ST CATHERINE'S COLLEGE - ½ YRLY - MATHS 2002
YEAR 12

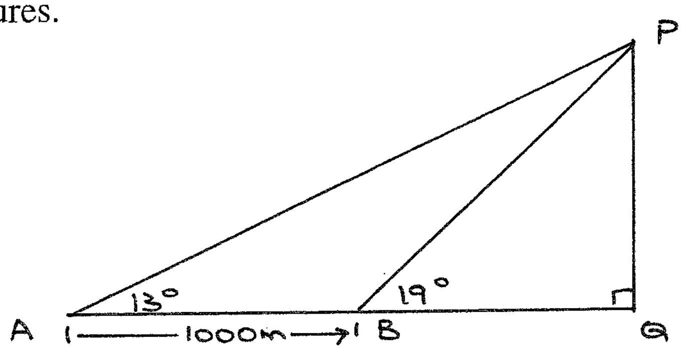
Question 1

a) Find:

i) $\int 4x+1.dx$ 1mark

ii) $\int x^2-3x+7.dx$ 2marks

b) From a point A , the angle of elevation of a mountain PQ is $13°$. After moving a distance of 1000m, over level ground, towards the mountain, the angle of elevation is $19°$. Calculate the height of the mountain to three significant figures. 3marks



c) If α and β are the roots of $3x^2-2x-1=0$, find :

i) $\alpha+\beta$ 1mark

ii) $\alpha\beta$ 1mark

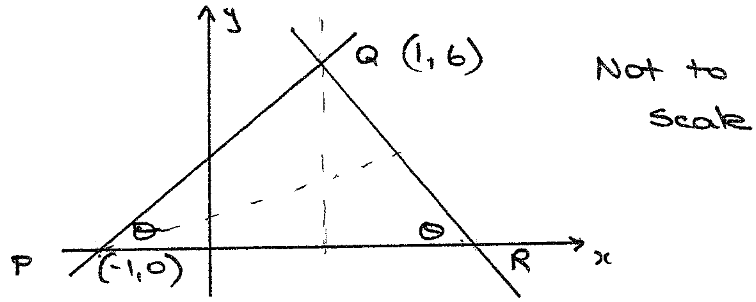
iii) $\frac{1}{\alpha}+\frac{1}{\beta}$ 1marks

iv) Form a quadratic equation where the roots are 2α *and* 2β 2marks

d) Find the primitive function of $2x+11$. 1mark

Question 2

In the diagram, P and Q have coordinates (-1,0) and (1,6), respectively ,R is a point on the x axis and $\angle QPR = \angle QRP = \theta$.



Copy the diagram into your examination booklet.

i) Find the coordinates of the midpoint of PQ — 1mark

ii) Show that PQ has equation $y = 3x + 3$. — 2marks

iii) Show that tan $\theta = 3$ — 1mark

iv) Show that the gradient of QR is –3. — 1mark

v) Show that the equation of QR is $3x + y - 9 = 0$ — 2marks

vi) Find coordinates of R. — 1mark

vii) Find the perpendicular distance from P to QR — 2marks

viii) Find the area of ΔPQR — 2marks

Question 3.

a)

i) Sketch the curve $y = 4x - x^2$. — 1mark

ii) Hence find the area between the curve and the x axis. — 3marks

b) Find $\int (5x+3)^6 .dx$ 2marks

c) Given the series:

$T_n = 8n - 5$, where T_n denotes the nth term,

i) find the first term 1mark

ii) find the common difference 1mark

iii) find the sum to 10 terms. 2marks

d) Express $0\cdot\dot{2}\dot{3}$ as a fraction . 2marks

<u>Question 4.</u>

a) Consider the equation $x^2 + (k+2)x + 4 = 0$.

Find what values of k does the equation have:

i) equal roots 2marks

ii) distinct,real roots. 2marks

b) On the number plane, shade the region given by the conditions

$x^2 + y^2 \leq 9 \ \ and \ \ y \leq 3 - x$ 3marks

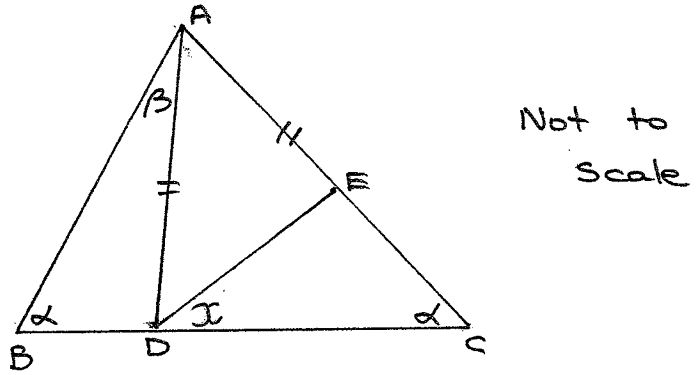
c) Solve : $9^x + 2.3^x - 15 = 0$ 3marks

d) Simplify: $\dfrac{\cos\alpha}{1-\sin\alpha} - \dfrac{\cos\alpha}{1+\sin\alpha}$ 2marks

