynomial whose roots are $\dfrac{1}{x_1^2}, \dfrac{1}{x_2^2}$ *and* $\dfrac{1}{x_3^2}$ (2m)

(v) Hence find the numerical value of $OP^2 + OQ^2 + OR^2$ (2m)

c) The polynomial equation $x^n - px + q = 0$ has a double root. Show that

$$\left(\frac{p}{n}\right)^n - \left(\frac{q}{(n-1)}\right)^{n-1} = 0$$

(3m)

End of Paper

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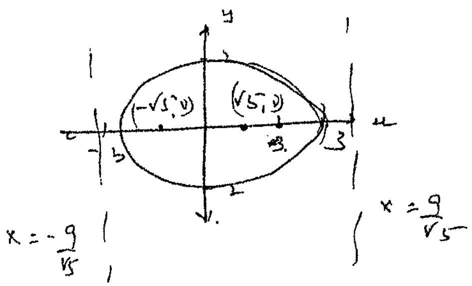
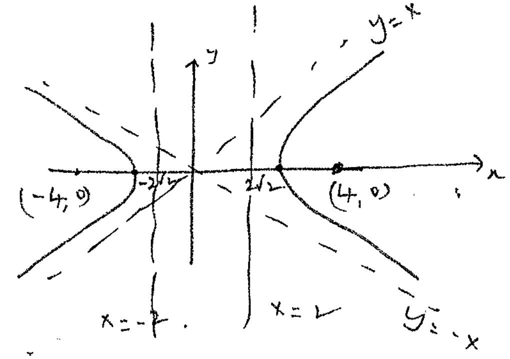
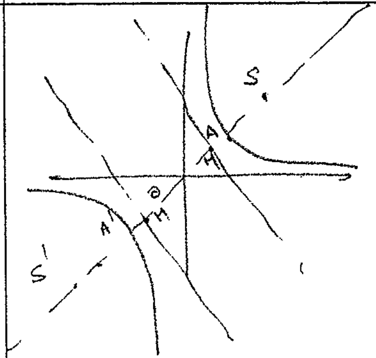
Extension II April 2007.

Qn	Solutions	Marks	Comments+Criteria
1a) (i)	$\frac{x^2}{9} + \frac{y^2}{4} = 1$ $4 = 9(1-e^2)$ $1-e^2 = \frac{4}{9}$ $e^2 = \frac{5}{9}$ $e = \frac{\sqrt{5}}{3} \quad e>0.$	1	
	foci: $(\pm 3 \cdot \frac{\sqrt{5}}{3}, 0)$	1	
	directrices: $x = \pm \frac{9}{\sqrt{5}}$.	1	
		1	
(ii)	$\frac{2x}{9} + \frac{2y}{4} y' = 0$ $y' = -\frac{2x}{9} \times \frac{4}{2y}$ $= \frac{-4x}{9y}$ y' at $(x_1,y_1) = \frac{-4x_1}{9y_1}$	1	
	Eqn. of tgt. is $y - y_1 = \frac{-4x_1}{9y_1}(x - x_1)$		

Qn	Solutions	Marks	Comments+Criteria
	$9yy_1 - 9y_1^2 = -4xx_1 + 4x_1^2$ $4xx_1 + 9yy_1 = 4x_1^2 + 9y_1^2$ $= 36$	1	$-\frac{1}{2}$ if $4x_1^2+9y_1^2$ is not simplified.
1b)	$x^2 - y^2 = 8$ $\frac{x^2}{8} - \frac{y^2}{8} = 1$ $8 = 8(e^2-1)$ $\therefore e^2 - 1 = 1$ $e^2 = 2$ $e = \sqrt{2} \quad (e>0)$	1	
			sketch 1 m.
	foci: $\pm(2\sqrt{2}\times\sqrt{2}, 0)$ $\pm(4, 0)$	1	~~Asymptote~~ 1 or shown!
	Directrices: $x = \pm\frac{2\sqrt{2}}{\sqrt{2}}$ $= \pm 2$	1	
	Asymptote $y = \pm x$	1	

Qn	Solutions	Marks	Comments+Criteria

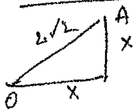


Note: vertices; foci lie on

$y = x$

the vertices:

A & A'



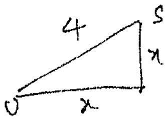
$x^2 + x^2 = 8$

$x^2 = 4$

$x = 2.$

$A:(2,2)$; $A'(-2,-2)$ — 1

S & S' are foci.



