St Catherine's School Year 12 Half-Yearly Exam '05

ST. CATHERINE'S
IN CHRISTO THESAURI SAPIENTIAE ET SCIENTIAE

Year: 12
Subject: Mathematics
Time Allowed: 2.5 hours
plus 5 min reading time
Date: March 2005

Exam number: 15227508

Tricia Khong

Directions to candidates:

- All questions are to be attempted.
- All questions are of equal value.
- All necessary **working** must be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Each question attempted should be started on a **new page**.
- Approved calculators and geometrical instruments maybe used.
- Write your exam number ~~name and teacher's name~~ on EACH bundle.
- Hand in your questions in 3 separate bundles:
  - 1. Include the question paper at the back of questions 1, 2, 3 and 4
  - 2. Questions 5, 6, 7
  - 3. Question 8, 9, 10

TEACHER'S USE ONLY
Total Marks

Q1	10	**/10**
Q2	10	**/10**
Q3	10	**/10**
Q4	10	**/10**
Q5	10	**/10**
Q6	10	**/10**
Q7	$9\frac{1}{2}$	**/10**
Q8	7	**/10**
Q9	10	**/10**
Q10	10	**/10**
Grand Total	$96\frac{1}{2}$	**/100**

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Question 1

(a) Calculate, correct to two decimal places: **1**

$$\sqrt[3]{\frac{3\times 25.4}{4\pi}}$$

(b) Factorise: **2**

$$q^2-16p^2$$

(c) Express with rational denominator: $\frac{1}{\sqrt{5}+3}$. **2**

(d) Solve the following:

2

(i) $\frac{3}{v-2}=6$ **3**

(ii) $|x+1|>5$, graphing your solution on a number line.

Question 2 Begin a new page.

(a) On a number plane, plot the points O(0,0), A(4,0), B(0,4) and C(6,6). Join AB and OC. Label the point of intersection D. **1**

(b) Show that AB is perpendicular to OC. **2**

(c) Find the lengths of AB and OC. **2**

(d) Use congruent triangles to show that OC bisects AB. **2**

(e) Show that the exact area of OACB is ~~48~~ 24 square units. **2**

(f) What shape is OACB? **1**

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Question 3 **Begin a new page.**

(a) Differentiate:

(i) $y = \dfrac{1}{\sqrt{x+2}}$ **2**

(ii) $f(x) = 2x(5-x)^3$ **3**

(b) Find $\int (2x+5)^4 \, dx$. **1**

(c) Evaluate $\int_{-1}^{2} 3y^2 dy$ **2**

(d) State the equation of the parabola with focus (0,3) and directrix $y = -3$. **2**

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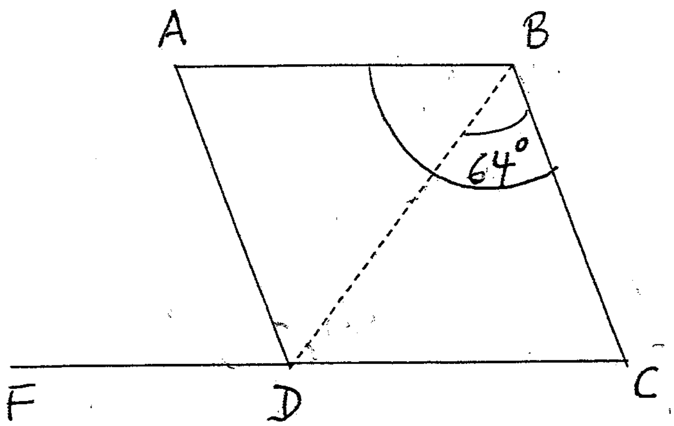
Question 4 **Begin a new page.**

(a)



In the diagram, ABCD is a rhombus and angle DBC is 64°. CD is produced to F. Copy the diagram.

Find the size of :

(i) angle ABC **1**

(ii) angle ADF, giving reasons. **2**

(b) The equation of a parabola is given by $(y-1)^2 = 4(x-3)$.