Question 5.

a) Find : $\int_0^1 \frac{1+x+2\sqrt{x}}{\sqrt{x}}\,dx$ 4marks

b)



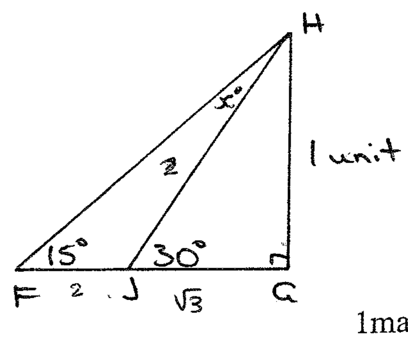
In the isosceles triangle ABC, $\angle ABC = \angle ACB = \alpha$. The points D and E lie on BC and AC respectively, so that AD= AE, as shown in the diagram. Let $\angle BAD = \beta$.

i) Copy the diagram into your examination booklet.

ii) Explain why $\angle ADC = \alpha + \beta$ 1mark

iii) Find $\angle DAC$ in terms of α and β. 1mark

iv) Hence, or otherwise, find $\angle EDC$ in terms of β. 2marks



c) In the diagram, ΔFHG and ΔGHJ are both right-angled at G with $\angle HFG = 15^\circ$ and $\angle HJG = 30^\circ$.

i) Find $\angle FHJ$ and hence show that FJ=JH. 1mark

ii) Given that GH=1 unit, show that JG=$\sqrt{3}$ and that JH=2. 1mark

(continued over page)

iii) Hence deduce that $\tan 15^\circ = 2 - \sqrt{3}$. 2marks

Question 6.

a) The first 3 terms of a particular series are $2000 + 3000 + 4500 + \ldots$

i) Show that it is a geometric series, and find the common ratio 2marks.

ii) What is the sum of the first 15 terms. 2marks

iii) Explain why the series does not have a limiting sum. 1mark

b) The tenth term of an arithmetic sequence is 29 and the fifteenth term is 44.

i) Find the value of the common difference and the value of the first term 3marks

ii) Find the sum of the first 75 terms 2marks

c) $1 + r + r^2 + \ldots\ldots$ is a geometric series.

i) For what value does it have a limiting sum. 1mark

ii) If its limiting sum is 25, find r. 1mark

Question 7

a) Given the equation of the curve $y = \frac{1}{3}x^3 - x^2 - 3x - 6$,

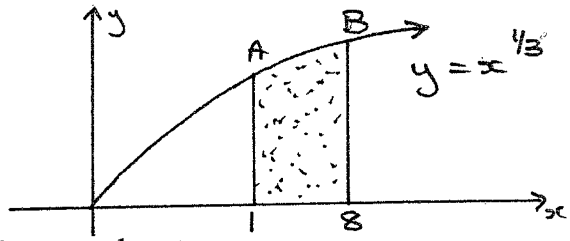
i) find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. 2marks

(continued over page)

ii) Find the value for x for which the curve is monotonic decreasing. 2marks

iii) Find the value for x for which the curve is concave up. 1mark

iv) Find a point of inflexion. 1mark



b) The sketch shows the curve $y = x^{\frac{1}{3}}$, in the first quadrant.

i) Determine the area bounded by the arc AB and the lines, x=1 and x=8 and the x axis. 3marks

ii) The arc AB is rotated around the x axis. Find the exact volume of the solids so generated. (leave your answer in terms of Π) 3marks

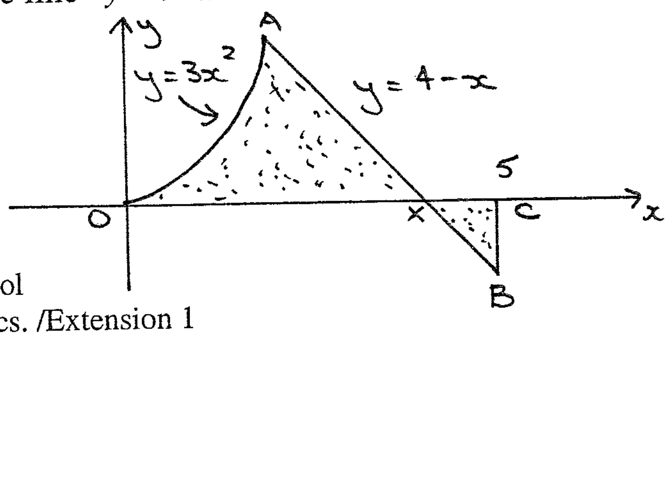
<u>Question 8.</u>

a) On the same axes, sketch $y = 6x - x^2$ and $y = x^2 - 4x$.