$OS: 4$

$x^2 + x^2 = 16$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

$S:(2\sqrt{2}, 2\sqrt{2})$; $S':(-2\sqrt{2}, -2\sqrt{2})$ — 2

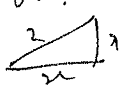
directrices are ⊥ to $y = x$

& therefore have grad: -1.

Also if H, is the pt. of intersection

between directrix & $y = x$

$OH: 2$



$x^2 + x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$

Qn	Solutions	Marks	Comments+Criteria

$\therefore$ $H:(\sqrt{2}, \sqrt{2})$ $H':(-\sqrt{2}, -\sqrt{2})$

Eq.n of directrices

$y - \sqrt{2} = -1(x - \sqrt{2})$

and

$y + \sqrt{2} = -1(x + \sqrt{2})$

i.e $x + y = 2\sqrt{2}$

&

$x + y = -2\sqrt{2}$. — 2m.

Q.2

$(2-i)z^2 - (1-i)z + 3 = 0$

$(2-i)i^2 - (1-i)i + 3$

$= -2 + \cancel{i} - \cancel{i} + i^2 + 3$

$= 0$ — 1

if $a + ib$ is another root

$a + ib + i = \dfrac{1-i}{2-i}$

also $i(a+ib) = \dfrac{3}{2-i} \cdot \dfrac{2+i}{2+i}$

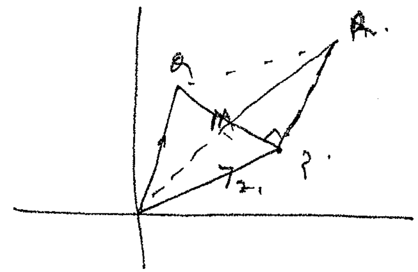
$-b + ia = \dfrac{3(2+i)}{5}$

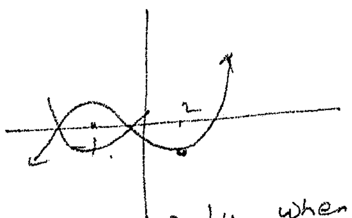
$\therefore$ $-b = \dfrac{6}{5}$

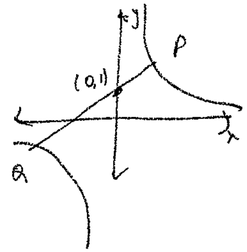
$a = \dfrac{3}{5}$

$\dfrac{3}{5} - \dfrac{6}{5}i$ is the other root. — 2m.

Qn	Solutions	Marks	Comments+Criteria
2b	$\text{Cis } 2\theta - 1$ $= (\cos 2\theta - 1) + i \sin 2\theta$ $= -2\sin^2\theta + i\, 2\sin\theta\cos\theta$ $= 2\sin\theta(-\sin\theta + i\cos\theta)$		
	$\frac{z+1}{z} = \text{Cis } \frac{2\pi}{7}$ $1 + \frac{1}{z} = 1 + 2\sin\frac{\pi}{7}\left(-\sin\frac{\pi}{7} + i\cos\frac{\pi}{7}\right)$ from (i) $\therefore \frac{1}{z} = 2\sin\frac{\pi}{7}\left(-\sin\frac{\pi}{7} + i\cos\frac{\pi}{7}\right)$ $z = \frac{1}{2\sin\frac{\pi}{7}} \cdot \frac{1}{-\sin\frac{\pi}{7} + i\cos\frac{\pi}{7}} \cdot \frac{-\sin\frac{\pi}{7} - i\cos\frac{\pi}{7}}{-\sin\frac{\pi}{7} - i\cos\frac{\pi}{7}}$ $= \frac{1}{2\sin\frac{\pi}{7}} \cdot \frac{-(\sin\pi/7 + i\cos\pi/7)}{1}$ $= \frac{-1}{2}(+1 + i\cot\pi/7)$		
2c)	continued.		

