St Catherine's School

Year: 12
Subject: Extension I Mathematics
Time Allowed: 55 minutes

Date: June 2006

Exam number: 16361275

Directions to candidates:

- All questions are to be attempted.
- All necessary **working** must be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Approved calculators and geometrical instruments are required.
- Hand in your work in **1** **bundle**:
- Attach the question paper

Extension I Mathematics

Q.1. (i) Find $\int \frac{x\,dx}{\sqrt{x-5}}$, use the substitution $x-5=u$ (3m)

(ii) Find $\int_0^3 \sqrt{9-x^2}\,dx$, using the substitution $x=3\sin\theta$ (5m)

(iii) $\int \frac{x+1}{(x^2+2x-1)^2}dx$, using the substitution $x^2+2x-1=u$ (3m)

Q.2. A spherical balloon is being deflated so that the radius is decreasing at a constant rate of 8 mm per second. Find the rate of change of volume when the radius is 5 mm. (Note that the volume of a sphere is given by $V=\frac{4\pi r^3}{3}$) (3m)

Q.3. The acceleration of a particle moving in Simple Harmonic motion is given by $\frac{d^2x}{dt^2}=-16x$. The particle starts at the centre of motion with a velocity of 3m/sec

(i) Using only the expression of acceleration and the initial conditions, show that $v^2=9-16x^2$ (2m)

(ii) Hence or otherwise, find an expression for x in terms of t, where x is the displacement from the origin at time t seconds. (3m)

(iii) Find the value of the acceleration at the end points of the motion. (2m)

P.T.O.

Q.4 . The cooling rate of a body is proportional to the difference between the temperature of the body and that of the surrounding medium and is represented by the equation $\frac{dT}{dt} = -k(T - M)$, where T is the temperature of the body and M is the surrounding temperature.

The original temperature of a body is 90^0 C and the temperature of the surrounding is 25^0C .It cools to 80^0C in 20 minutes.

(i) Show that $T = 25 + Ae^{-kt}$ is a solution to the given equation. (1m)

(ii) Show that A = 65 and k = 0.0084 (2 sig figs) (3m)

(ii) Find the temperature after 30 minutes. (1m)

(iii) Find the time taken to cool to 30^0C. (1m)

(iii) Sketch the graph of T **and** find the limiting value of the temperature. (2m)

Q.5 Sue hits a golf ball with a velocity of 50 metres per second and at an angle of α to the horizontal.

(i) Place the coordinate exes at the point of projection and show that the parametric expressions for x and y, the horizontal and vertical displacements respectively, in terms of t, are given by $x = 50\ \cos\alpha\ t$ *and* $y = -5t^2 + 50\ \sin\alpha\ t$

(Take g, the acceleration due to gravity as -10m/sec^2) (2m)

(ii) Find the value of α, the angle of projection, so that the ball just clears a wall 8 metres in height and 10 metres away.

(4m)

END OF PAPER

Question 1

1) i) $\log_6 27 + \log_6 4 - \log_6 3$

$$\frac{\log 27}{\log 6} + \frac{\log_{10} 4}{\log_{10} 6} - \frac{\log_{10} 3}{\log_{10} 6}$$