i)Find the points of intersection of the two curves. 2marks

ii)Find the area enclosed between the curves. 3marks

b)The shaded region OABC is bounded by the lines x=0, x=5, the curve $y = 3x^2$ and the line $y = 4 - x$ and the x axis, as in the diagram.(continued over page)



i) Show that A has co ordinates (1,3) 2marks

ii) What is the area of the shaded region OABC ? 3marks

c) Solve for x : $2x^2 - 7x + 6 < 0$. 2marks

Question 9.

a) Let A and B be fixed points (-1,0) and (2,0) respectively and let P be the variable point (x,y).

i) Write down the expressions for PA^2 *and* PB^2 in terms of x and y. 1mark

ii) Suppose that P moves so that $PA = 2PB$. Find the equation of the locus of points and show that P forms a circle. 3marks

iii) Find the centre and radius of the circle 1mark

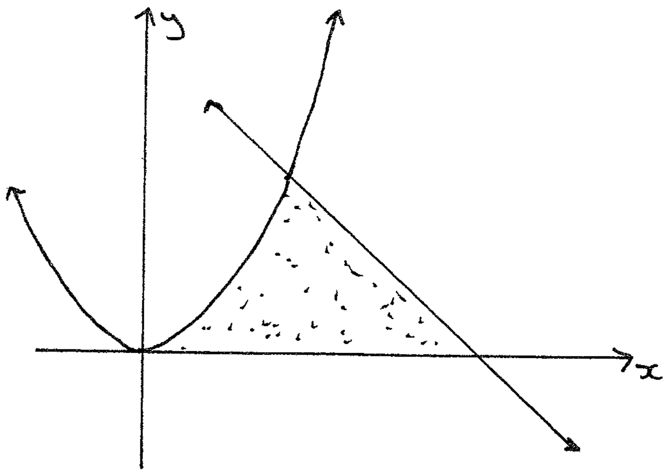
b) Josie invests $1000 into an account at the start of every year.
She makes 20 payments.
The interest is calculated at 12% p.a. compounded yearly.

i) What is the amount at the end of 20 years 4marks

ii) If the interest is calculated at 12%p.a., compounded monthly, what is the amount at the end of the 20 years 3marks

Question 10.

a)



In the sketch above, the shaded area is the area between the curves, $y = x^2$ *and* $x + y = 6$, and the x axis.

This area is rotated about the x axis. Find the volume generated. 5marks

b)

i) If f(x) is defined as an odd function and $f(x)=x^2,\ 0 \le x \le 2$, sketch y = f(x) on a number plane for the domain $-2 \le x \le 2$ 2marks

ii)Write down the value of

$\int_{-2}^{2} f(x)dx$, giving a brief reason for your answer.

2marks

c)Use Simpson's rule with three function values to approximate:

$$\int_1^3 \frac{dx}{x^2+1}$$ 3marks

Year 12 April 2002 Solutions :-

Q1 (12 marks)

(a) (i) $\int 4x+1\,dx$

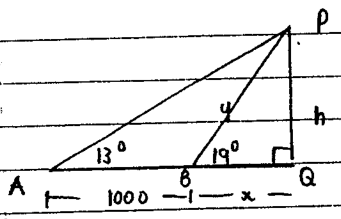
$= \frac{4x^2}{2} + x + c$

$= \boxed{2x^2 + x + c}$ ①

(ii) $\int x^2 - 3x + 7\,dx$

$= \boxed{\frac{x^3}{3} - \frac{3x^2}{2} + 7x + c}$ ②

(b)



$\angle ABP = 180 - 19$

$= 161$

$\angle APB = 180 - (161 + 13)$

$= 6°$

let $BP = y$ $\therefore \frac{y}{\sin 13°} = \frac{1000}{\sin 6°}$

$y = \frac{1000 \times \sin 13}{\sin 6°}$

$y = 2152.055501$

$\therefore$ In $\triangle BPQ$: $\sin 19° = \frac{h}{y}$

$y \times \sin 19° = h$

$\therefore \boxed{h = 701\text{m}}$ ③

3sf

(c) (i) $\alpha + \beta = \frac{-b}{a} = \frac{-(-2)}{3} = \boxed{\frac{2}{3}}$ ①

(ii) $\alpha\beta = \frac{c}{a} = \boxed{\frac{-1}{3}}$ ①

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{2/3}{-1/3} = \boxed{-2}$ ①

(iv) For $3x^2 - 2x - 1 = 0$

let roots be $2\alpha, 2\beta$

$\therefore \boxed{K(x - 2\alpha)(x - 2\beta) = 0}$ ½

$k(x^2 - 2\alpha x - 2\beta x + 4\alpha\beta) = 0$

$k(x^2 - 2x(\alpha+\beta) + 4\alpha\beta) = 0$

$k(x^2 - 2x(\frac{2}{3}) + 4(\frac{-1}{3})) = 0$

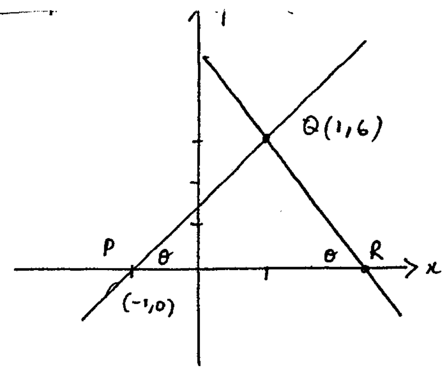
$k(x^2 - \frac{4x}{3} - \frac{4}{3}) = 0$

