St Catherine's School
Waverley, Sydney

Student Number: _____

Year 12
Assessment Task 2
2007

Mathematics

Time allowed: 3 hr

Reading time: 5 min

General Instructions

- Attempt ALL questions
- Write your Student NUMBER at the top of this page and on each writing booklet.
- Calculators may be used

Sections	**Marks**
Q1-5 in Booklet 1 Q6-10 in Booklet 2	12 marks per Question

Total marks
120

Question 1 (12 marks)

a) Evaluate $\sqrt{4.28+9.714}$ to 1 decimal place **1**

b) Simplify $\frac{x}{5}-\frac{x-1}{3}$ **2**

c) Solve for x: $3x^2-9x-12=0$ **3**

d) Solve and graph on the number line: **2**

$12-2x>4$

e) Factorise $9x^2-16$ **2**

f) Graph on the number plane: $y=|x+1|$ **2**

Question 2 starts on the next page

Question 2 (12 marks)
(Start a new page)

a) Find a if $\sqrt{75}-\sqrt{3}=\sqrt{a}$ **2**

b) Solve for x and graph on the number line: **2**

$$x^2-4x\leq 0$$

c) Express in simplest form: $\frac{x^3-8}{x^2-4}$ **2**

d) Simplify $\frac{4}{x-3}-\frac{x-2}{x^2-x-6}$ **3**

e) Prove that $\frac{\sin^3\theta}{\cos\theta}+\sin\theta\cos\theta=\tan\theta$ **3**

Question 3 starts on the next page

Question 3 (12 marks)
(Start a new page)

a) The line k cuts the x-axis at A(6,0) and has gradient $m=\frac{4}{3}$

i) Show that $4x - 3y - 24 = 0$ is the equation of line k **2**

ii) Find the co-ordinates of **B**, the y-intercept of line k **1**

iii) Draw a neat sketch showing line k and points **A** and **B** on the number plane. **1**

iv) Find the distance of the origin from line k **1**

v) Line j is perpendicular to line k and also passes through A(6,0). Find its equation. **2**

b) Sketch the region $y > 2x + 1$ **2**

c) Sketch the graph of the function: **3**

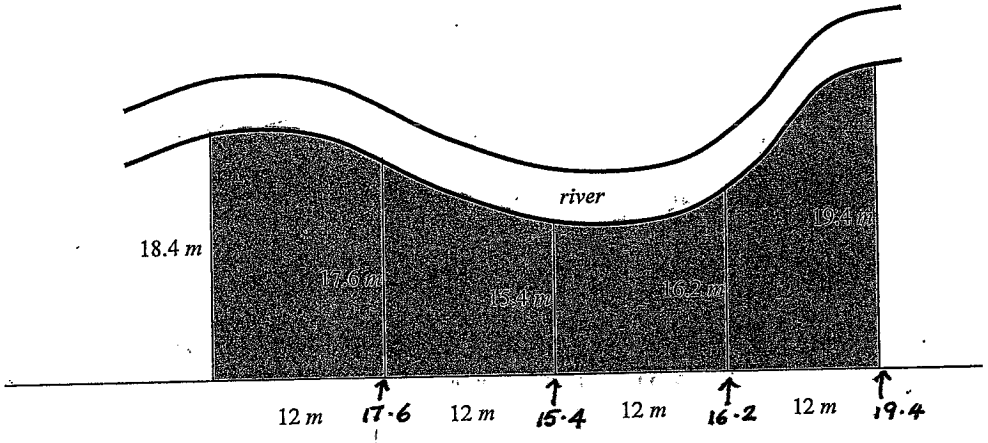
$$f(x)=\sqrt{9-x^2}$$

Specify the range and domain of this function

Question 4 starts on the next page

Question 4 (12 marks)
(Start a new page)

a) Farmer Brown's paddock, shown shaded below, is bounded by the river and 3 straight fences. Use Simpson's method to find the area of the paddock to the nearest m^2. 4



b) Differentiate:

i) $5x^{-4}$ 1

ii) $\dfrac{x^6-3x^2+4}{x^4}$ 2

iii) $\sqrt{3x-2}$ 2

c) Find the equation of the tangent to the curve $y=4x^2-6x$ at the point $P(2, 4)$ 3