(i) State the coordinates of the vertex. **1**
(ii) Find the focal length. **1**
(iii) Sketch the parabola. **1**
(iv) State the coordinates of the focus. **1**
(v) State the equation of the directrix. **1**
(vi) Find the x-intercept. **1**
(vii) State the length of the latus rectum. **1**

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Question 5 **Begin a new page.**

(a) Consider the equation $x^2 - 2x + 6 = 0$ with roots α and β.
Find the value of

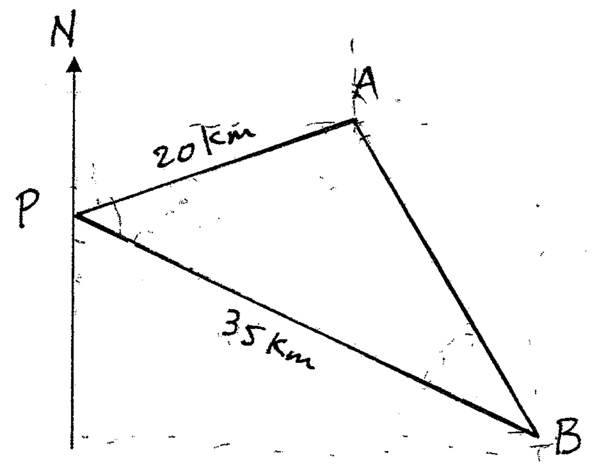
(i) $\alpha\beta$ 1
(ii) $\alpha + \beta$ 1
(iii) $\alpha^2 + \beta^2$ 2

(b) Find the values of k if the expression 2

$$x^2 - 2x + k - 4$$

is positive definite for all values of x.

(c)



In the diagram above, A is 20 km from P on a bearing of 074° T whereas B is 35 km from P on a bearing of 120° T.

(i) Find the size of <APB. 1

(ii) Find the length AB. 2

(iii) If the size of <PAB is 92°, state the bearing of B from A. 1

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Question 6 Begin a new page.

(a) A ball is dropped from a height of 45 m. It rises to two-thirds of that height on the next bounce then two thirds of that again on the following bounce, and so on.

(i) Develop a series to represent this situation. **1**

(ii) Find the total distance traveled by the ball before it comes to rest. **2**

(b) (i) Myfannwey invests her hard-earned \$1000 in an account which pays 9.5% per annum, compounded annually.

Find the amount accumulated if she invests it for 30 years. **2**

(ii) Rudy decides to invest \$1000 at the beginning of every year in the same type of account for 30 years.

Find the total amount of her investment at the end of 30 years. **3**

(c) Solve $\sin\theta\cos\theta - \sin\theta = 0$ for $0^0 \le \theta \le 360^0$. **2**

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Question 7 Begin a new page.

(a) The gradient function of a curve is given by $y' = 4x - 2$.
If the curve passes through the point (1, - 3) find the equation of the curve. **2**

(b) Consider the curves with equations $y = x^2 - 4x + 3$ and $y = x + 3$ which intersect at (0,3) and (5,8). **3**

(i) Sketch the curves and shade the region where
$y \geq x^2 - 4x + 3$ and $y \leq x + 3$
simultaneously ~~hold.~~

(ii) Find the area between the two curves.

Question 8 Begin a new page.

The equation of an ellipse with centre at the origin is given by $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

(i) Find the x-intercepts. **1**

(ii) This ellipse is rotated about the x-axis between $x = -3$ and $x = 3$.
Show that the volume generated is 16π units^3. **3**

(b) Use Simpson's Rule with 4 sub-intervals to find an approximation to the area under the curve $y = 3^x$ between $x=0$ and $x=4$. **4**

(c) Find the value of k if

$$\int_1^k \frac{1}{x^2}\, dx = 2$$

Question 9 Begin a new page.

(a) Explain why the curve $y = x^5 + 3x$ is increasing for all values of x. **2**

(b) Consider the points A(- 1,3) and B(5, 7). The point P (x, y) moves such that it is equidistant from A and B, i.e. AP=BP.