$\boxed{3x^2 - 4x - 4 = 0}$ 1½

(d) $\int 2x + 11\,dx$

$= \frac{2x^2}{2} + 11x + c$

$= \boxed{x^2 + 11x + c}$ ①



(i) $M = \left(\frac{-1+1}{2}, \frac{0+6}{2}\right)$

$\boxed{M = (0, 3)}$ ①

(ii) $P(-1,0)$ $Q(1,6)$

$m = \frac{6-0}{1-(-1)} = \frac{6}{2} = 3$

$y - 0 = 3(x+1)$.

$\boxed{y = 3x + 3.}$ ②

(iii) $m = \tan\theta$

since $m_{PQ} = 3$

$\therefore \boxed{3 = \tan\theta.}$ ①

(iv) as gradient $PQ = 3$.

& $\tan\theta = 3$.

$\therefore \angle QPR = \angle QRP$ ½ given

$\therefore \boxed{\tan\theta = -3.}$ ①

grad. QR = ~~grad tan θ~~ $\tan(180-\theta) = -\tan\theta$ ½

(v) $m = -3$ $Q(1,6)$.

$y - y_1 = m(x - x_1)$

$y - 6 = -3(x-1)$

$y - 6 = -3x + 3$.

$\boxed{3x + y - 9 = 0.}$ ②

(vi) R is where ... x-axis ...

let $y = 0$ $3x + 0 - 9 = 0$

$3x - 9 = 0$

$3x = 9$

$x = 3$.

$\boxed{R = (3, 0)}$ ①

(vii) $P(-1,0)$ $3x + y - 9 = 0$

$d = \left|\frac{3(-1) + 1(0) - 9}{\sqrt{3^2 + 1^2}}\right|$

$d = \left|\frac{-12}{\sqrt{10}}\right|$ $\boxed{d = \frac{12}{\sqrt{10}}\text{ u}}$

$= \frac{12}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

$= \frac{12\sqrt{10}}{10} = \boxed{\frac{6\sqrt{10}}{5}}$ ②

(viii) $A = \frac{1}{2} \times b \times h$

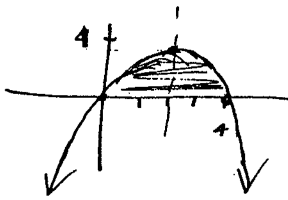
$= \frac{1}{2} \times 4 \times 6$

$\boxed{A = 12u^2}$ ②

(a) (i) $y = 4x - x^2$

$0 = x(4 - x)$

$x = 0 \quad x = 4$



at $x = 2$

$y = 4(2) - 2^2$

$= 8 - 4$

$\boxed{y = 4}$ (1)

(iii) $A = \int_0^4 4x - x^2 \, dx$

$= \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4$

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= \left(2(4)^2 - \frac{4^3}{3}\right) - (0)$

$= \left(32 - \frac{64}{3}\right) - 0$

$= \boxed{10\tfrac{2}{3}\ u^2}$ (3)

(b) $\int (5x+3)^6 \, dx$

$= \frac{(5x+3)^7}{5(7)} + c$

$= \boxed{\frac{(5x+3)^7}{35} + c}$ (2)

(c) (i) $T_n = 8n - 5$

$T_1 = 8(1) - 5$

$\boxed{T_1 = 3}$ (1)

(ii) $T_2 = 8(2) - 5$

$= 11$

$T_3 = 8(3) - 5$

$= 19$

$\therefore 3, 11, 19, \ldots$

$a = 3 \quad \boxed{d = 8}$ (1)

(iii) $T_{10} = 8(10) - 5$

$= 75$

$S_{10} = \frac{10}{2}(3 + 75)$

$= 5(78)$

$= \boxed{390}$ (2)

(d) let $x = 0.\dot{2}\dot{3}$

$x = 0.23232323\ldots$

$(\times 100)$

$100x = 23.23232323\ldots$

$99x = 23$

$\boxed{x = \frac{23}{99}}$ (2)

Qu:-4

(a) $x^2 + (k+2)x + 4 = 0$

(i) For equal roots let $\Delta = 0$.