Question 5 starts on the next page

Question 5 (12 marks)
(Start a new page)

a) Find the value of:

i) $\int_{-1}^{3}(x^2+5)dx$ 2

ii) $\int_{2}^{3}(9x-5)^3dx$ 2

iii) $\int_{4}^{9}\sqrt{x^3}dx$ 2

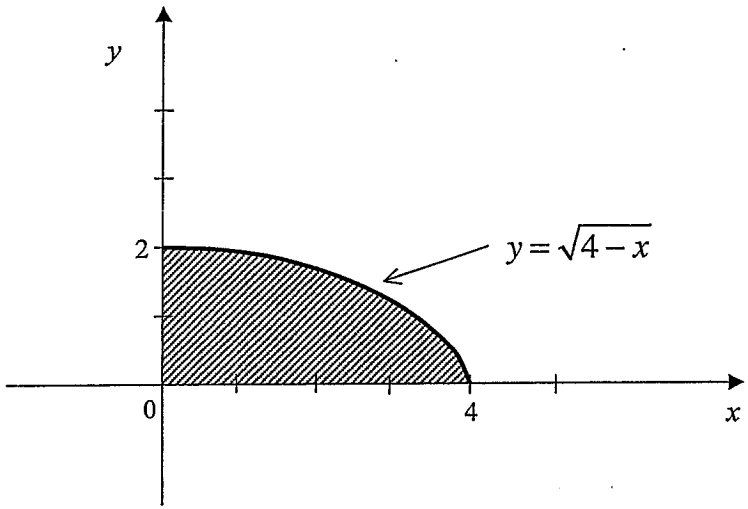
b)

i) Draw a neat sketch of the curve $y=x^3$ for $-2\le x\le 2$ 1

ii) Hence or otherwise evaluate $\int_{-2}^{2} x^3dx$ 3

Question 5 *continued*

c) A student is required to find the ~~area~~ Volume enclosed when the shaded area is rotated around the y-axis.



Her solution starts: 2

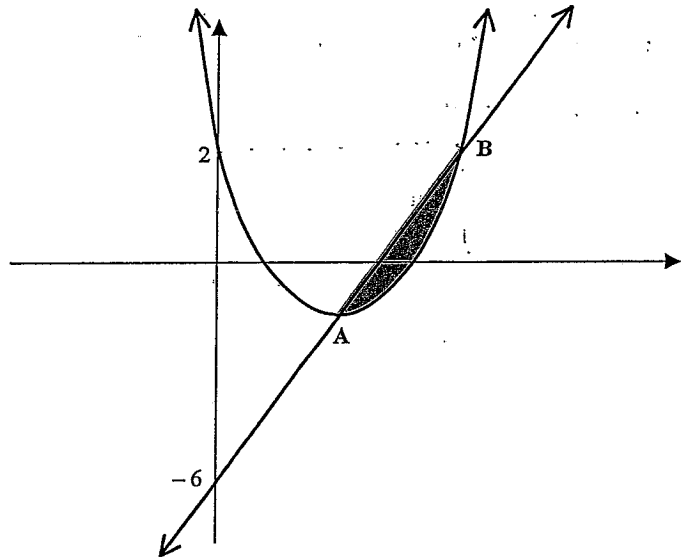
$$V=\pi\int_a^b x^2\,dy$$

$$\therefore V=\pi\int_0^4 (y-4)\,dy$$

The first line is correct but the second contains errors. Write the correct second line. *Do not continue the calculation.*

Question 6 starts on the next page

Question 6 (12 marks)
(Start a new booklet)



a) The diagram shows the parabola $y=x^2-4x+3$

i) Find the co-ordinates of A and B **2**

ii) Find the shaded area using integration. **4**

Question 6 *continued*

b) A parabola has its focus at (5,2) and its directrix is the line $y=8$.

