St Catherine's School

Year: 12
Subject: Mathematics
Time Allowed: 55 minutes
~~Date: June 2005~~

TOPICS : EXP & LOG FUNCTIONS
TRIG. FNS

Exam number: 15227508

Directions to candidates:

- All questions are to be attempted.
- All necessary **working** must be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Start new page when instructed
- Write your student number on the top right hand corner of every page as well as on exam paper
- Approved calculators and geometrical instruments are required.
- Hand in your work in **1 bundle:**
- Attach your question paper

TEACHER'S USE ONLY Total Marks	
Q.1	11
Q.2	7
Q.3	5
Q.4	4
Q.5	4
Q.6	4
TOTAL /35	35

Assessment Task 3 9/6/05

Q.1. Differentiate the following with respect to x.

a) $y = \sin 3x$ (1m)

b) $f(x) = e^{\tan x}$ (2m)

c) $y = \ln(x^2 - 4)$ (1m)

d) $y = x \ln x$ (2m)

e) $y = \sin^2 \frac{x}{2}$ (2m)

f) $y = \log_e \frac{\sqrt{x}}{x+1}$ (3m)

Q.2. Find

a) $\int \frac{5}{3x-2} dx$ (2m)

b) $\int e^{4x+1} dx$ (2m)

c) $\int \frac{x+2}{x^2} dx$ (3m)

(Start a new page)

Q.3. (i) Differentiate $x\, e^x$ (2m)

(ii) Hence find $\int_0^1 x\, e^x dx$ (3m)

Q.4. (i) State the amplitude and period of $y=3 \sin 2x$ (2m)

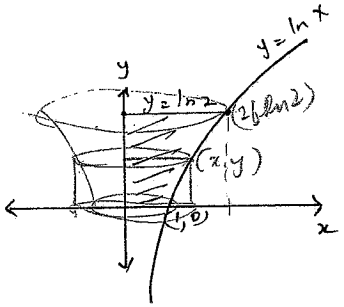
(ii) Hence sketch $y = 3 \sin 2x$ for $0 \le x \le 2\pi$ (2m)

Please Turn Over

(Start a new page)

Q.5. Find the exact area bounded by the curve $y = e^{\frac{x}{2}}$, the x axis and the lines x = ln 2 and x=ln 1
(4m)

Q.6 Find the volume generated when the area bounded by y=ln x , the x axis and y= ln2 is rotated about the y axis. Give your answer in exact form.
(4m)



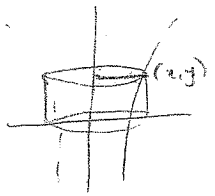
By slices.

$\Delta V = \pi x^2 \Delta y$

$\therefore V = \pi \int_0^{\ln 2} x^2 \, dy$

$= \pi \int_0^{\ln 2} e^{2y} \, dy$

etc.



By cylindrical shells.

