Student Number:

St. Catherine's School
Waverley

2007

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 1 – 15%
CLASS TEST: 12th February

Mathematics

General Instructions

- Working time – 55 minutes
- Start each question on a new page in your answer booklet.
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work.

Total marks – 45

- Attempt Questions 1–3
- All questions are of equal value

Question 1

a) The 3rd term of an arithmetic progression is 16, and the 12th term is 79

(i) Find the first term and common difference. 2

(ii) Find the sum of the first 25 terms. 2

b) Evaluate $\sum_{k=1}^{10}(3k+2)$ 2

c) How many terms of the series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\ldots$ give a sum of $\frac{1023}{1024}$? 3

d) Find the limiting sum of the series $1+\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+\ldots$ 2

e) Find the values of x for which $2x+2$, $5x+1$, $10x+2$ form:

(i) An arithmetic progression 2

(ii) A geometric progression 2

Question 2 Marks

a) Differentiate each of the following with respect to x.

(i) $y = 3x^2 - 7x$ 1

(ii) $y = \dfrac{x^2 + 3x}{x^5}$ 2

(iii) $y = \sqrt{3x+4}$ 2

(iv) $y = (x^3 - 5)(3x^2 + 1)$ 3

b) A curve has the equation: $y = 2x^4 - 5x^2 - 1$.
Find:

(i) The gradient of the tangent to the curve at the point $(-1\ ;\ -4)$ 2

(ii) Show that the equation of the normal at $x = -1$ is given by: 2

$$x + 2y + 9 = 0$$

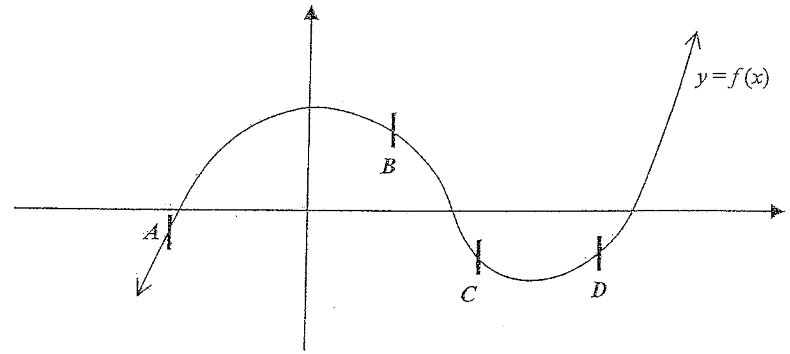
c) Find $f''(x)$ for the function: 3

$$f(x) = \frac{x}{2x+1}$$

Question 3 Marks

a) State which point on the sketch below fits all of the following descriptions: 1

$$y < 0 \quad , \quad \frac{dy}{dx} > 0 \quad , \quad \frac{d^2y}{dx^2} < 0$$



b) A right-angled triangle needs to be created subject to the condition that the sum of the perpendicular sides is to be 26 cm.

(i) Show that the area of the triangle is given by: 2

$$A = \frac{26x - x^2}{2}$$

(ii) Find the lengths of the sides that will give a maximum area. 3

c) For the curve $y = x^3 + 6x^2 + 9x + 1$:

(i) Find **all** turning points on the curve and determine their nature. 4

(ii) Find any points of inflexion 2

(iii) Find the absolute maximum point in the domain $-4 \le x \le 1$ 1

(iv) Sketch and label the curve on a number plane. 2

End of Paper

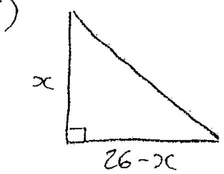
Qu	Solutions	Marks	Comments+Criteria
	a)		
	(i) $T_3 \Rightarrow \quad a+(3-1)d=16$		
	$a+2d=16 \quad \ldots\ldots (1)$	$\frac{1}{2}$	
	$T_{16} \Rightarrow \quad a+(12-1)d=79$		
	$a+11d=79 \quad \ldots\ldots (2)$	$\frac{1}{2}$	
	$(2)-(1) \quad 9d=63$		
	$\underline{\underline{d=7}}$	$\frac{1}{2}$	
	substitute into (1)		
	$a+2\times 7=16$		
	$a+14=16$		
	$\underline{\underline{a=2}}$	$\frac{1}{2}$	
	(ii) $S_{25} = \frac{n}{2}\left(2a+(n-1)d\right)$		
	$= \frac{25}{2}\left(2\times 2+(25-1)\times 7\right)$	1	
	$= \underline{\underline{2150}}$	1	
	b) $\sum_{k=1}^{10}(3k+2)$ $\quad a=3\times 1+2$		
	$\underline{\underline{a=5}}$	$\frac{1}{2}$	
	$\underline{\underline{d=3}}$	$\frac{1}{2}$	
	$\underline{\underline{n=10}}$	$\frac{1}{2}$	
	$S_{10} = \frac{n}{2}\left(2a+(n-1)d\right)$		
	$= \frac{10}{2}\left(2\times 5+(10-1)\times 3\right)$		
	$= \underline{\underline{185}}$	$\frac{1}{2}$	

