Student Number:________________

Teacher: ______________

St Catherine's School

Waverley, Sydney

Year 12 Extension 1 Mathematics

Half Yearly Examination

Task #2

April 2010

Time allowed: *2 hours*

Reading time: 5 minutes

General Instructions

- Working time – 120 minutes
- Write using black or blue pen
- Board-approved calculators may be used.
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value
- Section A- Questions 1-3 are to be completed in one booklet
- Section B - Questions 4-7 are to be completed in one booklet

The following list of standard integrals may be used:

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\ dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\ dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\ dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\ dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\ dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\ dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\ dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\ dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

SECTION A – Questions 1-3 (Write your answers in the one booklet)

Question 1 – 12 Marks **Marks**

a) Solve the inequality: $\dfrac{x^2-9}{x} \geq 0$. 3

b) The point $P(-3,8)$ divides the interval AB externally in the ratio $k:1$. If A is the point $(6,-4)$ and B is the point $(0,4)$, find the value of k. 3

c) Prove that: for $0 < x < \dfrac{\pi}{4}$,

$$\tan\left(\frac{\pi}{4}+x\right)=\frac{\cos x+\sin x}{\cos x-\sin x}$$ 2

d) Differentiate the following with respect to x:

i. $y=\cos^{-1}\left(\dfrac{1}{x}-1\right)$ 2

ii. $y=\tan^{-1}\sqrt{x^2-1}$ 2

Question 2 – 12 Marks (START A NEW PAGE)

a) Let α, β and γ are the roots of the polynomial $2x^3-5x-1=0$.
Find $\alpha^{-1}\beta^{-1}\gamma^{-1}$. 2

b) Show that $x+4$ is a factor of $P(x)=x^3+2x^2-23x-60$ and hence factorise $P(x)$. 3

c) i. Evaluate $\int (x^2+1)^3 .2x dx$, using the substitution $u=x^2+1$. 2

ii. Evaluate $\int \sqrt{1-x^2}\, dx$, using the substitution $x=\sin\theta$. 3

d) Evaluate $\int_{-1}^{0} x\sqrt{1+x}\, dx$, using the substitution $u=1+x$. 4

Question 3 – 12 Marks (START A NEW PAGE) **Marks**

a) Let $f(x)=\dfrac{x}{x^2-1}$.

i. For what values of x is $f(x)$ undefined? 1

ii. Show that $y=f(x)$ is an odd function. 1

iii. Show that $f'(x)<0$ at all values of x for which the function is defined. 2

iv. Hence sketch $y=f(x)$. 2

b) Consider the parabola $y=(x-2)^2-3$.

i. Sketch the parabola $y=(x-2)^2-3$, showing its vertex and y-intercept. 2

ii. Find the largest positive domain such that the graph defines a function $f(x)$ which has an inverse function. 1

iii. Find the inverse function, stating its domain. 2

iv. Sketch the inverse function. 1

SECTION B – Questions 4 – 7 (START A NEW BOOKLET)

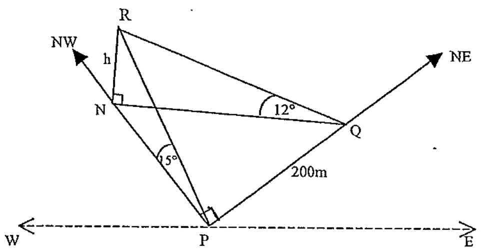
Question 4 – 12 Marks) **Marks**

a) Evaluate, in terms of π,

i. $\cos^{-1}\left(-\frac{1}{2}\right)-\sin^{-1}\left(-\frac{1}{2}\right)$ **2**

ii. $2\tan^{-1}(1)+\tan^{-1}\left(-\sqrt{3}\right)$ **2**

b) A bushwalker on a horizontal straight road that runs north-east, observes from a point P that a hill bears north-west and its peak R has an angle of elevation of 15°. On walking 200m further along this road to a point Q, the angle of elevation of R is now 12°.
What is the height h of the hill to the nearest metre? **4**



c) In surveying, when determining a reference point, the expression $\frac{1}{2\cos^2\alpha}-\tan^2\alpha$ occurs.
Show that this expression is equal to $\frac{\cos 2\alpha}{2\cos^2\alpha}$. **2**

d) Find $\int \cos^2 4x\,dx$. **2**

Question 5 – 12 Marks (START A NEW PAGE) **Marks**

a) The two points $P(2ap,ap^2)$ and $Q(2aq,aq^2)$ are on the parabola $x^2=4ay$.

i. The equation of the tangent to $x^2=4ay$ at a point $T(2at,at^2)$ is given as $y=tx-at^2$.
Show that the tangents at the points P and Q meet at R, where R is the point $\left(a(p+q),apq\right)$. **2**

ii. As P varies, the point Q is always chosen so that $\angle POQ$ is a right angle, where O is the origin.
Find the locus of R. **2**

b) Consider the polynomial $P(x)=x^3+2x-4$.

