Student Number:__

St. Catherine's School
Waverley

2008

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 1 – 15%
CLASS TEST: 20th February

Q1	15	/15
Q2		/18
Q3		/14
TOT		/47

Mathematics

General Instructions

- Working time – 55 minutes
- Start each question on a new page in your answer booklet.
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary **working** must be shown.
- Marks may be deducted for careless or badly arranged work.

Total marks – 50

- Attempt Questions 1–3
- Questions are not of equal value

Question 1

a) Melanie is saving to buy a bike costing $480

The first week, she saved $5, the second week she saved $7, the third week she saved $9, and so on, until she had enough money.

Her savings form an arithmetic sequence.

(i) What is the **first term** and **common difference**? **1**

(ii) How much does she save in the tenth week? **2**

(ii) After how many weeks will she have saved $480? **3**

b) Evaluate $\sum_{k=2}^{5} 10^k$ **2**

c) Find the limiting sum of the series $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+...$ **2**

d) Find the common ratio of the Geometric Sequence which has 18 as its 3rd term and 144 as its 6th term. **2**

e) **i)** What is the condition for a series $1+a+ar+ar^2.....$ to have a limiting sum? **1**

ii) For which values of x does the limiting sum of this series exist? **2**

$$(2-5x)+(2-5x)^2+(2-5x)^3.....$$

Question 2 **Marks**

a) Differentiate each of the following with respect to x.

(i) $f(x)=3x^5-5x^3$ **1**

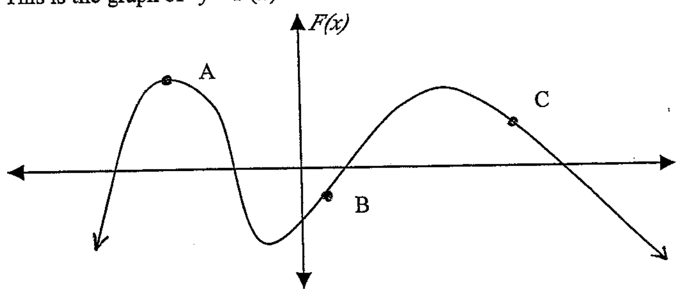
(ii) $f(x)=\dfrac{1}{x^6}$ **2**

(iii) $f(x)=\dfrac{5}{\sqrt{x+4}}$ **2**

(iv) $f(x)=x^4\left(3x^2+1\right)$ **2**

(v) $f(x)=\dfrac{x}{x^2+1}$ **3**

b) This is the graph of $y=F(x)$



From the graph, describe the value (positive, negative or zero) of each of these:

i) $F'(x)$ at point A **1**

ii) $F(x)$ at point B **1**

iii) $F''(x)$ at point C **1**

c) Find the equation of the tangent to the curve $y=x^3+\dfrac{3}{x^2}$ at the point $(1, 4)$ **2**

d) Show that the curve $y=x^3+\dfrac{3}{x^2}$ is concave up at the point $(1, 4)$ **2**

Question 3 **Marks**

a) "When Susie and Michael started to save for a holiday, they each had the same amount of money in the bank.

Susie's savings are increasing, but at a decreasing rate, while

Michael's savings are increasing at a constant rate".

Sketch a graph with Time on the horizontal axis, and Savings on the vertical axis, to illustrate the savings of each person. **2**

b) For the curve $f(x)=3x^4-8x^3$

(i) Find the stationary points and determine their nature. **3**

(ii) Sketch the curve $y=f(x)$ for $0\le x\le 5$ showing any points of inflexion. **3**

c) For the curve $F(x)=x^5-9x^4+26x^3-34x^2+21x-5$

(i) Show that there is a stationary point at $x=1$ **2**

(ii) Show that $F''(1)=0$ **2**

(ii) What kind of stationary point is at $x=1$? Justify your answer. **2**

End of Paper

Year 12 Mathematics Assessment Task 1 2008 SOLUTIONS

Q1.

a) i) 5, 7, 9.... $a=5$ $d=2$

ii) $T_{10} = a+9d$

$= 5+9\times 2$

$= 23$ $\therefore$ \$23 saved in week 10

iii) Total \$480 $\therefore$ $S_n = 480$

$S_n = \frac{n}{2}(2a+(n-1)d)$

$480 = \frac{n}{2}(10+(n-1)\times 2)$

$480 = \frac{n}{2}(8+2n)$

$480 = 4n+n^2$

$\therefore \quad 0 = n^2+4n-480$

$= (n+24)(n-20)$

$\therefore$ $n=-24$ or $n=20$

Ignore -ve, as $n \geq 0$

$\therefore$ She takes 20 weeks to save \$480

b) $\sum_{k=2}^{5} 10^k = 100+1000+10000+100000$

$= 111100$ (or use formula)

c) $S_\infty = \frac{a}{1-r}$

$= \frac{1}{1-\frac{-2}{3}} = 1 \div \frac{5}{3} = \frac{3}{5}$

1d) ~~$T_3 = 18 \therefore ar^2 = 18$~~

$T_6 = 144 \quad \therefore ar^5 = 144$

$r^3 = 8 \quad \therefore r = 2$

e) i) $|r| < 1$

ii) $|2-5x| < 1 \quad \therefore \quad -1 < 2-5x < 1$

$-3 < -5x < -1$

$\frac{3}{5} > x > \frac{1}{5}$

or $\frac{1}{5} < x < \frac{3}{5}$.