Qu	Solutions	Marks	Comments+Criteria
	c) $a=\frac{1}{2}$, $r=\frac{1}{2}$, $S_n=\frac{1023}{1024}$	$\frac{1}{2}$	
	$S_n = \frac{a(1-r^n)}{1-r}$	$\frac{1}{2}$	
	$\frac{1023}{1024} = \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$	$\frac{1}{2}$	
	$\frac{1023}{1024} = 1-\left(\frac{1}{2}\right)^n$		
	$\left(\frac{1}{2}\right)^n = \frac{1}{1024}$	$\frac{1}{2}$	
	$\therefore 2^n = 1024$	$\frac{1}{2}$	
	$\therefore n=10$	$\frac{1}{2}$	
	$\therefore$ 10 terms of the G.P. give a sum of $\frac{1023}{1024}$		
	D) $a=1$, $r=\frac{3}{4}$	$\frac{1}{2}$	
	$S_\infty = \frac{a}{1-r}$	$\frac{1}{2}$	
	$= \frac{1}{1-\frac{3}{4}}$		
	$= \frac{1}{\frac{1}{4}}$		
	$= 4$	1	

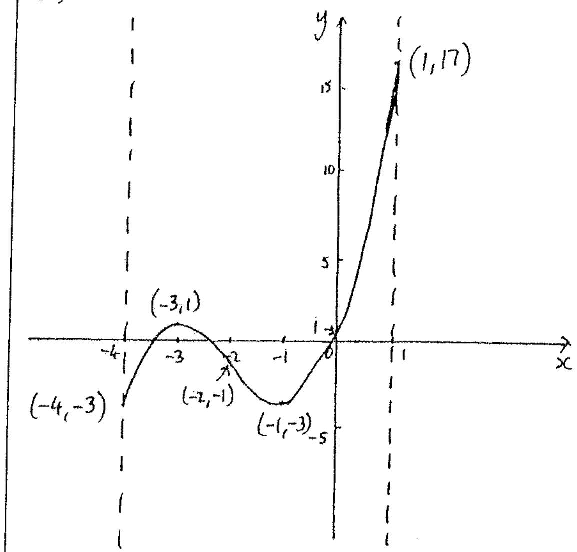
Qn	Solutions	Marks	Comments+Criteria
	e) (i) For Arithmetic		
	$5x+1-(2x+2)=10x+2-(5x+1)$	1	
	$5x+1-2x-2=10x+2-5x-1$		
	$3x-1=5x+1$	$\frac{1}{2}$	
	$-2=2x$		
	$\underline{\underline{x=-1}}$	$\frac{1}{2}$	
	(ii) For Geometric		
	$\frac{5x+1}{2x+2} = \frac{10x+2}{5x+1}$	$\frac{1}{2}$	
	$(5x+1)^2=(10x+2)(2x+2)$	$\frac{1}{2}$	
	$25x^2+10x+1=20x^2+24x+4$		
	$5x^2-14x-3=0$		
	$(5x+1)(x-3)=0$	$\frac{1}{2}$	
	$\therefore x=-\frac{1}{5}$ or 3	$\frac{1}{2}$	
	* $x\neq-\frac{1}{5}$ as this makes the denominator of $\frac{10x+2}{5x+1}$ equal 0.		
	$\therefore \underline{\underline{x=3}}$		
	NB $\frac{5x+1}{2x+2}=\frac{2\cancel{(5x+1)}}{\cancel{5x+1}}$		
	$5x+1=4x+4$		
	$\underline{\underline{x=3}}$ — Full marks also given for this method.		