(i) Derive the equation of the locus of P. **3**

(ii) Describe this locus geometrically. **1**

(c)

(i) Show that $\dfrac{d}{dx}\left(\dfrac{9-x^2}{9+x^2}\right) = \dfrac{-36x}{\left(9+x^2\right)^2}$. **2**

(ii) Hence evaluate $\displaystyle\int_0^1 \frac{x\ dx}{(9+x^2)^2}$ **2**

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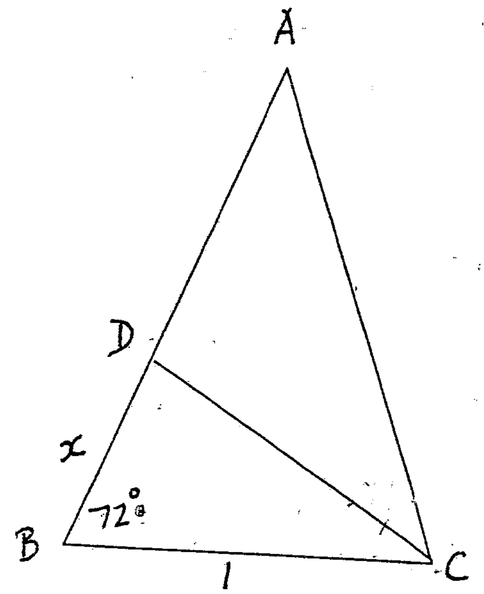
Question 10 Begin a new page.

(a) Simplify $\cos\alpha\tan\alpha$. **1**

(b)



Consider the triangle ABC with AB=AC. Angle ABC=72°. CD bisects angle ACB. Let the lengths BC=1 unit and BD=x units.

(i) Prove $\Delta ABC\,|||\,\Delta CDB$. **2**

(ii) Hence find the length of AC in terms of x. **2**

Question 10 continued

c)

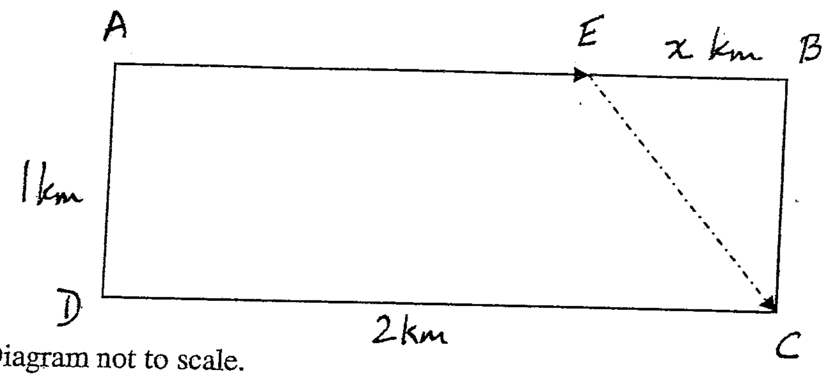


Diagram not to scale.

A rectangular park ABCD is 2 km long and 1 km wide. Gabby wants to run from one corner A, of the park to the opposite corner C. If she goes along the perimeter she runs at 10 km/h on concrete but if she cuts across the park she squelches through mud at 6 km/h. She decides to cut across the park at E, which is x km from B, to save time.

The total time, T hours, for her run is given by $T = \dfrac{\sqrt{x^2+1}}{6} + \dfrac{2-x}{10}$.

(i) Show that $\dfrac{dT}{dx} = \dfrac{x}{6\sqrt{x^2+1}} - \dfrac{1}{10}$. **2**

(ii) Hence find the value of x so that Gabby's time is the quickest. **3**

END!!

SECTION A

Question 1.

a. $\sqrt[3]{\dfrac{3\times 25.4}{4\pi}} = 1.8235\ldots$

$= 1.82 \text{ (2dp)}$ 1

b. $q^2 - 16p^2$ 2

$= (q-4p)(q+4p)$

c. $\dfrac{1}{\sqrt{5}+3} \times \dfrac{\sqrt{5}-3}{\sqrt{5}-3}$

$= \dfrac{\sqrt{5}-3}{5-9}$

$= \dfrac{\sqrt{5}-3}{-4}$

$= \dfrac{3-\sqrt{5}}{4}$ 2

d. i) $\dfrac{3}{v-2} = 6$

$3 = 6v - 12$

$15 = 6v$ 2

$2\frac{1}{2} = v$

ii) $|x+1| > 5$

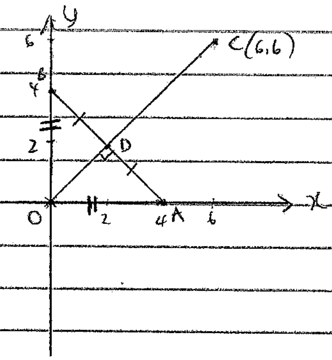
$x+1 > 5$ or $-x-1 > 5$

$x > 4$ $\quad -6 > x$ 3

$\longleftarrow\circ$ -6 $\quad 0 \quad$ 4 $\circ\longrightarrow$

Question 2.

a.