$\Delta V = 2\pi xy \, \Delta y$

$V = 2\pi \int_0^{\ln 2} xy \, dy$

$= 2\pi \int_0^{\ln 2} e^y \cdot y \, dy$

$u = y \quad v = e^y$

$u' = 1 \quad v' = e^y$

$= 2\pi \left[uv - \int_0^{\ln 2} vu' \, dx \right]$

$= 2\pi \left[\left[ye^y \right]_0^{\ln 2} - \int_0^{\ln 2} e^y \, dx \right]$

$= 2\pi \left(\ln 2 e^{\ln 2} - \left[e^y \right]_0^{\ln 2} \right)$

$= 2\pi \left(2\ln 2 - \ln 2 + 1 \right)$

15227508

1a. $y = \sin 3x$

$\frac{dy}{dx} = 3\cos 3x$ ✓

b. $f(x) = e^{\tan x}$

$f'(x) = \sec^2 x \cdot e^{\tan x}$ ✓

c. $y = \ln(x^2 - 4)$

$y' = \frac{2x}{x^2 - 4}$ ✓

d. $y = x \ln x$

$y' = uv' + vu'$

$= x\left(\frac{1}{x}\right) + \ln x (1)$

$= 1 + \ln x$ ✓

e. $y = \sin^2 \frac{x}{2} = \left(\sin \frac{x}{2}\right)^2$

$y' = 2 \sin \frac{x}{2} \cdot \frac{1}{2} \cos \frac{x}{2}$

$= \sin \frac{x}{2} \cos \frac{x}{2}$ ✓

f. $y = \ln \frac{\sqrt{x}}{x+1}$

now $\frac{d}{dx}\left(\frac{\sqrt{x}}{x+1}\right) = vu' - uv'$

$= (x+1)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - \sqrt{x}$

$= \frac{x+1}{2\sqrt{x}} - \frac{2x}{2\sqrt{x}}$

$= \frac{1-x}{2\sqrt{x}}$

$= \frac{1-x}{2\sqrt{x}(x+1)^2}$

4. $y = \ln \sqrt{x} - \ln(x+1)$

$y' = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{x}} - \frac{1}{x+1}$

$= \frac{1}{2x} - \frac{1}{x+1}$ ✓

2a. $\int \frac{5}{3x-2} \, dx$

$= \frac{5}{3} \int \frac{3}{3x-2} \, dx$

$= \frac{5}{3} \ln |3x-2| + C$ ✓

b. $\int e^{4x+1} \, dx$

$= \frac{1}{4} \int 4 e^{4x+1} \, dx$

$= \frac{1}{4} e^{4x+1} + C$ ✓

c. $\int \frac{x}{x^2} + \frac{2}{x^2} \, dx$

$= \int \frac{1}{x} + 2x^{-2} \, dx$

$= \ln x + \frac{2x^{-1}}{-1} + C$

$= \ln x - \frac{2}{x} + C$ ✓

15227508

3. i) $y = xe^x$

$u = x \qquad v = e^x$

$u' = 1 \qquad v' = e^x$

$\therefore y' = uv' + vu'$

$= xe^x + e^x$

$= xe^x + e^x$ ✓ 2

ii) $\int_0^1 xe^x + e^x \, dx = [xe^x]_0^1$ ✓

$\therefore \int_0^1 xe^x \, dx = [xe^x]_0^1 - \int_0^1 e^x \, dx$

$= [1e^1 - 0] - [e^x]_0^1$

$= e - (e-1)$

$= 1$ ✓ 3

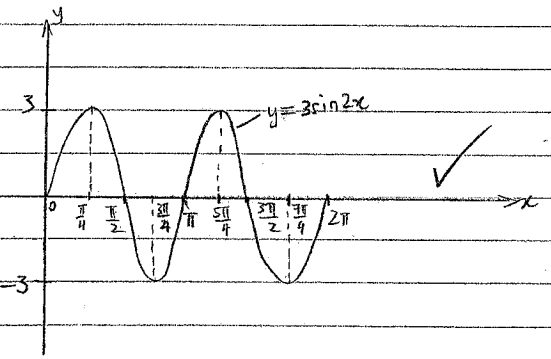
4. i) $y = 3\sin 2x$

$a = 3$ ✓

$T = \frac{2\pi}{n} = \frac{2\pi}{2}$

$= \pi$ secs ✓

ii)



✓ 4

15227508

5. $\int_{\ln 1}^{\ln 2} e^{\frac{x}{2}} \, dx$

$= \frac{4}{2}$

$2\int_{\ln 1}^{\ln 2} \frac{1}{2} e^{\frac{x}{2}} \, dx$

$= 2\left[e^{\frac{x}{2}}\right]_{\ln 1}^{\ln 2}$

$= 2\left(e^{\frac{\ln 2}{2}} - e^{\frac{\ln 1}{2}}\right)$

$= 2(\sqrt{2} - 1)$

$= 2\sqrt{2} - 2$ units^2 ✓ 4

6. $y = \ln x$

$e^y = x$

$V = \pi \int_a^b x^2 \, dy$

$= \pi \int_0^{\ln 2} (e^y)^2 \, dy$

$= \pi \int_0^{\ln 2} e^{2y} \, dy$

$= \frac{\pi}{2} \int_0^{\ln 2} 2e^{2y} \, dy$

$= \frac{\pi}{2} \left[e^{2y}\right]_0^{\ln 2}$

$= \frac{\pi}{2} \left(e^{2\ln 2} - e^{2\times 0}\right)$

$= \frac{\pi}{2}(4 - 1)$

$= \frac{3\pi}{2}$ units^3 ✓ 4