i) Sketch the parabola giving the co-ordinates of its vertex. **2**

ii) What is the focal length of this parabola? **1**

iii) What is the equation of this parabola? **2**

iv) Write down the equation of another parabola with the same vertex and focal length. **1**

Question 7 starts on the next page

Question 7 (12 marks)
(Start a new page)

a) The tenth term of an arithmetic series is 93 and the fifteenth term is 68.
Find the first term a and common difference d. **3**

b) Find the sum of the first 20 terms of this series:

8,15,22..... **2**

c) $2+\sqrt{3}$, x, $2-\sqrt{3}$ ..... forms a geometric series

Find x **2**

d) For which values of k does a limiting sum exist for:

$1 + (2k-1) + (2k-1)^2 + ...$ **2**

3

e) A tree was planted at the beginning of the year 2000, when it was 70 cm tall. At the beginning of 2001 it was 92 cm tall and at the beginning of 2002 it was 114 cm tall.
Assuming it continues to grow at this rate, find its height at the beginning of 2010.

Question 8 starts on the next page

Question 8 (12 marks)
(Start a new page)

a) Prove that $\frac{1}{\sin\theta\cos\theta} - \tan\theta = \cot\theta$ **2**

b) From Town A two straight roads run to Town B and Town C. The bearing of B from A is $200°T$ and the bearing of C from A is $310°T$. A is 40 km from B and 55 km from C.

i) Draw a diagram showing the relative positions of the towns. **1**

ii) Use trigonometry to calculate the distance BC to the nearest kilometre. **3**

c) Evaluate $\sum_{h=0}^{6} 5h-1$ **2**

d) For the curve $f(x)=4x^3-6x^2+5$

i) Find the stationary points on the curve and determine their nature. **2**

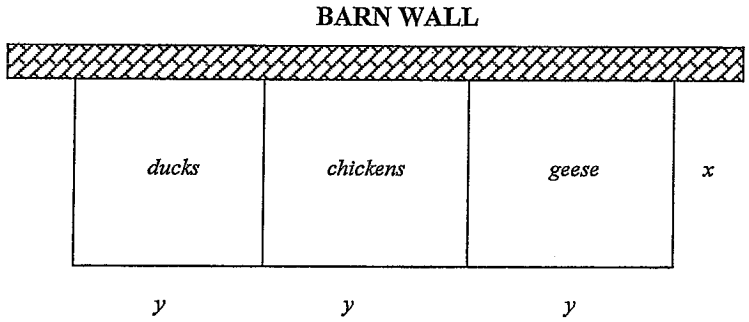
ii) Locate the point of inflection. **1**

iii) Sketch the curve **1**

Question 9 starts on the next page

Question 9 (12 marks)
(Start a new page)

a) Farmer Brown is building exercise pens for his ducks, chickens and geese. The pen is built against the barn wall and each kind of fowl has the same area of pen, as shown in the diagram. Farmer Brown has 60 m of fencing mesh.



i) Show that the area A in m^2 he can enclose is given by **2**

$A = x(60-4x)$

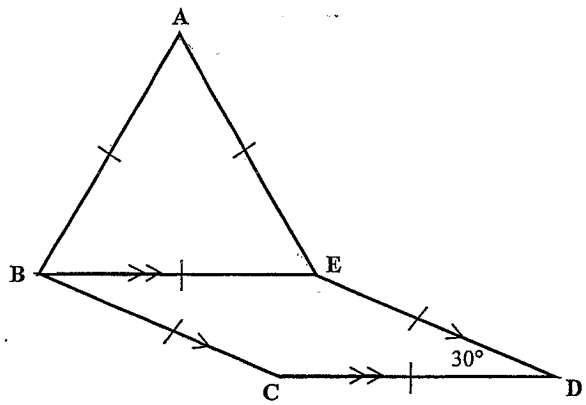
ii) Find the maximum area he can enclose. **4**

b) The quadratic equation $2x^2+9x-12=0$ has roots α and β **3**

Use the values of $\alpha+\beta$ and $\alpha\beta$ to find the value of $|\alpha-\beta|$