1

b. $m_{AB} = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{4-0}{0-4}$

$= -1$

$m_{OC} = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{6-0}{6-0}$ 2

$= 1$

$m_{AB} \times m_{OC} = 1 \times -1 = -1$

$\therefore AB \perp OC$

c. $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)}$

$= \sqrt{(0-4)^2+(4-0)^2}$

$= \sqrt{32} = 4\sqrt{2}$ units 2

$OC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(6-0)^2+(6-0)^2}$

$= \sqrt{72} = 6\sqrt{2}$ units

d. In $\triangle ODA$ and $\triangle ODB$

OD is common

$OA = OB =$ ~~4~~ units (from diagram)

$\angle ODA = \angle ODB = 90°$ ($AB \perp OC$)

$\therefore \triangle ODA \equiv \triangle ODB$ ( ) ?

$\therefore$ $AD = BD$ (corresp. sides on cong $\triangle s$)

2

$\therefore$ OC bisects AB.

e $A = \frac{1}{2}xy$

$= \frac{1}{2}(OC)(AB)$

$= \frac{1}{2}(6\sqrt{2})(4\sqrt{2})$

$= 24$ units^2

2

f. A kite (diagonals bisect at $\perp$, 1 pair of adjacent sides are equal in length)

1

Question 3.

a. i) $y = (x+2)^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}(x+2)^{-\frac{3}{2}}$

$= -\frac{1}{2\sqrt{(x+2)^3}}$

2

ii) $f(x) = 2x(5-x)^3$

$f'(x) = uv' + vu'$

$u' = 2$

$v' = 3(5-x)^2 \cdot -1$

$= -3(5-x)^2$

$\therefore f'(x) = 2x(-3(5-x)^2) + (5-x)^3 \cdot 2$

$= 2(5-x)^2(-3x + 5 - x)$

$= 2(5-x)^2(5-4x)$

3

b. $\int (2x+5)^4\, dx$

$= \frac{(2x+5)^5}{5 \times 2} + c$

$= \frac{(2x+5)^5}{10} + c$

1

c. $\int_{-1}^{2} 3y^2\,dy$

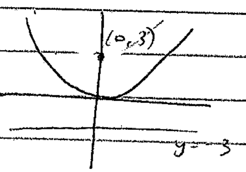
$= \left[\frac{\cancel{3}y^3}{\cancel{3}}\right]_{-1}^{2}$

$= \left[y^3\right]_{-1}^{2}$

$= 2^3 - (-1)^3$
$= 9$

2

d.



$a = 3$

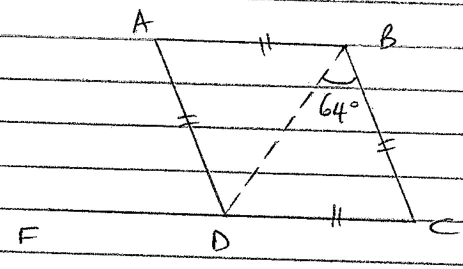
$x^2 = 4ay$

2

$\therefore x^2 = 12y$

Question 4

a)



i) $\angle ABC = 2 \times \angle DBC$

(diagonals of rhombus bisect angles)

$\therefore \angle ABC = 2(64)$

$= 128°$

1

ii) ~~∠ABA~~

$\angle ADC = \angle ABC$ (opp. ∠'s of rhombus are equal)

$\therefore \angle ADC = 128°$

$\therefore \angle ADF = 180° - 128°$ (st. line)

$= 52°$

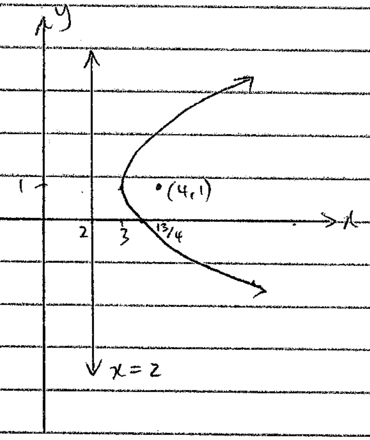
2

b. $(y-1)^2 = 4(x-3)$

i) vertex: $(3,1)$

ii) focal Length $= 1$

iii)



iv) focus: $(4,1)$

v) directrix: $x = 2$

vi) x int when $y = 0$

$$(0-1)^2 = 4(x-3)$$

$$1 = 4x - 12$$

$$13 = 4x$$

$$x = \frac{13}{4}$$

$\therefore$ x int: $\left(\frac{13}{4}, 0\right)$

vii) latus rectum $= 4a$

SECTION B

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Question 5.

