St Catherine's School

Year: **12**
Subject: Mathematics
Time allowed: 55 minutes

Date: 17th February 2006

Name: ______________________________

Directions to candidates:

- All questions are to be attempted.
- Marks may be deducted for careless or badly arranged work.
- Answer all questions.
- All necessary **working** must be shown in every question.
- Start a new page for every question clearly labelling the questions.

GOOD LUCK ☺

Teacher's Use Only

Question 1	/11
Question 2	/7
Question 3	/12
Question 4	/8
TOTAL	/38

Question 1 (Start a new page) 11 Marks

(a) Find the equation of the tangent to the curve $y = x^3 - x$ at the point where $x = 2$. **3**

(b) Differentiate with respect to x:

(i) $f(x) = \sqrt{25 - x^2}$ **3**

(ii) $y = \dfrac{(3x-2)}{(2-x)}$ **3**

(iii) $y = \dfrac{1}{\sqrt[3]{x^2}}$ **2**

Question 2 Locus (Start a new page) 7 Marks

(a) $P(x, y)$ is a point which moves so that its distance from A(2,1) is always equal to its distance from the line $y = -1$.

(i) Show that the equation of the locus of P is: $(x-2)^2 = 4y$. **2**

(ii) Sketch the locus of P clearly labelling the vertex, focus and the directrix. **2**

(b) Find the equation of the parabola in which the focus is $(1,5)$ and the equation of the directrix is $y = -1$. **3**

Question 3 *(Start a new page)* **12 Marks**

(a) Consider the function $f(x) = (x-5)^2(x+1)$

(i) Show that $f'(x) = 3(x^2 - 6x + 5)$ $3x^2-18x+15$ **2**

(ii) Find the co-ordinates of the stationary points of the curve $y = f(x)$ and determine their nature. **3**

(iii) Find the x- and y-intercepts **2**

(iv) Sketch the graph of the curve $y = f(x)$ showing all intercepts, and stationary points for the domain $-3 \le x \le 7$. **3**

(v) What is the minimum value of the function in this domain. **1**

(vi) For what values of x, in the given domain, is the function increasing? **1**

Question 4 *(Start a new page)* **8 Marks**

(a) In an arithmetic sequence, the fourth term is 12 and the fourteenth term is 62.

(i) Find the first term and the common difference. **3**

(ii) Calculate the sum of the first 50 terms of this sequence. **2**

(b) Find the number of terms in the geometric series 4, 12, 36,....whose sum is 1456. **3**

108 324 972

END OF TASK

✓ means 1 mark | ✗ means ½ mark

Question 1:

(a) $y = x^3 - x$

$y' = 3x^2 - 1$

when $x = 2$, $y' = 3(2)^2 - 1$

$y' = 11$

ie gradient of the tangent is 11

and when $x = 2$

$y = 2^3 - 2$

$= 8 - 2$

$y = 6$

Equation of the tangent is

$y - y_1 = m(x - x_1)$

$y - 6 = 11(x - 2)$

$y - 6 = 11x - 22$

$11x - y - 16 = 0$

(b) (i) $f(x) = \sqrt{25 - x^2}$

$f(x) = (25 - x^2)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(25 - x^2)^{-\frac{1}{2}} \times (-2x)$

$= -x(25 - x^2)^{-\frac{1}{2}}$

$= \dfrac{-x}{(25 - x^2)^{\frac{1}{2}}}$

$= \dfrac{-x}{\sqrt{25 - x^2}}$

(ii) $y = \dfrac{(3x - 2)}{2 - x}$

let $u = 3x - 2$, $u' = 3$ | $v = 2 - x$, $v' = -1$

$y' = \dfrac{u'v - v'u}{v^2}$

$= \dfrac{3(2 - x) - (-1)(3x - 2)}{(2 - x)^2}$

$= \dfrac{6 - 3x + 3x - 2}{(2 - x)^2}$

$= \dfrac{4}{(2 - x)^2}$

(iii) $y = \dfrac{1}{\sqrt[3]{x^2}}$

$y = \dfrac{1}{x^{2/3}}$

$y = x^{-2/3}$

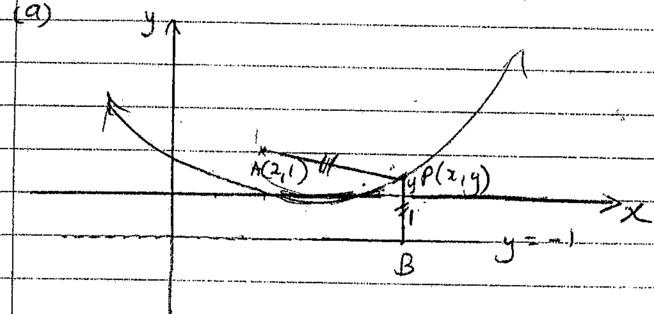
$y' = -\dfrac{2}{3} x^{-5/3}$

$= \dfrac{-2}{3x^{5/3}}$

$= \dfrac{-2}{3\sqrt[3]{x^5}}$

Question 2:

(a)



$PA = PB$

$\left(\sqrt{(x-2)^2 + (y-1)^2}\right)^2 = (y+1)^2$

$(x-2)^2 + (y-1)^2 = (y+1)^2$

$x^2 - 4x + 4 + y^2 - 2y + 1 = y^2 + 2y + 1$

$x^2 - 4x + 5 - 2y - 2y - 1 = 0$

$x^2 - 4x + 4 - 4y = 0$

$x^2 - 4x + 4 = 4y$

$(x-2)^2 = 4y$

(ii) $(x-2)^2 = 4y$

$(x-h)^2 = 4a(y-k)$

$(x-2)^2 = 4 \times \frac{1}{4}(y-0)$

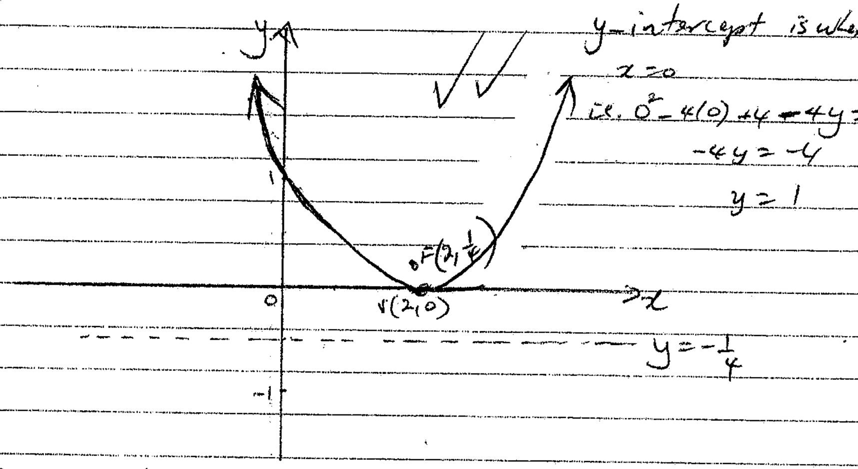
$V(h,k)$

$\therefore V(2,0)$

Focal length is $\frac{1}{4}$

$\therefore$ focus $(2, \frac{1}{4})$

and directrix $y = -\frac{1}{4}$



y-intercept is when $x=0$

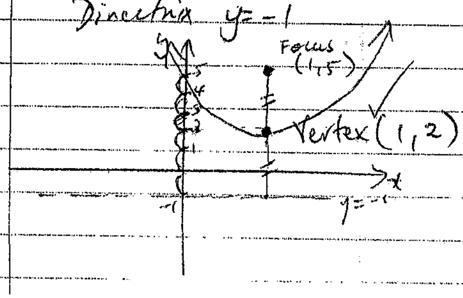
ie. $0^2 - 4(0) + 4 - 4y = 0$

$-4y = -4$

$y = 1$

(b) Focus $(1,5)$

Directrix $y = -1$



focal length $= 3$ ✓

$(x-h)^2 = 4a(y-k)$

$(x-1)^2 = 4 \times 3(y-2)$

$(x-1)^2 = 12(y-2)$ ✓

Question 3.

(a) $f(x) = (x-5)^2(x+1)$

(i) $f'(x) = u'v + v'u$

let $u = (x-5)^2$, $u' = 2(x-5)$ | $v = x+1$, $v' = 1$

$= 2(x-5)(x+1) + 1(x-5)^2$

$= 2(x^2 + x - 5x - 5) + x^2 - 10x + 25$

$= 2x^2 - 8x - 10 + x^2 - 10x + 25$

$= 3x^2 - 18x + 15$

$= 3(x^2 - 6x + 5)$ ✓✓

(ii) Stat. pts occur at $f'(x) = 0$

ie. $3(x^2 - 6x + 5) = 0$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$\therefore x = 5, 1$ ✓

Test for $x = 5$

x	5^-	5	5^+
$f'(x)$	<0	0	>0

min

$\therefore (5, 0)$ minimum turning pt ✓

when $x = 5$

$y = (5-5)^2(5+1)$

$= (0)(6)$

$= 0$

Test for $x = 1$

x	1^-	1	1^+
$f'(x)$	>0	0	<0

max. ✓

$\therefore (1, 32)$ maximum turning pt

when $x = 1$

$y = (1-5)^2(1+1)$

$= (-4)^2(2)$

$= (16)(2)$

$= 32$

(iii) The x-intercept (is when $y=0$)