Qn	Solutions	Marks	Comments+Criteria
	 P: z_1 Q: z_2 R: z_3. now PQ: $z_2 - z_1$ PR: $z_3 - z_2$ $z_2 - z_1 = i(z_3 - z_2)$ $\Rightarrow$ PQ $\perp$ PR $\therefore$ ΔPQR is right angled at P.		also $\left\lvert\frac{z_2 - z_1}{z_3 - z_2}\right\rvert = \lvert i \rvert = 1$ $\therefore \lvert z_2 - z_1\rvert = \lvert z_3 - z_2\rvert$ $\therefore$ isos.
2d.	$y = 2x^3 - 3x^2 - 12x + 6k$ $\frac{dy}{dx} = 6x^2 - 6x - 12$ $\frac{d^2y}{dx^2} = 12x - 6$ Stat. pts when $\frac{dy}{dx} = 0$ $x^2 - x - 2 = 0$ $(x-2)(x+1) = 0$ $x = 2$; $x = -1$		

Qn	Solutions	Marks	Comments+Criteria
	at $x = 2$; $y = 16 - 12 - 24 + 6k$ $= -20 + 6k$. also $\frac{d^2y}{dx^2}$ at $x = 2$ > 0 $\therefore (2, -20+6k)$ is a minimum turning point. at $x = -1$ $y = -2 - 3 + 12 + 6k$ $= 7 + 6k$ $\therefore (-1, 7+6k)$ is a maximum turning point.		
(ii)	when the curve has 3 distinct real roots, it has 3 distinct points of intersection with the x axis.  possible only when $7 + 6k > 0$ and $-20 + 6k < 0$ $k > -\frac{7}{6}$ and $k < \frac{10}{3}$ $-7/6$ $10/3$ $-\frac{7}{6} < k < \frac{10}{3}$		

Qn	Solutions	Marks	Comments+Criteria
Q.3	$P : (cp, \frac{c}{p})$ $Q : (cq, \frac{c}{q})$  grad. of PQ : $\dfrac{\frac{c}{q} - \frac{c}{p}}{cq - cp}$ $= \frac{p-q}{pq} \times \frac{1}{q-p}$ $= -\frac{1}{pq}$	1	
	$\therefore$ Eqn. of PQ: $y - \frac{c}{p} = -\frac{1}{pq}(x - cp)$ $pqy - cq = -x + cp$ $\therefore x + pqy = cp + cq$.	1	
(ii)	sub: $(0,1)$ $0 + pq = c(p+q)$ $\therefore pq = c(p+q)$	1	
(iii)	$xy = c^2$ $xy' + y = 0$ $y' = -\frac{y}{x}$		

Qn	Solutions	Marks	Comments+Criteria
	y' at P $= \frac{-c/p}{cp} = -\frac{1}{p^2}$	1	
	$\therefore$ Eqn. of tangent at P is		
	$y - \frac{c}{p} = -\frac{1}{p^2}(x - cp)$		
	$p^2y - pc = -x + cp$		
	$\therefore\ x + p^2y = 2cp$ — (1)	1	
	Eqn. at Q: $x + q^2y = 2cq$ — (2)		
	Solve (1) & (2) simultaneously		
	(1) − (2): $(p^2-q^2)y = 2c(p-q)$		
	$y = \frac{2c}{p+q}$ $(p \neq q)$		
	Sub in (1)		
	$x + p^2\cdot\frac{2c}{p+q} = 2cp$		
	$px + qx + 2cp^2 = 2cp^2 + 2cpq$		
	$x = \frac{2cpq}{p+q}$.		
	$\therefore\ R: \left(\frac{2cpq}{p+q}, \frac{2c}{p+q}\right)$		
(iv)	Locus of R:		