a) $x^2 - 2x + 6 = 0$

i) $\alpha\beta = \frac{c}{a}$

$= \frac{6}{1}$

$= 6$ ✓

5 − 10

6 − 10

$7 - 9\frac{1}{2}$

ii) $\alpha + \beta = -\frac{b}{a}$

$= \frac{2}{1}$

$= 2$ ✓

iii) $\alpha^2 + \beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$

$= (2)^2 - 2(6)$

$= -8$ ✓

4

b) pos. definite: $\Delta < 0$

$b^2 - 4ac < 0$

$(-2)^2 - 4(k-4) < 0$

$4 - 4k + 16 < 0$

$20 - 4k < 0$

$5 - k < 0$

$5 < k$

$\therefore \underline{k > 5}$ ✓ 2

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c. i) $\angle NPB - \angle NPA = \angle APB$

$\therefore \angle APB = 120 - 74$

$= 46°$ ✓ 1

ii) $a^2 = b^2 + c^2 - 2bc \cos A$

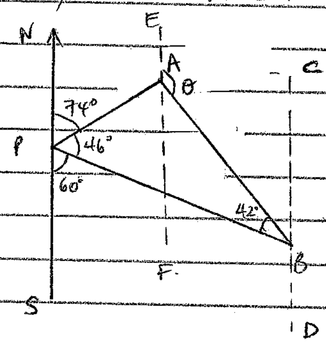
$(AB)^2 = 20^2 + 35^2 - 2(20)(35)\cos 46$

$= 652.478\ldots$

$\therefore AB = 25.543\ldots$

$= 25.54$ km (2dp) ✓ 2

iii) $\angle ABP = 42°$ ($\angle$ sum of $\Delta = 180°$)



$\angle BPS = 60°$ (st. line)

$\therefore \angle CBP = 60°$ (altern $\angle$'s, NS // CD)

$\therefore \angle CBA = 60 - 42$

$= 18°$

$\therefore$ ~~KABA~~

$\therefore \theta = 180 - 18°$ (cointerior $\angle$'s, EF // CD)

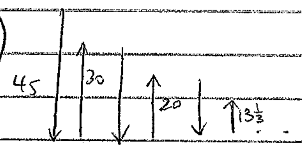
$= 162°$

$\therefore$ Bearing of B from A is

162° T ✓ 1

Question 6.

a) i)



$45 + 30 + 30 + 20 + 20 + 13\frac{1}{3} + 13\frac{1}{3} + \dots$

$= 45 + 2(30 + 20 + 13\frac{1}{3} + \dots)$ ✓

ii) $45 + 2(\underbrace{30 + 20 + 13\frac{1}{3} + \dots}_{})$

$a = 30$

$r = \frac{2}{3}$

$\text{dist} = 45 + 2\left(\frac{a}{1-r}\right)$ ✓

$= 45 + 2\left(\frac{30}{1-\frac{2}{3}}\right)$

$= 45 + 2(90)$

$= 225\text{ m}$ ✓ 3

b i) $A = P(1+r)^n$

$= 1000(1 + 0.095)^{30}$

$= \$15\ 220.312\dots$

$= \$15\ 220.31$ (2dp) ✓ 2

ii) let A_n be the amount at the end of n y

$A_1 = 1000(1.095)$

$A_2 =$

$=$

$= A_1(1.095) + 1000(1.095)$

$= 1000(1.095)^2 + 1000(1.095)$

$= 1000(1.095 + 1.095^2)$

$A_3 = 1000(1.095 + 1.095^2 + 1.095^3)$

$\therefore A_{30} = 1000(\underbrace{1.095 + 1.095^2 + \dots + 1.095^{30}}_{})$

GP. $a = 1.095$ ✓

$r = 1.095$

$n = 30$

$\therefore A_{30} = 1000\left(\frac{a(r^n - 1)}{r-1}\right)$

$= 1000\left(\frac{1.095(1.095^{30} - 1)}{1.095 - 1}\right)$

$= \$163\ 907.814\dots$

$= \$163\ 907.81$ (2dp) ✓ 3

c. $\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(\cos\theta - 1) = 0$

$\therefore \sin\theta = 0 \qquad \cos\theta = 1$

$\theta = 0°, 180°, 360° \qquad \theta = 0, 360°$

$\therefore \theta = 0°, 180°, 360°$ ✓

2

10

Question 7.