Question 9 *continued*

c) ΔAEB is an equilateral triangle and quadrilateral $EBCD$ is a rhombus with $\angle EDC = 30°$



i) Copy the diagram and mark in the size of all internal angles **1**

ii) Hence find the size of $\angle EDA$ **2**

Question 10 starts on the next page

Question 10 (12 marks)

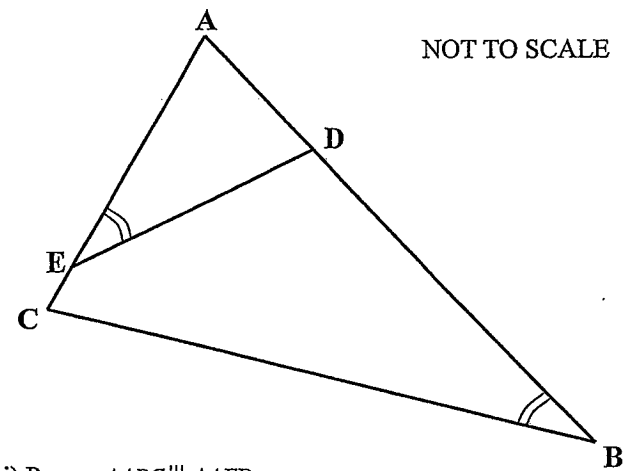
(Start a new page)

a) $ABCD$ is a quadrilateral with diagonals AC and BD intersecting at 90° at point E. AC bisects $\angle BCD$.

i) Draw a neat sketch to represent $ABCD$ and mark in all given information. **1**

ii) Using congruent triangles, prove that ΔBCD is isosceles. **3**

b) In the diagram below: $\angle ABC = \angle AED$



i) Prove $\Delta ABC \,|||\, \Delta AED$ **2**

ii) If $AB = 12$ cm, $AE = 8$ cm and $AC = 10$ cm, find AD **2**

c) $A(2,0)$ and $B(8,0)$ are points on the number plane. **4**
P is the point (x,y)
The locus of P is defined by $AP = 2BP$, so that the point P is always twice as far from A as it is from B.
Find the equation of the locus of P and describe the locus in words.

End of Paper

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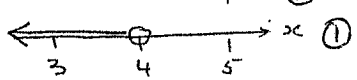
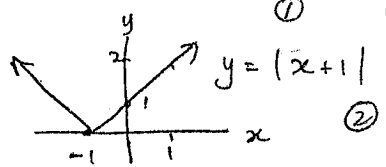
Question	Method	Comments
1 a)	$\sqrt{4.28+9.714} = 3.7$ (1 dp)	①
b)	$\frac{x}{5} - \frac{x-1}{3} = \frac{3x-5(x-1)}{15}$ (½ ½) $= \frac{-2x+5}{15}$ ①	
c)	$3x^2-9x-12=0$ $3(x^2-3x-4)=0$ ① $3(x-4)(x+1)=0$ ① $\therefore x-4=0$ or $x+1=0$ $x=4$ or $x=-1$ (½ ½)	
d)	$12-2x>4$ $-2x>-8$ (½) $x<4$ (½)  ①	
e)	$9x^2-16=(3x+4)(3x-4)$ ① ①	
f)	 $y=\lvert x+1\rvert$ ②	① for graph $y=x+1$ $-\frac{1}{2}$ no arrow heads. $-\frac{1}{2}$ no intercepts.