Qn	Solutions	Marks	Comments+Criteria
	$x = \frac{2cpq}{p+q}$ — (1)		
	$y = \frac{2c}{p+q}$ — (2)		
	— (3)		
	also $pq = c(p+q)$		
	Combining the above we get,		
	$x = 2c\cdot\frac{c(p+q)}{p+q}$		
	$x = 2c^2$ is the locus of R.		
3b)	(i) $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$; $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$		
	$\cos 4\theta = \frac{1-\tan^2 2\theta}{1+\tan^2 2\theta}$		
	$= \frac{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2}{1+\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2}$		
	Let $t = \tan\theta$		
	$\cos 4\theta = \frac{1-\left(\frac{2t}{1-t^2}\right)^2}{1+\left(\frac{2t}{1-t^2}\right)^2}$		
	$= \frac{(1-t^2)^2-4t^2}{(1-t^2)^2+4t^2} = \frac{1+t^4-6t^2}{1+t^4+2t^2}$		
	$= \frac{1+\tan^4\theta-6\tan^2\theta}{1+\tan^4\theta+2\tan^2\theta}$		

Qn	Solutions	Marks	Comments+Criteria
3b (iii)	$x^4 - 6x^2 + 1 = 0$ if $x = \tan\theta$ $\tan^4\theta - 6\tan^2\theta + 1 = 0$ ~~has~~ is the solution to $\cos 4\theta = 0$ $4\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$ $\theta = \pm\frac{\pi}{8}, \pm\frac{3\pi}{8}$ $\therefore$ The solution to $x^4 - 6x^2 + 1 = 0$ are $\tan\frac{\pi}{8}, -\tan\frac{\pi}{8}, \tan\frac{3\pi}{8}, -\tan\frac{3\pi}{8}$.		
(iv)	Product of roots taken two at a time: $-\tan^2\frac{\pi}{8} - \tan^2\frac{3\pi}{8} + \tan\frac{\pi}{8}\tan\frac{3\pi}{8} - \tan\frac{\pi}{8}\tan\frac{3\pi}{8} - \tan\frac{\pi}{8}\tan\frac{3\pi}{8} + \tan\frac{\pi}{8}.\tan\frac{3\pi}{8}$ $= -6$ $\therefore \tan^2\frac{\pi}{8} + \tan^2\frac{3\pi}{8} = 6$.		
Q.4	$y = \cos^{-1}x^2$ D: ~~Note: $x^2 \geq 0$~~ ~~We need $x^2 \leq 1$~~ ~~$\therefore -1 \leq x \leq 1$~~		

$z^3 = 1$

$(z^3 - 1) = 0$

$(z-1)(z^2+z+1) = 0$

$z = 1$; $z = \frac{-1 \pm \sqrt{1-4}}{2}$

$= \frac{-1 \pm \sqrt{3}i}{2}$

if $w = \frac{-1+\sqrt{3}i}{2}$

$w^2 = \left(\frac{-1+\sqrt{3}i}{2}\right)^2$

$= \left(\frac{1 - 3 - 2\sqrt{3}i}{2}\right)$

$= -\frac{2 - 2\sqrt{3}i}{2}$

$= -1 - \sqrt{3}i$ — the other root.

$1, w, w^2$ are root of $z^3 = 1$

$1 + w + w^2$ is sum of roots $= 0$

$2 + w + 2 + w^2 = 4 + w + w^2$

$= 4 - 1$

$= 3$

$(2+w)(2+w^2)$

$= 4 + 2(w + w^2) + w^3$

$= 4 - 2 + 1$

$= 3$.