$= 2$ 2/2

ii) $e^{3\ln 2}$

$= 8$ 1/1

Question 2

i) $\int -\cos 4x \, dx$ 1/1

$= -\frac{1}{4}\sin 4x + c$

ii) $\frac{dy}{dx}$ if $y = \ln x^3$

$y = 3\ln x$

$\therefore y' = \frac{3}{x}$ $\therefore \frac{dy}{dx} = \frac{3}{x}$ 1/1

iii) $f'(x)$ if $f(x) = x^3 e^{3x}$

$f'(x) = 3x^2 e^{3x} + 3x^3 e^{3x}$ 2/2

$= 3x^2 e^{3x}(1 + x)$

Question 3

i) $f'\left(\frac{\pi}{6}\right)$ $f(x) = \sin 3x$

$f'(x) = 3\cos 3x$ ✓

$f\left(\frac{\pi}{6}\right) = 3\cos\frac{3\pi}{6}$ ✓

$= 3\cos\frac{\pi}{2}$ ✓ $\frac{2}{2}$

$= 0$ ✓

ii) $\int_0^{\frac{\pi}{2}} \sec^2\left(\frac{x}{2}\right) dx$ ✓

$= \left(2\tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$ ✓

$= \left(2\tan\frac{\frac{\pi}{2}}{2} - 2\tan 0\right)$ ✓ ✓

$= 2$ ✓ $- 0$ ✓ $\frac{2}{2}$

$= 2$ ✓

iii $\int \frac{x^2}{x^3+1} dx$ $\frac{2}{2}$

$= \frac{f'(x)}{f(x)}$ $= \frac{1}{3}\ln\left|x^3+1\right| + c$ ✓

iv) $\int \frac{e^{3x} + e^{x}}{e^{2x}} dx$

$= \int \frac{e^{3x}}{e^{2x}} + \frac{e^{x}}{e^{2x}} dx$

$= \int e^{x} + e^{-x} dx$

$= e^{x} - e^{-x} + C$

$= e^{x} - \frac{1}{e^{x}} + c$

$\frac{2}{2}$

Question 4

i) ~~Area~~

Arc $= 10$

$\theta = 2^{c}$

i) $\cancel{A} = r\theta$

$r = \frac{l}{\theta}$

$r = \frac{10}{2^{c}}$

$\therefore r = 5$ cm

$\frac{1}{1}$

ii) area) sector

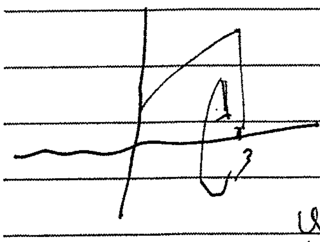
$A = \left(\frac{1}{2} r^{2} \theta\right)$

$A = \frac{1}{2}(25 \cdot 2)$

$= 25 \text{ cm}^2$

$\frac{1}{1}$

Questions:



y^2:

R.T.P $V = 3\pi(e-1)\ u^3$

$y = e^{\frac{x}{6}}$

$y^2 = e^{\frac{x}{3}}$

$V = \pi\int_0^3 e^{\frac{x}{3}}\,dx$

$V = \pi\left(3e^{\frac{x}{3}}\right)_0^3$

$V = \pi\left(3e^1 - 3e^0\right)$ 3/3

$\therefore V = 3\pi(e-1)\ u^3$ as req

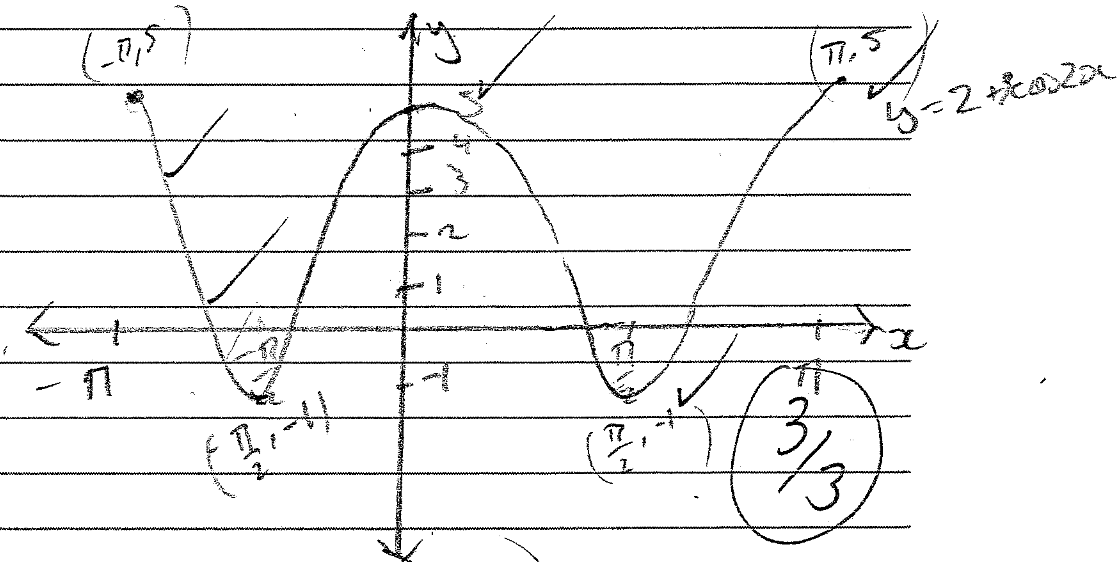
Question 6:

i) amp : 3 ✓
period : π ✓

2/2

$y = 2 + 3\cos 2x$

ii)



3/3

iii) R: $-1 \leq y \leq 5$ ✓

1/1

Question 7:

i) $y = \sin x$
$y = \cos x$

point ~~of~~ intersection

$$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$$

$\therefore \tan x = 1$

$\therefore \quad x = \frac{\pi}{4}$ ✓

$\therefore \quad y = \sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$ ✓

$\therefore$ they intersect at $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ as req'd.