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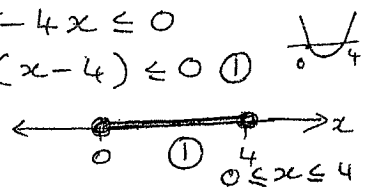
Question	Method	Comments
2 a)	$\sqrt{a} = \sqrt{75}-\sqrt{3}$ $= 5\sqrt{3}-\sqrt{3}$ (½) $= 4\sqrt{3}$ (½) $= \sqrt{48}$ (½) $\therefore a=48$ (½)	
b)	$x^2-4x \le 0$ $x(x-4) \le 0$ ① 	
c)	$\frac{x^3-8}{x^2-4} = \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$ ① (½) $= \frac{x^2+2x+4}{x+2}$ (½)	
d)	$\frac{4}{x-3} - \frac{x-2}{x^2-x-6} = \frac{4(x+2)}{(x-3)(x+2)}$ (½) $- \frac{(x-2)}{(x-3)(x+2)}$ (½) ① $= \frac{3x+10}{(x-3)(x+2)}$ ①	
e)	LHS = $\frac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta$ $= \frac{(1-\cos^2\theta)\cdot\sin\theta}{\cos\theta}$ ① $+ \sin\theta\cos\theta$ $= \frac{\sin\theta}{\cos\theta} - \sin\theta\cos\theta + \sin\theta\cos\theta$ ① $= \frac{\sin\theta}{\cos\theta} = \tan\theta = $ LHS	

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Question	Method	Comments

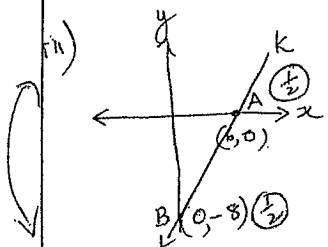
3a) i) Through $(6,0)$ with grad $\frac{4}{3}$

$y-0=\frac{4}{3}(x-6)$ $\frac{1}{2}$

$3y=4x-24$ 1

$\therefore 4x-3y-24=0$ $\frac{1}{2}$

ii)



$\frac{1}{2}$ $\frac{1}{2}$

iii) $4x-3y-24=0$

if $x=0$, $-3y=24$ $\frac{1}{2}$ $\therefore y=-8$

$\therefore B(0,-8)$ $\frac{1}{2}$

iv) $d=\left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

$=\left|\frac{0+0-24}{\sqrt{3^2+4^2}}\right|$ $\frac{1}{2}$

$=\frac{24}{5}=4.8$ u $\frac{1}{2}$

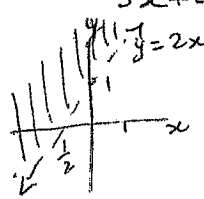
v) $j \perp k \therefore m=-\frac{3}{4}$ $\frac{1}{2}$ through $(0,6)$

$y-0=-\frac{3}{4}(x-6)$ $\frac{1}{2}$

$4y=-3x+18$ $\frac{1}{2}$

$3x+4y-18=0$ $\frac{1}{2}$

3b)



$\frac{1}{2}$ – intercepts.

$\frac{1}{2}$ – dotted correct line

1 correct region

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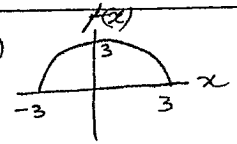
Question	Method	Comments

3b) i)



1

ii) domain: $-3 \le x \le 3$ 1

range: $0 \le f(x) \le 3$ 1

NB marks given if correct domain/range for wrong graph

Q4a)

fx	w	$w \cdot fx$
18.4	1	18.4
17.6	4	70.4
15.4	2	30.8
16.2	4	64.8
19.4	1	19.4
		203.8

$A=\frac{h}{3}$ 1 $\times 203.8$ 1

$=\frac{12}{3}\times 203.8$

$=815.2\,m^2$

$\therefore$ Area is $815\,m^2$ 1

correct h 1

$\div 3$ 1

No marks deducted for incorrect rounding or units

b) i) $\frac{d}{dx}(5x^{-4})=-20x^{-5}$ 1

ii) $\frac{d}{dx}\left(\frac{x^6-3x^2+4}{x^4}\right)$

$=\frac{d}{dx}(x^2-3x^{-2}+4x^{-4})$

$=2x+6x^{-3}-16x^{-5}$ 2

if done by quotient rule, $\frac{2x^9+6x^5-16x^3}{x^8}$ $1\frac{1}{2}$ because this exp. simplifies.