$\Delta = b^2 - 4ac$

$\Delta = (k+2)^2 - 4(1)(4)$

$0 = k^2 + 4k + 4 - 16$

$0 = k^2 + 4k - 12$

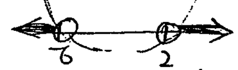
$0 = (k+6)(k-2)$

$\boxed{k = -6} \quad \boxed{k = 2}$ (2)

(ii) For real & diff. $\Delta > 0$

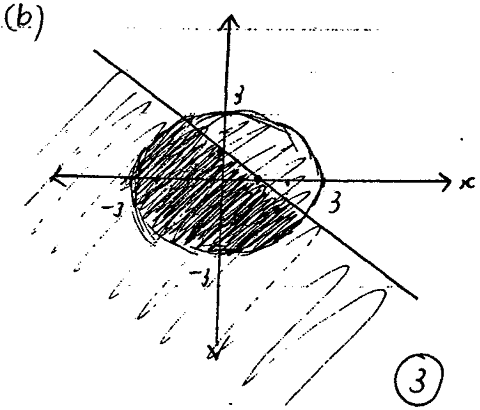
$k^2 + 4k - 12 > 0$

$(k+6)(k-2) > 0$



$\boxed{k > 2,\ k < -6}$ (2)

(b)



(3)

(c) $9^x + 2 \cdot 3^x - 15 = 0$

$(3^2)^x + 2(3^x) - 15 = 0$

$(3^x)^2 + 2(3^x) - 15 = 0$

let $m = 3^x$

$m^2 + 2m - 15 = 0$

$(m+5)(m-3) = 0$

$\boxed{m = -5} \quad \boxed{m = 3}$

but ~~$3^x = -5$ no soln~~ $\quad 3^x = 3$

$\boxed{x = 1}$ (1)

(d) $\frac{\cos\alpha}{1 - \sin\alpha} - \frac{\cos\alpha}{1 + \sin\alpha}$

$\frac{\cos\alpha(1 + \sin\alpha) - \cos\alpha(1 - \sin\alpha)}{1 - \sin^2\alpha}$

$= \frac{\cos\alpha + \cos\alpha\sin\alpha - \cos\alpha + \cos\alpha\sin\alpha}{\cos^2\alpha}$

$= \frac{2\sin\alpha\cos\alpha}{\cos^2\alpha}$

$= \frac{2\sin\alpha}{\cos\alpha}$

$= \boxed{2\tan\alpha}$ (2)

(12 marks)

(a) $\int_0^1 \frac{1 + x + 2\sqrt{x}}{\sqrt{x}}\,dx$

$= \int_0^1 \frac{1}{\sqrt{x}} + \frac{x + 2\sqrt{x}}{\sqrt{x}}\,dx$ (½)

$= \int_0^1 x^{-1/2} + x^{1/2} + 2\,dx$ (½)

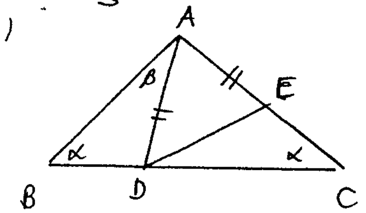
$= \left[\frac{x^{1/2}}{1/2} + \frac{x^{3/2}}{3/2} + 2x + c\right]_0^1$ (1)

$= \boxed{2\sqrt{x} + \frac{2}{3}x^{3/2} + 2x + c}_0^1$ (1)

$= 2 + \frac{2}{3} + 2$ (4)

$= 4\frac{2}{3}$ (1)

(b) (i)



(ii) $\angle ADB = 180 - (\alpha + \beta)$

(angle sum $\Delta = 180$) ~~exterior angle~~

$\angle ADC = 180 - [180 - (\alpha + \beta)]$

straight ∠

$= 180 - 180 + (\alpha + \beta)$

$\boxed{\angle ADC = (\alpha + \beta)}$ (1)

OR

From ΔADB,

$\angle ADC = \angle DBA + \angle BAD$

(ext. ∠ of a Δ)

$= \alpha + \beta$

(iii) In ΔABC

$\alpha + \alpha + \beta + \angle DAC = 180^\circ$

(angle sum $\Delta = 180^\circ$)

$2\alpha + \beta + \angle DAC = 180^\circ$

$\boxed{\angle DAC = 180 - (2\alpha + \beta).}$ (1)

(iv) $\angle ADE = \frac{180 - [180 - (2\alpha + \beta)]}{2}$

(angle sum of Isos Δ ADE

$= \frac{180 - 180 + (2\alpha + \beta)}{2}$

$= \frac{2\alpha + \beta}{2}$

$\angle ADE = \alpha + \frac{\beta}{2}$

from (ii) $\angle ADC = \alpha + \beta.$

$\therefore \angle EDC = \angle ADC - \angle ADE$

$= \alpha + \beta - (\alpha + \frac{\beta}{2})$

$= \alpha + \beta - \alpha - \frac{\beta}{2}$

$\boxed{\angle EDC = \frac{\beta}{2}}$ (2)

[NB lost ½ mark for no reasons]

(c) (i) $\angle FHJ = 30 - 15$

$= 15^\circ$ (exterior ∠ of Δ) (½)

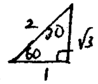
$\therefore$ In Δ FJH base ∠'s = 15°

$\therefore FJ = JH$ (½)

(equal side of Isos Δ).