a) $y' = 4x - 2$

$y = \int(4x-2)\,dx$

$= \frac{4x^2}{2} - 2x + c$

$= 2x^2 - 2x + c$ ✓

Sub $(1, -3)$ in

$-3 = 2 - 2 + c$

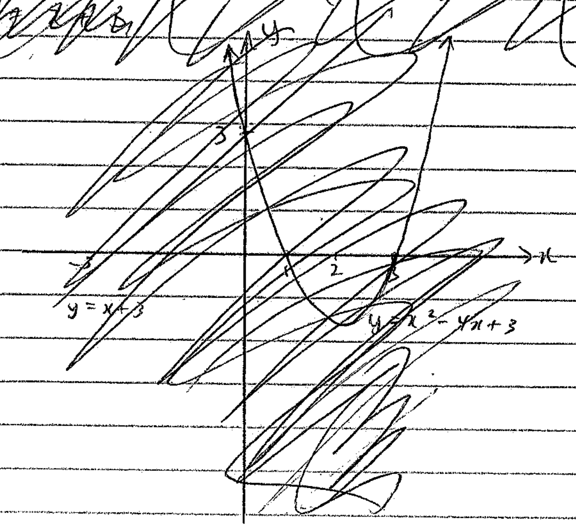
$\therefore c = -3$ ✓

$\therefore$ eqn:

$y = 2x^2 - 2x - 3$ ✓

3

~~b) $y = x^2 - 4x + 3 = (x-3)(x-1)$~~



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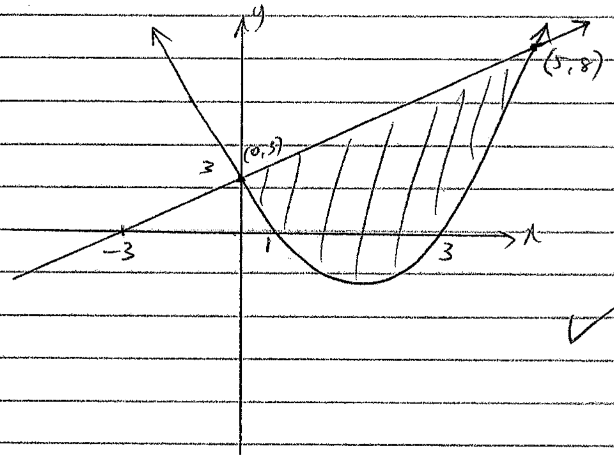
b) i) $y = x^2 - 4x + 3$

$= (x-3)(x-1)$



3 ✓

$y = x+3$ test $(0,0)$ $0 \leq 3$ ✓

$y = x^2 - 4x + 3$ test $(0,0)$ $9 \geq 3$ ✗

ii) $\int_0^5 \left[(x+3) - (x^2 - 4x + 3)\right] dx$

$= \int_0^5 (x + 3 - x^2 + 4x - 3)\, dx$

$= \int_0^5 (-x^2 + 5x)\, dx$ $3\frac{1}{2}$

$= \left[-\frac{x^3}{3} + \frac{5x^2}{2}\right]_0^8$

$= \left(-\frac{8^3}{3} + \frac{5(8)^2}{2}\right) - (0)$

$= -10\frac{2}{3}$, however Area is +ve,

$\therefore$ Area $= 10\frac{2}{3}$ units^2

SECTION C

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Question 8.