iii) $\frac{d}{dx}\left((3x-2)^{\frac{1}{2}}\right)=\frac{1}{2}(3x-2)^{-\frac{1}{2}}\cdot 3$ 1

$=\frac{3}{2}(3x-2)^{-\frac{1}{2}}$ 1

OR $\frac{3}{2\sqrt{3x-2}}$

c) $y=4x^2-6x$

$y=8x-6$ 1

at $x=2$, $y=10$

$\therefore m=10$ 1

eq: $y-4=10(x-2)$ $\frac{1}{2}$

$y=10x-16$ $\frac{1}{2}$

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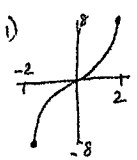
Question	Method	Comments
Q5a	i) $\int_{-1}^{3}(x^2+5)\,dx = \left[\frac{1}{3}x^3+5x\right]_{-1}^{3}$	(½) mark for reasonable attempt at raising power by 1 & dividing.
	$= 24 - -5\frac{1}{3}$	(½) mark for correct substitution into wrong integral.
	$= 29\frac{1}{3}$ or $\frac{88}{3}$	
	ii) $\int_{2}^{3}(9x-5)^3 = \left[\frac{1}{9}\cdot\frac{1}{4}(9x-5)^4\right]_{2}^{3}$ (1)	
	$= \frac{1}{36}\times 22^4 - \frac{1}{36}\times 13^4$	
	$= 5713\frac{3}{4}$ (1)	
	iii) $\int_{4}^{9} x^{3/2}\,dx = \left[\frac{2}{5}x^{5/2}\right]_{4}^{9}$ (1)	
	$= \frac{2}{5}\times 243 - \frac{2}{5}\times 32$	
	$= 84.4$ (1)	
b)	i)  ii) $\int_{-2}^{2} x^3\,dx = 0$	(½) - correct shape (½) - correct domain/range (1) - sensible attempt at integral (2) if correctly found area not integral
c)	$V = \pi\int_{a}^{b} x^2\,dy$ $\quad y^2 = 4-x$	(½) show $x = 4-y^2$
	$= \pi\int_{0}^{2}(4-y^2)^2\,dy$ $\quad x = 4-y^2$	(½) show $x^2 = (4-y^2)^2$
	or $= \pi\int_{0}^{2}(16-8y^2+y^4)\,dy$. $\quad \therefore x^2 = (4-y^2)^2$	(½) correct bounds.

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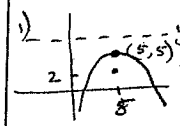
Question	Method	Comments
Q6a i)	solve simultaneously $y = x^2-4x+2$ & $y = 2x-6$	solve simultaneously (1) correct points (1).
	$x^2-4x+2 = 2x-6$	
	$x^2-6x+8 = 0$ (1)	
	$(x-2)(x-4) = 0$	
	$\therefore x = 2$ or $x = 4$	
	A (2, −2) B (4, 2) (1)	
ii)	$A = \int_{2}^{4} 2x-6 - \int_{2}^{4} x^2-4x+2$ dx	↖ can integrate separately giving $= (-8 - -8) - (-2\frac{2}{3} - -1\frac{1}{3})$
	$= \int_{2}^{4} -x^2+6x-8\,dx$	$= 1\frac{1}{3}$
	$= \left[-\frac{1}{3}x^3+3x^2-8x\right]_{2}^{4}$	correct subtraction (upper − lower) (½) correct limits (1) correct integration (1) correct substitution (1) correct answer (½)
	$= -5\frac{1}{3} - -6\frac{2}{3} = 1\frac{1}{3}$ u² //	
b) i)	 (1) Vertex (5,5) (1)	i) diagram (1) vertex (1)
ii)	focal length 3 (1)	ii) focal length (1)
iii)	$(x-5)^2 = -12(y-5)$ (½) (1) (½)	iii) correct eq. (2)
iv)	$(x-5)^2 = 12(y-5)$	iv) correct eq (1)
	or $(y-5)^2 = 12(x-5)$ (1)	
	or $(y-5)^2 = -12(x-5)$	