Qn	Solutions	Marks	Comments+Criteria
2	A) (i) $y=3x^2-7x$		
	$\underline{y'=6x-7}$	1	
	(ii) $y=\frac{x^2+3x}{x^5}$		
	$y=x^{-3}+3x^{-4}$	$\frac{1}{2}$	
	$y'=-3x^{-4}-12x^{-5}$	$\frac{1}{2}$	
	$\underline{\underline{y'=\frac{-3}{x^4}-\frac{12}{x^5}}}$	1	
	(iii) $y=\sqrt{3x+4}=(3x+4)^{\frac{1}{2}}$	$\frac{1}{2}$	
	$y'=\frac{1}{2}(3x+4)^{-\frac{1}{2}}\times 3$	1	
	$=\frac{3}{2\sqrt{3x+4}}$ ← NO MARKS HAVE BEEN DEDUCTED IF NOT IN THIS FORM	$\frac{1}{2}$	
	(iv) $y=(x^3-5)(3x^2+1)$		
	$=3x^5+x^3-15x^2-5$		
	$y'=15x^4+3x^2-30x$	3	
	OR		
	$u=x^3-5$ $\quad v=3x^2+1$		
	$u'=3x^2$ $\quad v'=6x$		
	$y'=vu'+uv'$		
	$=(3x^2+1)3x^2+(x^3-5)6x$		
	$=9x^4+3x^2+6x^4-30x$		
	$=15x^4+3x^2-30x$		
	$=3x(5x^3+x-10)$		← not needed for full marks

Qn	Solutions	Marks	Comments+Criteria
	b) $y = 2x^4 - 5x^2 - 1$		
	$y' = 8x^3 - 10x$		
	(i) $(-1, -4)$		
	$m_T = 8 \times (-1)^3 - (10 \times -1)$	1	
	$= 2$	1	
	(ii) $m_n = -\frac{1}{m_T}$		
	$= -\frac{1}{2}$	$\frac{1}{2}$	
	$\therefore m = -\frac{1}{2}, \quad (-1, -4)$		
	$y - y_1 = m(x - x_1)$		
	$y + 4 = -\frac{1}{2}(x + 1)$	$\frac{1}{2}$	
	$2y + 8 = -x - 1$		
	$\underline{\underline{x + 2y + 9 = 0}}$	1	
	c) $f(x) = \frac{x}{2x+1}$		
	$u = x \quad v = 2x+1$		
	$u' = 1 \quad v' = 2$		
	$f'(x) = \frac{2x - (2x+1)}{(2x+1)^2}$	1	
	$= \frac{-1}{(2x+1)^2} = -(2x+1)^{-2}$	1	
	$f''(x) = 2(2x+1)^{-3} \times 2$		
	$= \underline{\underline{\frac{4}{(2x+1)^3}}}$	1	

Qn	Solutions	Marks	Comments+Criteria
3	A) A	1	
	B) (i)		
			
	Area $= \frac{1}{2} \times (26 - x) \times x$	1	
	$= \frac{x(26-x)}{2}$		
	$= \frac{26x - x^2}{2}$	1	
	(ii) For Max/Min, $dA/dx = 0$	1	
	$dA/dx = 13 - x$		
	$\therefore 13 - x = 0$		
	$x = 13$		
	$\therefore$ short side lengths are 13cm & 13cm	1	
	hypotenuse $= \sqrt{338}$ units		
	≈ 18.4 units		
	$d^2A/dx^2 = -1$		-1 if A'' is not considered.
	$\therefore < 0 \quad \therefore$ The area is max.		
	c) $y = x^3 + 6x^2 + 9x + 1$		
	$y' = 3x^2 + 12x + 9$		
	$y'' = 6x + 12$		
	(i) Turning points : $y' = 0$		
	$3x^2 + 12x + 9 = 0$	1	
	$3(x^2 + 4x + 3) = 0$		
	$3(x+3)(x+1) = 0$		
	$\therefore x = -1 \quad x = -3$	1	

Qn	Solutions	Marks	Comments+Criteria
	(i) Cont.		no y-coord $\left(-\frac{1}{2}\right)$
	when $x=-1$, $y=-3$ $\therefore (-1,-3)$	$\frac{1}{2}$	
	$y''=6\times -1+12$ $=6$		
	$\therefore y''>0$ $\therefore$ Min	1	
	when $x=-3$, $y=1$ $\therefore (-3,1)$		
	$y''=6\times -3+12$ $=-6$		
	$\therefore y''<0$ $\therefore$ Max	1	
	(ii) Inflexion Point: $y''=0$ change of concavity		
	$\therefore 6x+12=0$ $6x=-12$ $\underline{\underline{x=-2}}$	1	
	when $x=-2$, $y=-1$ $\therefore (-2,-1)$		
	test/check $x=-3$, $y''<0$, $x=-1$, $y''>0$. $\therefore$ concavity changes	1	
	(iii) ABSOLUTE MAX BOUNDARY VALUES		
	when $x=-4$, $y=-3$ $(-4,-3)$ when $x=1$, $y=17$ $(1,17)$	$\frac{1}{2}$	
	$\therefore$ absolute max point $=(1,17)$	$\frac{1}{2}$	

Qn	Solutions	Marks	Comments+Criteria
3	(IV) 	2	