The quad eq^n

$x^2 - 3x + 3 = 0$

(iv) $\omega^2(c + a\omega + b\omega^2)$

$= c\omega^2 + a\omega^3 + b\omega^4$

$= c\omega^2 + a + b\omega$ $\quad (\omega^3 = 1,\ \omega^4 = \omega)$

b) $y = \sin^{-1}(\cos x)$

Dom. all x

Range $-1 \le \cos x \le 1$

$-\frac{\pi}{2} \le \sin^{-1}(\cos x) \le \pi/2$.

(i) $\frac{dy}{dx} = \frac{1}{\sqrt{1-\cos^2 x}} \cdot -\sin x$

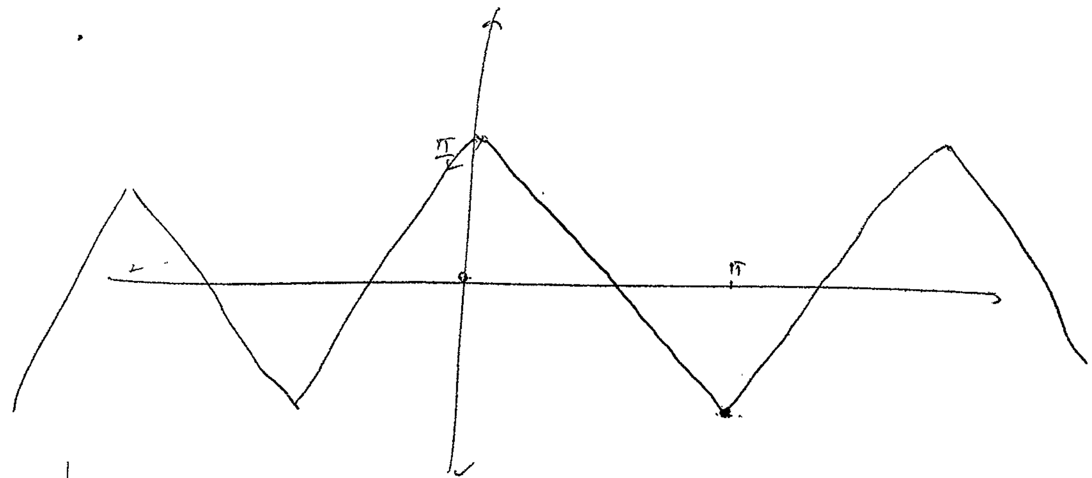
$= \frac{-\sin x}{|\sin x|}$

$= \frac{-\sin x}{\sin x}$ or $\frac{-\sin x}{-\sin x}$

$= -1$ or $+1$.

(iii). $\frac{dy}{dx} = -1$ when $\sin x > 0$
$0 \le x \le \pi$, $2\pi \le x \le 3\pi$ etc

$= +1$ when $\sin x < 0$
π to 2π; 3π — 4π etc.



$x = 0$; $\cos 0 = 1$; $\sin^{-1}(1) = \frac{\pi}{2}$

$x = \pi$; $\cos\pi = -1$; $\sin^{-1}(-1) = -\pi/2$.

c). Let $\sin^{-1}x = \alpha$; $\sin\alpha = x$ $\quad \alpha$ is acute given.

$\cos^{-1}x = \beta$; $\cos\beta = x$; $\quad \beta$ is acute given

$\sin(\alpha - \beta)$

$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= x^2 - \sqrt{1-x^2}\cdot\sqrt{1-x^2}$

$= x^2 - (1-x^2)$

$= 2x^2 - 1$

$\sin(\sin^{-1}x - \cos^{-1}x) = 3x - 2$ $\quad x = 1$

$2x^2 - 1 = 3x - 2$ $\quad x = \frac{1}{2}$.