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Question	Method	Comments
Q7a	$T_{10} = 93 \quad a+9d = 93$ $T_{15} = 68 \quad a+14d = 68$ ① $5d = -25$ $d = -5$ ① $\therefore a - 45 = 93$ $a = 138$ ① $a = 138,\ d = -5$	
b)	$S_n = \frac{n}{2}(2a+(n-1)d)$ $S_{20} = \frac{20}{2}(16 + 19\times 7)$ ① $= 1490$ ①	
c)	$\frac{x}{x+\sqrt{3}} = \frac{2-\sqrt{3}}{x}$ ① $x^2 = 2^2 - 3$ $x^2 = 1 \quad \therefore x = \pm 1$ ①	
d)	$\lvert 2k-1 \rvert < 1$ ① $-1 < 2k-1 < 1$ $0 < 2k < 2$ $0 < k < 1$ ①	
e)	70, 92, 114 ... Ap with $a = 70$, $d = 22$ 2000 $T_1 = 70$ 2001 $T_2 = 70 + 22$ 2002 $T_3 = 70 + 22\times 2$ 2010 $\therefore T_{11} = 70 + 22\times 10$ $= 290$ cm. ③	gwe ② for T_{10} 268

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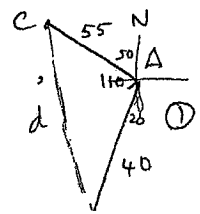
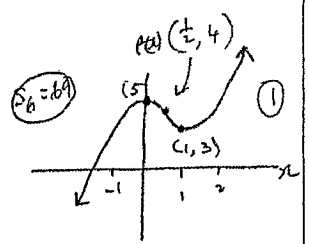
Question	Method	Comments
Q8a)	LHS $= \frac{1}{\sin\theta\cos\theta} - \tan\theta$ $= \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta}$ ① $= \frac{1-\sin^2\theta}{\sin\theta\cos\theta}$ $= \frac{\cos^2\theta}{\sin\theta\cos\theta}$ ① $= \frac{\cos\theta}{\sin\theta}$ $= \cot\theta =$ RHS.	
b)	 $d^2 = a^2+b^2-2cb\cos A$ ① $= 40^2+55^2-2\times 40\times 55\cos 110$ $= 6129.9$ ① $d = 78.3$ km $\therefore$ distance $\doteq$ 78 km. (nearest km) ①	
c)	$-1 + 4 + 9 \ldots + 29$ (7 terms) $S_7 = \frac{7}{2}(-1+29) = 98$ ① ①	 ①
d) i)	$f(x) = 4x^3 - 6x^2 + 5$ $f'(x) = 12x^2 - 12x$ $f'(x) = 0$ when $x = 0,\ x = 1$ $\therefore$ st pts at $(0, 5)$ and $(1, 3)$ ① $f''(x) = 24x - 12$ at $x = 0$ $f''(x) = -12$ $\therefore \cap$ max tp ① at $x = 1$ $f''(x) = 12$ $\therefore \cup$ min tp Pt of inflex when $f''(x) = 0$ $\therefore x = \frac{1}{2}$ $(\frac{1}{2}, 4)$ is ① pt	

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Question	Method	Comments
9a)	$3y + 4x = 60$	
	$\therefore y = \frac{60 - 4x}{3}$	(1)
	Area $= 3y \times x$	
	$= 3 \times \frac{60-4x}{3} \cdot x$	
	$= x(60-4x)$	(1)
	$A = 60x - 4x^2$	
	$\frac{dA}{dx} = 60 - 8x$	(1)
	max/min when $\frac{dA}{dx} = 0$	
	$60 - 8x = 0$	
	$8x = 60$	
	$x = 7\frac{1}{2}$	
	$\frac{d^2A}{dx^2} = -8 < 0 \therefore \cap$	(1)
	$\therefore$ max area $= 7\frac{1}{2} \times (60-30)$	(1)
	$= 225\ m^2$	(1)