$7 + 10 + 10$ $\frac{27}{30}$

a). $\frac{x^2}{9} + \frac{y^2}{4} = 1$

i) x int when $y = 0$

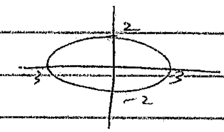
$\frac{x^2}{9} + 0 = 1$

$x^2 = 9$

$\therefore x = \pm 3$

$\therefore x$ int: $(3,0), (-3,0)$ 1 ✓

ii) $V = \pi \int_{-3}^{3} y^2\, dx$



Now $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\frac{y^2}{4} = 1 - \frac{x^2}{9}$

$= \frac{9 - x^2}{9}$

$y^2 = \frac{4(9 - x^2)}{9}$

$= \frac{4}{9}(9 - x^2)$ 3

$\therefore V = \pi \int_{-3}^{3} \frac{4}{9}(9 - x^2)\, dx$ ✓

$= \frac{4}{9}\pi \left[9x - \frac{x^3}{3}\right]_{-3}^{3}$

$= \frac{4}{9}\pi \left(\left(9(3) - \frac{3^3}{3}\right) - \left(9(-3) - \frac{(-3)^3}{3}\right)\right)$

$= \frac{4}{9}\pi(36) = 16\pi$ units^3

b. $y = 3^x$

x	0	$1\frac{1}{3}$	$2\frac{2}{3}$	4
y	1	$3\frac{1}{3}$	$9\frac{2}{3}$	81

function value

Simpson's Rule:

$$A \doteq \frac{h}{3}\left(y_0 + 4(y_1 + y_3 + \dots) + 2(y_2 + y_4 + \dots) + y_n\right)$$

$$= \frac{1\frac{1}{3}}{3}\left(1 + 4\left(3\tfrac{1}{3}\right) + 2\left(9\tfrac{2}{3}\right) + 81\right)$$

$$= \frac{1\frac{1}{3}}{3}\left(114\tfrac{2}{3}\right)$$

$$= \frac{4}{9}\left(114\tfrac{2}{3}\right)$$

$$= 50\tfrac{26}{27}$$

$$= 50.922\ldots$$

$$= 50.92 \text{ units}^2 \text{ (2dp)}$$

$\frac{1}{4}$

c. $\int_1^k x^{-2}\,dx = 2$

$$\left[\frac{x^{-1}}{-1}\right]_1^k = 2$$

$$\left[-\frac{1}{x}\right]_1^k = 2$$

$$\left[-\frac{1}{k} - -\frac{1}{1}\right] = 2$$

$$-\frac{1}{k} + 1 = 2$$

$$-\frac{1}{k} = 1$$

$$-1 = k$$

$\frac{7}{10}$

Question 9

a) $y = x^5 + 3x$

$y' = 5x^4 + 3$

now $x^4 \geq 0$, $\therefore 5x^4 + 3 > 0$

$\therefore$ increasing for all values of x as y' is always greater than 0

ie. $y' > 0$

b) i) $AP = BP$

let $P = (x, y)$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(x+1)^2 + (y-3)^2} = \sqrt{(x-5)^2 + (y-7)^2}$$

$$\therefore \cancel{x^2} + 2x + 1 + \cancel{y^2} - 6y + 9 = \cancel{x^2} - 10x + 25 + \cancel{y^2} - 14y + 49$$

$$2x - 6y + 10 = -10x - 14y + 74$$

$$12x + 8y = 64$$

$$3x + 2y = 16$$

$$\therefore 0 = 3x + 2y - 16$$

ii) $2y = -3x + 16$

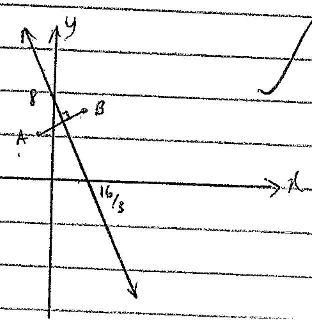
$y = -\frac{3}{2}x + 8$

$\therefore$ grad $= -\frac{3}{2}$

y int $= 8$

x int $= \frac{16}{3}$

locus is ~~b~~ bisector of pts A and B.



c. i) $y = \dfrac{9 - x^2}{9 + x^2}$

$$\therefore \frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$$= \frac{(9+x^2)(-2x) - (9-x^2)(2x)}{(9+x^2)^2}$$

$$= \frac{2x(-9 - x^2 - 9 + x^2)}{(9+x^2)^2}$$

$$= \frac{2x(-18)}{(9+x^2)^2}$$

$$= \frac{-36x}{(9+x^2)^2}$$

ii) $$\int_0^1 \frac{x}{(9+x^2)^2}\,dx$$

$$= -\frac{1}{36}\int_0^1 \frac{-36x}{(9+x^2)^2}\,dx$$

$$= -\frac{1}{36}\left[\frac{9-x^2}{9+x^2}\right]_0^1$$

$$= -\frac{1}{36}\left(\frac{9-1}{9+1} - \frac{9}{9}\right)$$

$$= -\frac{1}{36}\left(-\frac{1}{5}\right)$$

$$= \frac{1}{180}$$

10/10

Question 10.