2/2

ii) area:

$$\int_0^{\frac{\pi}{4}} \cos x - \sin x \, dx$$

$$= \left(\sin x + \cos x\right)_0^{\frac{\pi}{4}}$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1\right)$$

$= \left(\frac{2}{\sqrt{2}} - 1\right)$

$= \left(\frac{2\sqrt{2}}{2} - 1\right)$

$= \cancel{\sqrt{2}}$

$= \frac{2\sqrt{2}}{2} - 1$

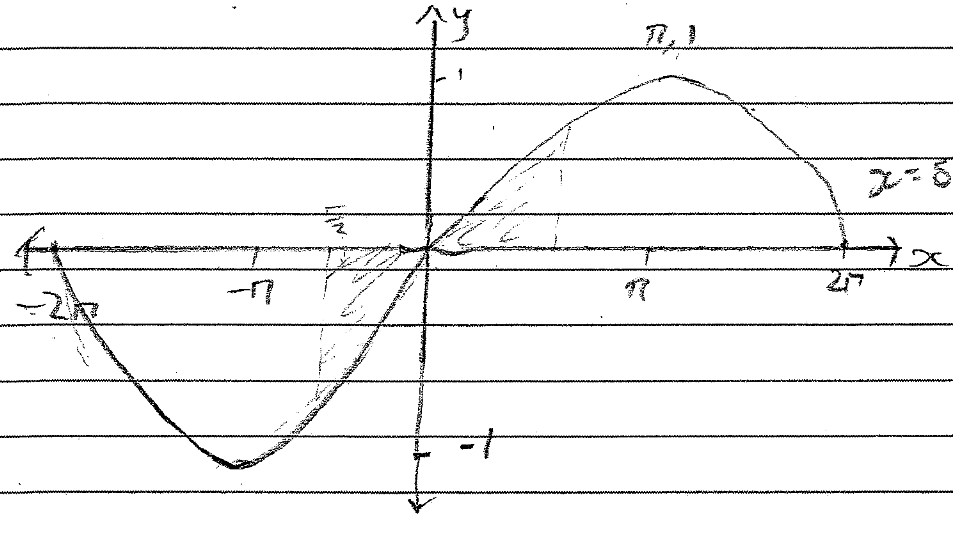
$= \sqrt{2} - 1 \quad u^2$ ✓

$= 0.4142 \quad u^2$

$= 0.41u^2 \,(2.\cancel{d}.p)$

3/3

8 i)



✓ 1/1

$A = 2\int_0^{\frac{\pi}{2}} \sin\frac{x}{2}\,dx$ ✓

$= 2\left(-2\cos\frac{x}{2}\right)_0^{\frac{\pi}{2}}$

$= 2\left(-2\cdot\frac{1}{\sqrt{2}} + 2\right)$

$= 2\left(-\frac{2\sqrt{2}}{2} + 2\right)$

$= 2\left(2 - \sqrt{2}\right)$

$= 4 - 2\sqrt{2} \; u^2$ ✓

$= 1.17157 \; u^2$

$= 1.172 \,(3.d.p)\, u^2$

4/4

Question 9:

i) R.T.P $\frac{d}{dx}(x\sin 3x) = \sin 3x + 3x\cos 3x$

$y = x\sin 3x$

$u' = 1 \quad u = x$

$v' = 3\cos 3x \quad v = \sin 3x$ ✓

$y' = x \cdot 3\cos 3x + \sin 3x$ ✓

$\therefore \frac{d}{dx} = \sin 3x + 3x\cos 3x$ as req'd

$\frac{2}{2}$

ii) $\int x\cos 3x \, dx$

$u = x \qquad v' = \cos 3x$

$u' = 1 \qquad v = \frac{1}{3}\sin 3x$

$\int x\cos 3x = uv - \int u'v$

$= \frac{x}{3}\sin 3x - \int \frac{1}{3}\sin 3x \, dx$

$= \frac{x}{3}\sin 3x - \frac{1}{3} \cdot \frac{1}{3}(-\cos 3x) + c$

$= \frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x + c$ ✓

$\frac{2}{2}$