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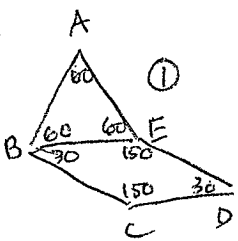
Question	Method	Comments
9b)	$2x^2 + 9x - 12 = 0$	
	$\alpha + \beta = -\frac{9}{2}$	(1)
	$\alpha\beta = -\frac{12}{2} = -6.$	
	now $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$	
	$= \alpha^2 + 2\alpha\beta + \beta^2 - 4\alpha\beta$	
	$= (\alpha+\beta)^2 - 4\alpha\beta$	
	$= \left(-\frac{9}{2}\right)^2 - 4 \times -6$	
	$= 20\frac{1}{4} + 24$	
	$= 44\frac{1}{4}$	(2)
	$\therefore \lvert\alpha - \beta\rvert = \sqrt{44\frac{1}{4}}$	
	$= \sqrt{\frac{177}{4}} = \frac{\sqrt{177}}{2}$	
9c)	 (1)	AE = ED (given) $\therefore$ $\triangle$AED is isosceles with one angle = 150°
	$\therefore \angle EDA = \frac{1}{2}(180-150)$	$\angle AED = 360° - (60+150)$
	$= 15°$	$= 150°$ (½) Creve at (½)
	(1)	

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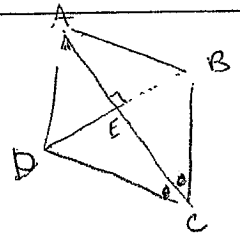
Question	Method	Comments
10 a)	 (1)	draw a general quadrilateral
	In $\triangle$s CED, CEB $\angle ECD = \angle ECB$ (given) EC is common $\angle DEC = \angle BEC$ (AC $\perp$ DB) (1) $\therefore \triangle CED \equiv \triangle CEB$ (AAS) (1) or $\angle EDC = \angle EBC$ (2 prs of angles + corresp sides equal) $\therefore CD = CB$ (corresp sides of cong. $\triangle$s) $\therefore \triangle BCD$ is isosceles (1)	
10b)	In $\triangle$s ABC, AED $\angle A$ is common $\angle ABC = \angle AED$ (given) (1) $\angle ACB = \angle ADE$ ($\angle$ sum $\triangle$) $\therefore \triangle ABC \,\|\|\|\, \triangle AED$ (equiangular) (1)	
ii)	$\therefore \frac{AB}{AE} = \frac{AC}{AD}$ (corresp sides of similar $\triangle$s) ($\frac{1}{2}$) ($\frac{1}{2}$) $\frac{12}{8} = \frac{10}{AD}$ $\rightarrow AD = \frac{10 \times 8}{12}$ $= 6\frac{2}{3}$ cm. (1)	

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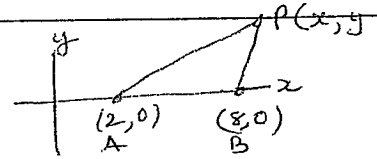
Question	Method	Comments
10c)	 $AP = 2PB$ $\sqrt{(x-2)^2+y^2} = 2\sqrt{(x-8)^2+y^2}$ (1) $x^2-4x+4+y^2 = 4(x^2-16x+64+y^2)$ $x^2-4x+4+y^2 = 4x^2-64x+256+4y^2$ $0 = 3x^2-60x+252+3y^2$ (1) $0 = x^2-20x+84+y^2$ $-84 = x^2-20x+y^2$ $-84+100 = x^2-20x+100+y^2$ (1) $16 = (x-10)^2+y^2$ $\therefore$ Locus of P is $(x-10)^2+y^2 = 4^2$ which is a circle, (1) centre (10, 0) radius 4	