(ii) $\tan 30^\circ = \frac{HG}{JG}$

$\tan 30^\circ = \frac{1}{JG}$



$\frac{1}{\sqrt{3}} = \frac{1}{JG}$ (½)

$\therefore \boxed{JG = \sqrt{3}.}$

$JH^2 = JG^2 + HG^2$

$JH^2 = (\sqrt{3})^2 + 1^2$

$JH^2 = 3 + 1$

$JH^2 = 4$

$JH = \sqrt{4}$

$\boxed{JH = 2.}$ (½)

(iii) In Δ FHG

$\tan 15 = \frac{HG}{FG}$

$= \frac{1}{FJ + JG}$

$= \frac{1}{FJ + \sqrt{3}}$

as $FJ = JH$ (proven before Isos Δ).

$\tan 15^\circ = \frac{1}{2 + \sqrt{3}.}$ (1)

$\therefore \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}.} = \frac{2 - \sqrt{3}}{4 - 3}$

$= \frac{2 - \sqrt{3}}{1}$ (1)

$\therefore \boxed{\tan 15^\circ = 2 - \sqrt{3}.}$ (2)

Qu: 6

(a) $2000 + 3000 + 4500 + \ldots$

(i) $\frac{T_3}{T_2} = \frac{T_2}{T_1}$

$\frac{4500}{3000} = \frac{3000}{2000}$ (2)

$\frac{3}{2} = \frac{3}{2}$ $\therefore$ GP $\boxed{r = \frac{3}{2}}$ or 1.5

(ii) $S_{15} = \frac{a(r^{15} - 1)}{r - 1}$

$= \frac{2000\left(\frac{3}{2}^{15} - 1\right)}{\frac{3}{2} - 1}$

$\boxed{S_{15} = 1747575.561}$ (2)

(iii) For limiting sum $-1 < r < 1$ (1)

as $r = \frac{3}{2}$ $\quad \frac{3}{2} > 1$ $\therefore$ $\boxed{\text{no limiting sum}}$

Qu: 6 con't

(b)(i) $T_{10} = 29 \Rightarrow a + 9d = 29$ — ①
$T_{15} = 44 \Rightarrow a + 14d = 44$ — ②

② − ① $5d = 15$

$d = 3.$

$\therefore a + 9(3) = 29$
$a + 27 = 29$
$a = 2.$ ③

The first term is 2 and the common diff is 3.

(ii) $S_{75} = \frac{75}{2}[2(2) + (75-1)3]$

$S_{75} = \$8475$ ②

c) $1 + r + r^2 + \ldots$

(i) $-1 < r < 1$ ①

(ii) $S_\infty = 25$

$S_\infty = \frac{a}{1-|r|}$

$25 = \frac{1}{1-|r|}$ ①

$25(1-r) = 1$
$25 - 25|r| = 1$
$24 = 25|r|$ $r = \pm\frac{24}{25}$

NB. still gave ① if $r = \frac{24}{25}$

Qu: 7

(a) $y = \frac{1}{3}x^3 - x^2 - 3x - 6.$

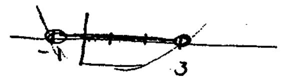
(i) $y' = 3(\frac{1}{3})x^2 - 2x - 3.$

$y' = x^2 - 2x - 3$ ①

$y'' = 2x - 2.$ ①

(ii) For decreasing let $y' < 0$.

$\therefore x^2 - 2x - 3 < 0$
$(x-3)(x+1) < 0$



$-1 < x < 3$ ②

(iii) For concave up ∪

let $y'' > 0$

$2x - 2 > 0$
$2x > 2$
$x > 1$ ①

(iv) For pt of inflexion let $y'' = 0$

$2x - 2 = 0$
$2x = 2$
$x = 1$
$y = -9\frac{2}{3}$

pt $(1, -9\frac{2}{3})$ ①

check concavity: $f''(0) = -2 < 0$, $f''(2) > 0$ ...
as it changes then there ...