$f(x) = (x-5)^2(x+1)$

$0 = (x-5)(x-5)(x+1)$

$\therefore x = 5, 5, -1$

The y-intercept (when $x=0$)

$f(x) = (0-5)^2(0+1)$

$= (25)(1)$

$= 25$

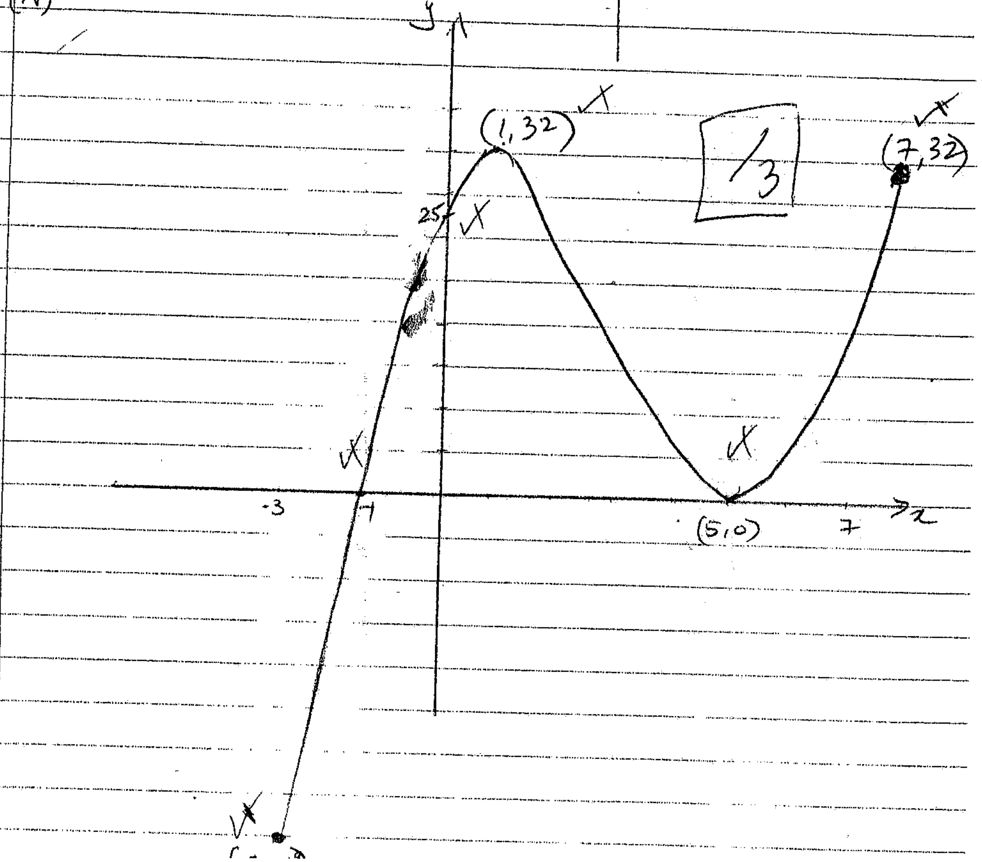
(v) The minimum value of the function is $y = -128$

(vi) $-3 \leq x < 1$

and

$5 < x \leq 7$

(iv)



$\boxed{1/3}$

Question 4:

(a) (i) $T_4 = 12$ and $T_{14} = 62$

$T_n = a + (n-1)d$

$12 = a + (4-1)d$

$12 = a + 3d$ — (1)

$62 = a + 13d$ — (2)

$a + 3d = 12$ — (1)

$-\ a + 13d = 62$ — (2)

$-10d = -50$

$d = 5$

$a + 3d = 12$

$a + 3(5) = 12$

$a + 15 = 12$

$a = 12 - 15$

$a = -3$

$\therefore$ first term is -3 and the common difference is 5

(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{50} = \frac{50}{2}[2(-3) + (50-1)5]$

$= 25[-6 + (49 \times 5)]$

$= 25[239]$

$= 5975$

(b) $S_n = 1456$, $a = 4$, $r = 3$, $n = ?$

$S_n = \frac{a(r^n - 1)}{r-1}$

$\frac{4(3^n - 1)}{3-1} = 1456$

$\frac{4(3^n - 1)}{2} = 1456$

$2(3^n - 1) = 1456$

$3^n - 1 = \frac{1456}{2}$

$3^n - 1 = 728$

$3^n = 729$

$\therefore n = 6$