15227508

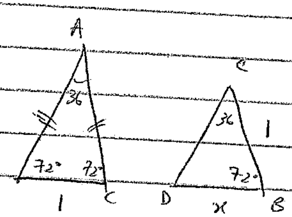
a) $\cos\alpha \tan\alpha$

$= \cancel{\cos\alpha} \cdot \dfrac{\sin\alpha}{\cancel{\cos\alpha}}$

$= \sin\alpha$

1

b) i) ~~In △ABC and △CDB~~



① → $\angle ACB = 72°$ (base ∠'s of isos △ABC are =)

② → $\angle CDB = 72°$ (base ∠'s of isos △CDB are =)

∴ In △ABC and △CDB

$\angle ABC = \angle CDB$ (proven in ①)

$\angle ACB = \angle CBD$ (proven in ②)

∴ $\angle BAC = \angle DCB$ (∠ sum of △ = 180°)

∴ ~~△ABC~~

△ABC ||| △CDB (3 = ∠'s)

ii) $\dfrac{AC}{CB} = \dfrac{BC}{DB}$ (sides in ||| △s in same proportion)

∴ $\dfrac{AC}{1} = \dfrac{1}{x}$

∴ $AC = \dfrac{1}{x}$

2

1522

c i) $T = \dfrac{\sqrt{x^2+1}}{6} + \dfrac{2-x}{10}$

$= \frac{1}{6}(x^2+1)^{\frac{1}{2}} + \frac{1}{10}(2-x)$

$= \frac{1}{6}(x^2+1)^{\frac{1}{2}} + \frac{1}{5} - \frac{1}{10}x$

$T' = \left(\frac{1}{6} \cdot \frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}} \cdot \cancel{2}x\right) - \frac{1}{10}$

$= \frac{1}{6}(x^2+1)^{-\frac{1}{2}} \cdot x - \frac{1}{10}$

$= \dfrac{x}{6\sqrt{x^2+1}} - \dfrac{1}{10}$

2

ii) max/min value when $T' = 0$

$0 = \dfrac{x}{6\sqrt{x^2+1}} - \dfrac{1}{10}$

$\dfrac{1}{10} = \dfrac{x}{6\sqrt{x^2+1}}$

$6\sqrt{x^2+1} = 10x$

$36(x^2+1) = 100x^2$

$36x^2 + 36 = 100x^2$

$36 = 64x^2$

$\dfrac{9}{16} = x^2$

∴ $x = \pm\dfrac{3}{4}$

10/10

~~Test~~ however, time must be positive,

∴ $x = \dfrac{3}{4}$

15227508

Check concavity at $x = \frac{3}{4}$

$$v = (x^2+1)^{\frac{1}{2}}$$
$$v' = \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x$$
$$= x(x^2+1)^{-\frac{1}{2}}$$

$$T'' = \frac{vu' - uv'}{v^2}$$

$$= \frac{1}{6}\left(\frac{(\sqrt{x^2+1})(1) - x\left(\frac{x}{\sqrt{x^2+1}}\right)}{x^2+1}\right)$$

$$= \frac{1}{6}\left(\frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1}\right)$$

Check at $x = \frac{3}{4}$

$$T'' = \frac{1}{6}\left(\frac{\sqrt{\left(\frac{3}{4}\right)^2+1} - \frac{\left(\frac{3}{4}\right)^2}{\sqrt{\left(\frac{3}{4}\right)^2+1}}}{\left(\frac{3}{4}\right)^2+1}\right)$$

$$= \frac{1}{6}\left(\frac{\frac{5}{4} - \frac{9}{20}}{\frac{25}{16}}\right)$$

$$= \frac{32}{375}$$

3

which is positive

$\therefore$ min pt

~~i.e at $x = \frac{3}{4}$, turn pt~~

$\therefore$ so that Gabby's time is the quickest,

$x = \frac{3}{4}$ km