(b)(i) $A = \int_1^8 x^{1/3}\,dx$

$= \left[\frac{x^{4/3}}{4/3}\right]_1^8$

$= \left[\frac{3}{4}x^{4/3}\right]_1^8$

$= \frac{3}{4}(8)^{4/3} - \frac{3}{4}(1)^{4/3}$

$= 12 - \frac{3}{4}$

$A = 11\frac{1}{4}\,u^2$ ③

(ii) $V = \pi\int_a^b [f(x)]^2\,dx$

$V = \pi\int_1^8 (x^{1/3})^2\,dx$

$V = \pi\int_1^8 x^{2/3}\,dx$

$= \pi\left[\frac{x^{5/3}}{5/3}\right]_1^8$

$= \pi\left[\frac{3}{5}x^{5/3}\right]_1^8$

$= \pi\left[\frac{3}{5}(8)^{5/3} - \frac{3}{5}(1)^{5/3}\right]$

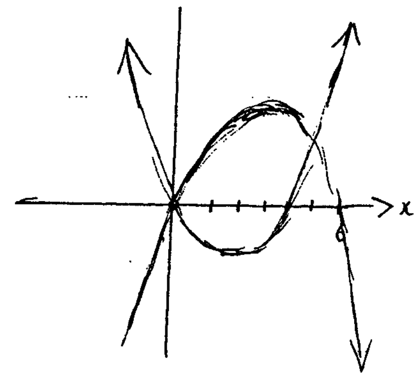
$= \pi\left[\frac{96}{5} - \frac{3}{5}\right]$

$= \frac{93\pi}{5}\,u^3$ ③

$(18\frac{3}{5}\pi\,u^3)$

Qu: 8

(a) $y = 6x - x^2$ $y = x^2 - 4x$
$y = x(6-x)$ $y = x(x-4).$



(i) $6x - x^2 = x^2 - 4x$
$0 = 2x^2 - 10x$
$0 = 2x(x-5).$

$x = 0$ $x = 5$
$y = 0$ $y = 5.$ ②

pts of int: $(0,0)$ $(5,5)$

(ii) $A = \left|\int_0^5 6x - x^2 - (x^2 - 4x)\,dx\right|$

$= \int_0^5 6x - x^2 - x^2 + 4x\,dx$

$= \int_0^5 10x - 2x^2\,dx$

$= \left[\frac{10x^2}{2} - \frac{2x^3}{3}\right]_0^5$

$= \left(5(5)^2 - \frac{2}{3}(5)^3\right) - 0$

$= 41\frac{2}{3}\,u^2$ ③

(i) $y = 3x^2 \quad y = 4 - x$

$$3x^2 = 4 - x$$
$$3x^2 + x - 4 = 0$$
$$(3x + 4)(x - 1) = 0$$

~~$x = -\frac{4}{3}$~~ $\quad x = 1$, $y = 3(1)^2$ } in first quad

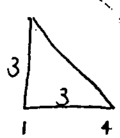
A = (1, 3) (2)

(ii) $A_1 = \int_0^1 3x^2\,dx$

$$= \left[\frac{3x^3}{3}\right]_0^1$$
$$= 1^3 - 0^3$$
$$= \boxed{1}$$

$y = 4 - x$, $0 = 4 - x$, $x = 4$

$A_2 =$



$$= \frac{3 \times 3}{2}$$
$$= \boxed{\tfrac{9}{2}}$$

$A_3 =$

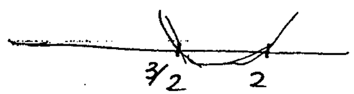
$$= \frac{1 \times 1}{2}$$
$$= \boxed{\tfrac{1}{2}}$$

Total = $6u^2$ (3)

(c) $2x^2 - 7x + 6 < 0$

$$(2x - 3)(x - 2) < 0$$

$x = 3/2 \quad x = 2$



(2)

$$\boxed{\tfrac{3}{2} < x < 2}$$

Qu: 9

(a) $P(x, y)$ $A(-1, 0)$ $B(2, 0)$

$$(PA)^2 = \left(\sqrt{(x+1)^2 + (y-0)^2}\right)^2$$
$$(PB)^2 = \left(\sqrt{(x-2)^2 + (y-0)^2}\right)^2$$
$$\boxed{PA^2 = (x+1)^2 + y^2}$$
$$\boxed{PB^2 = (x-2)^2 + y^2}$$

(1)

(ii) $PA = 2\,PB$.

dist PA = 2 × dist PB

$$\sqrt{\quad} = 2\sqrt{\quad}$$
$$(x+1)^2 + y^2 = 4\left[(x-2)^2 + y^2\right]$$
$$x^2 + 2x + 1 + y^2 = 4\left[x^2 - 4x + 4 + y^2\right]$$
$$x^2 + 2x + 1 + y^2 = 4x^2 - 16x + 16 + 4y^2$$

(3)

$$\boxed{0 = 3x^2 + 3y^2 - 18x + 15.}$$

(iii) **Locus is a circle**

$$0 = 3(x^2 + y^2 - 6x + 5)$$
$$0 = 3\left((x-3)^2 + y^2 + 5 - 9\right)$$
$$0 = (x-3)^2 + y^2 - 4$$
$$4 = (x-3)^2 + y^2$$

$\boxed{C = (3, 0)}$ $\boxed{r = 2}$ (1)