Question 2

a) i) $\frac{d}{dx}(3x^5 - 5x^3) = 15x^4 - 15x^2$

ii) $\frac{d}{dx}\left(\frac{1}{x^6}\right) = \frac{d}{dx} x^{-6} = -6x^{-7}$ or $\frac{-6}{x^7}$

iii) $\frac{d}{dx}\left(\frac{5}{\sqrt{x+4}}\right) = \frac{d}{dx} 5(x+4)^{-\frac{1}{2}} = -\frac{5}{2}(x+4)^{-3/2}$

$\left(\text{or } \frac{-5}{\sqrt{(x+4)^3}}\right)$

iv) $\frac{d}{dx} x^4(3x^2+1) = \frac{d}{dx}(3x^6 + x^4)$

$= 18x^5 + 4x^3$

(v) $\frac{d}{dx}\left(\frac{x}{x^2+1}\right) = \frac{(x^2+1)\cdot 1 - x \cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$

Year 12 Mathematics Assessment Task 1 2008 SOLUTIONS

2b) i) $F'(x) = 0$ zero slope.

ii) $F(x) < 0$ below x-axis

iii) $F''(x) < 0$ conc down

c) $y = x^3 + 3x^{-2}$.

$y' = 3x^2 - 6x^{-3}$

at $x = 1$, $y' = 3 - 6 = -3 \quad \therefore$ grad -3.

eq: $y - 4 = -3(x-1)$

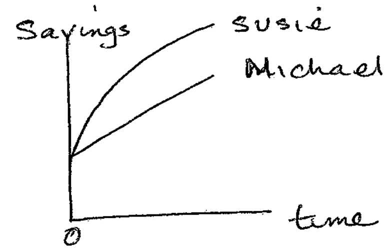
$y = -3x + 7$

d) $y'' = 6x + 18x^{-4}$

at $x = 1$, $y'' = 6 + 18 > 0 \quad \therefore$ conc up at $(1,4)$

Question 3

a)



Q3 b) $y = 3x^4 - 8x^3$ $\qquad y'' = 36x^2 - 48x$

$y' = 12x^3 - 24x^2$

Stationary pts: $y' = 0$ when $12x^3 - 24x^2 = 0$.

$12x^2(x-2) = 0$

$\therefore\ x = 0$ or $x = 2$.

at $x=0$, $y=0$ $\therefore$ $(0,0)$ is a st pt

at $x=0$, $y''=0$ $\therefore$ could be hpi

check l+r

x	-1	0	1
$f'(x)$	-36	0	-12

$\therefore$ horizontal pt of inflection at $(0,0)$

at $x=2$, $y=-16$ $\quad (2,-16)$ is a st pt

at $x=2$ $y'' = 48 > 0$ $\therefore$ $(2,-16)$ is a min tp

c) locate any inflection points: $f''(x) = 0$.

$36x^2 - 48x = 0$

$12x(3x-4) = 0$

$\therefore\ x = 0 \quad x = \frac{4}{3}$.

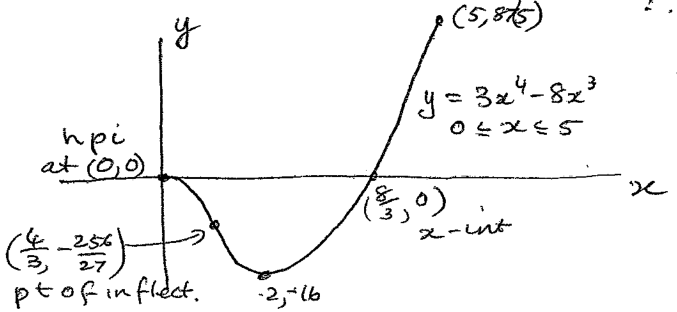
$(0,0)$ is hpi — see above.

$(\frac{4}{3}, \frac{-256}{27})$ is pt of inflex { check l+r

x	1	$\frac{4}{3}$	2
$f''(x)$	-12	0	48

$\therefore$ concavity chan

at $x=5$ $y=875$



3c) $f(x) = x^5 - 9x^4 + 26x^3 - 34x^2 + 21x - 5$

i) $f'(x) = 5x^4 - 36x^3 + 78x^2 - 68x + 21$

$f'(1) = 5 - 36 + 78 - 68 + 21$

$= 0$

$\therefore$ st pt at $x = 1$ as $f'(1) = 0$.

ii) $f''(x) = 20x^3 - 108x^2 + 156x - 68$.

$f''(1) = 20 - 108 + 156 - 68$

$\therefore f''(1) = 0$ ☺ as required.

iii) at $x=1$, $f'(x) = 0$ and $f''(x) = 0$

$\therefore$ check $f'(x)$ l+r of $x=1$

x	0	1	2
$f'(x)$	21	0	-11

$\therefore$ max tp at $x = 1$