$2x^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$

Qn	Solutions	Marks	Comments+Criteria
Q.5	$y = mx + c$ is a tgt. to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ when the discriminant of $b^2x^2 - a^2(mx+c)^2 - a^2b^2 = 0$ is zero: $x^2(b^2 - a^2m^2) - 2mca^2x - c^2a^2 - a^2b^2 = 0.$ $\Delta: \quad 4m^2c^2a^4 + 4(b^2 - a^2m^2)(c^2a^2 + a^2b^2) = 0.$ $m^2\cancel{c^2}a^2 + b^2c^2 + b^4 - a^2\cancel{m^2c^2} - a^2m^2b^2 = 0$ $c^2 + b^2 = a^2m^2$ or $c^2 = a^2m^2 - b^2.$		
(ii)	$y = mx + c$ is a tgt. to $\frac{x^2}{16} - \frac{y^2}{9} = 1$ when $c^2 = 16m^2 - 9$; from (1) — (1) also $2 = \sqrt{3}m + c$ — (2) Combine (1) & (2) $16m^2 - 9 = (2 - \sqrt{3}m)^2$ $16m^2 - 9 = 4 - 4\sqrt{3}m + 3m^2$		

Qn	Solutions	Marks	Comments+Criteria
	$16m^2 - 9 - 4 - 3m^2 + 4\sqrt{3}m = 0$ $13m^2 + 4\sqrt{3}m - 13 = 0$ if m_1 & m_2 are the roots here $m_1m_2 = \frac{-13}{13} = -1$ $\therefore$ The two tangents are at right angles		
5)	$xy = -1$ meets $y = x^2 - 9$ when $x^2 - 9 = \frac{-1}{x}$ $x^3 - 9x + 1 = 0.$		
(i)	$OP^2 = x_1^2 + \frac{1}{x_1^2}$ or $\cancel{x_1^2 + x_1^2 - 9}$ $= \cancel{2x_1^2 - 9}.$		
(ii)	x_1, x_2, x_3 are roots of $x^3 - 9x + 1 = 0,$ x_1^2, x_2^2, x_3^2 are roots of $(\sqrt{x})^3 - 9\sqrt{x} + 1 = 0$ $\sqrt{x}(x - 9) = -1$ $x(x-9)^2 = 1.$ $x^3 - 18x^2 + 81x - 1 = 0$		

$x_1^2+x_2^2+x_3^2=18$

Qn	Solutions	Marks	Comments+Criteria
	$OP^2 = 2x_1^2 - 9$ (iv) $\left(\frac{1}{x}\right)^3 - 18\left(\frac{1}{x}\right)^2 + 81\left(\frac{1}{x}\right) - 1 = 0$		
	$OP^2 + OQ^2 + OR^2$ $\quad 1 - 18x + 81x^2 - x^3 = 0$		
	$= 2(x_1^2+x_2^2+x_3^2) - 27$ $\quad$ or $x^3 - 81x^2 + 18x - 1 = 0$		
	$= 2(18) - 27$ $\quad \frac{1}{x_1^2} + \frac{1}{x_2^2} + \frac{1}{x_3^2}$		
	$= 9$ $\quad = 81$		
	$\Sigma OP^2 = 81 + 18 = 99$		
c)	Let α be the double root		
	$\alpha^n - p\alpha + q = 0$ — (1)		
	also α is a root of		
	$nx^{n-1} - p = 0$		
	$n\alpha^{n-1} - p = 0$ — (2)		
	from (2) $\alpha^{n-1} = \frac{p}{n}$		
	(1) $\Rightarrow \alpha(\alpha^{n-1} - p) + q = 0$		
	$\alpha\left(\frac{p}{n} - p\right) + q = 0$		
	$\alpha p\left(\frac{1-n}{n}\right) + q = 0$		
	$\alpha = \frac{qn}{(n-1)p}$		

Qn	Solutions	Marks	Comments+Criteria
	$\alpha^{n-1} \div \left(\frac{qn}{(n-1)p}\right)^{n-1} = \frac{p}{n}$		
	$\left(\frac{q}{n-1}\right)^{n-1} = \left(\frac{p}{n}\right)\left(\frac{p}{n}\right)^{n-1}$		
	$= \left(\frac{p}{n}\right)^n$		
	$\therefore \left(\frac{p}{n}\right)^n - \left(\frac{q}{n-1}\right)^{n-1} = 0$		