(b) (i) $A_1 = 1000\left(1 + \frac{12}{100}\right)^{20}$

$A_2 = 1000(1.12)^{19}$

$\vdots$

$A_{20} = 1000(1.12)^1$

$$T = A_1 + A_2 + \dots + A_{20}$$
$$T = 1000(1.12)^{20} + 1000(1.12)^{19} + \dots + 1000(1.12)$$
$$= 1000(1.12)\left[1.12^{19} + 1.12^{18} + \dots + 1\right]$$
$$= 1000(1.12)\left[1 + 1.12 + \dots + 1.12^{19}\right]$$
$$= 1000(1.12) \times \left[\frac{1(1.12^{20} - 1)}{1.12 - 1}\right]$$
$$= \$80698.74$$

(4)

$\frac{12}{100} \div 12 = 0.01 \qquad 20 \times 12 = 240$

(ii) $A_1 = 1000(1.01)^{240}$

$A_2 = 1000(1.01)^{239}$

$\vdots$

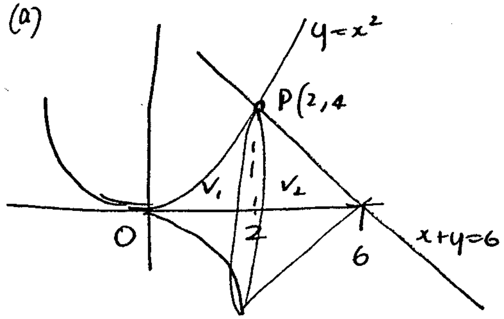
$A_{240} = 1000(1.01)^1$

$$T = A_1 + A_2 + \dots + A_{240}$$
$$T = 1000(1.01)^{240} + 1000(1.01)^{239} + \dots + 1000(1.01)^1$$
$$= 1000(1.01)\left[1.01^{239} + 1.01^{238} + \dots + 1\right]$$
$$= 1000(1.01)\left[1 + 1.01^1 + \dots + 1.01^{239}\right]$$
$$= 1000(1.01)\left[\frac{1(1.01^{240} - 1)}{1.01 - 1}\right]$$
$$= \$999\,147.92$$

(3)

Qu:-10

(a)



$y=x^2$
$y=6-x$

$x^2=6-x$
$x^2+x-6=0$
$(x+3)(x-2)=0$
$x=-3$ $\boxed{x=2.}$ in 1st quad.
$y=4.$

$$V_1=\pi\int_0^2 (x^2)^2\,dx + V_2=\pi\int_2^6 (6-x)^2\,dx$$

$$=\pi\int_0^2 x^4\,dx + \pi\int_2^6 36-12x+x^2\,dx$$

$$=\pi\left[\frac{x^5}{5}\right]_0^2 + \pi\left[36x-\frac{12x^2}{2}+\frac{x^3}{3}\right]_2^6$$

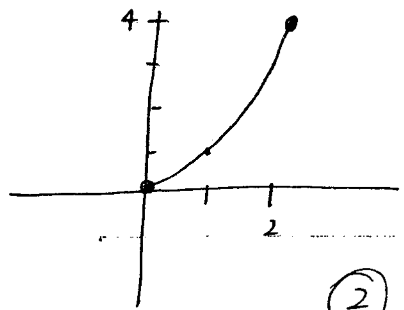
$$=\pi\left[\frac{32}{5}-0\right]+\pi\left[\left(216-216+\frac{216}{3}\right)-\left(72-24+\frac{8}{3}\right)\right]$$

$$\frac{32\pi}{5}+\frac{64\pi}{3}$$

$$=\boxed{\frac{416}{15}\pi}$$

(5)

(b) (i) $y=x^2$

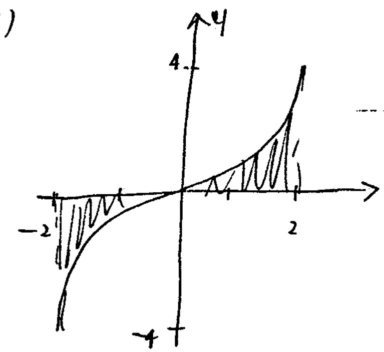


(2)

~~Df = 0≤x≤2.~~

~~Rf = 0≤y≤4~~

(ii)



$$\int_{-2}^{2} f(x)\,dx$$

(2)

will = $\boxed{0}$ as odd functi

have pt symmetry which means they cancel each other out

(c) ... 2.

$h=\frac{2}{2}$

$h=1$

$$\int_1^3 \frac{dx}{x^2+1}$$

x	1	2	3
f(x)	1/2	1/5	1/10
	y_0	y_1	y_n

$$A \doteq \frac{h}{3}\left[y_0+y_n+2(\text{evens})+4(\text{odds})\right]$$

$$\doteq \frac{1}{3}\left[\frac{1}{2}+\frac{1}{10}+2(0)+4\left(\frac{1}{5}\right)\right].$$

$$\boxed{A \doteq \frac{7}{15}\,u^2}$$

$$\boxed{A \doteq 0.4\dot{6}\,u^2}$$

(3)