i. Show that $P(x)=0$ has only one real root and that it lies in the interval $1<x<1.5$. **2**

ii. Taking $x_1=1.2$ as a first approximation to this root, use one step of Newton's Method to find a better approximation x_2, correct to two decimal places. **2**

c) By substitution $t=\tan\frac{\theta}{2}$, or otherwise, show that $\frac{1-\cos\theta}{\sin\theta}=\tan\frac{\theta}{2}$. **2**

d) Sketch the graph of $f(x)=2\cos^{-1}x$, indicating clearly the coordinates of the endpoints of the graph. **2**

Question 6 – 12 Marks (START A NEW PAGE) **Marks**

a) Find the exact values of x and y which satisfy the simultaneous equations **3**

$$\sin^{-1}x+\frac{1}{2}\cos^{-1}y=\frac{\pi}{3} \quad \text{and}$$

$$3\sin^{-1}x-\frac{1}{2}\cos^{-1}y=\frac{2\pi}{3}.$$

b) The cubic polynomial $P(x)=x^3+rx^2+sx+t$, where r, s and t are real numbers,

has three real zeros, 1, α and $-\alpha$.

i. Find the value of r. **1**

ii. Find the value of $s+t$. **2**

c) i. Write $8\cos x+6\sin x$ in the form $A\cos(x-\alpha)$, where $A>0$ and $0\le\alpha\le\frac{\pi}{2}$. **2**

ii. Hence, or otherwise, solve the equation $8\cos x+6\sin x=5$ for $0\le x\le 2\pi$. **2**

Give your answers correct to three decimal places.

d) Suppose $x^3-2x^2+a\equiv(x+2)Q(x)+3$ where $Q(x)$ is a polynomial.

Find the value of a. **2**

Question 7 – 12 Marks (START A NEW PAGE) **Marks**

a) Prove by mathematical induction that:

$1\times2^0+2\times2^1+3\times2^2+\ldots.+n\times2^{n-1}=1+(n-1)2^n$, for all $n\ge1$. **4**

b) Let $f(x)=\sin^{-1}(x+5)$.

i. State the domain and range of the function $f(x)$. **2**

ii. Find the gradient of the graph of $y=f(x)$ at the point where $x=-5$. **2**

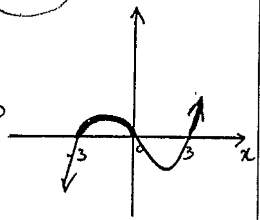
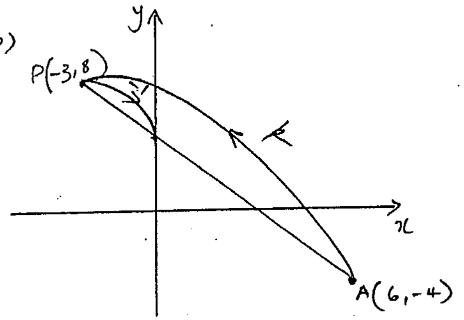
iii. Sketch the graph of $y=f(x)$. **2**

c) Let $5x=90^\circ$. Show that $\sin 2x=\cos 3x$ **2**

END OF PAPER

YEAR 12 – Extension 1 Mathematics – Midcourse Exam – APRIL 2010

①

Qn	Solutions	Marks	Comments: Criteria
1	(a) $\dfrac{x^2-9}{x} \geq 0 \quad (\times x^2)$ ✓		
	$\dfrac{x^2(x^2-9)}{x} \geq 0$		
			
	$x(x^2-9) \geq 0$		
	$x(x-3)(x+3) \geq 0$ ✓	3	
	$-3 \leq x < 0$ or $x \geq 3$ ✓		
	(b) 		2 if correct working but internal instead of external!
	The coordinates of P are given by		
	$x_p = \dfrac{kx_2 + lx_1}{k+l}$ ✓, $y = \dfrac{ky_2+ly_1}{k+l}$	3	
	$-3 = \dfrac{k(0)+(-1)(6)}{k+(-1)}$ ✓		
	$-3 = \dfrac{-6}{k-1}$		
	$-3(k-1) = -6$		
	$-3k+3 = -6$		
	$-3k = -9$		
	$\therefore k = 3$ ✓		

②

Qn	Solutions	Marks	Comments: Criteria
1(c)	$\tan\left(\frac{\pi}{4}+x\right) = \dfrac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\cdot\tan x}$ ✓		the statement is true for all x where $\tan(\frac{\pi}{4}+x)$ is defined so answer ignores required given domain.
	$= \dfrac{1+\tan x}{1-\tan x}$		
	$= \dfrac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}$	2	
	$= \dfrac{\frac{\cos x+\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}}$		
	$= \dfrac{\cos x+\sin x}{\cos x-\sin x}$ ✓		
(d)	(i) $y = \cos^{-1}\left(\frac{1}{x}-1\right)$		
	let $u = \frac{1}{x}-1$ $\qquad \therefore y = \cos^{-1} u$		
	$u = x^{-1}-1$ $\qquad \dfrac{dy}{du} = \dfrac{-1}{\sqrt{1-u^2}}$ ✓		
	$\dfrac{du}{dx} = -x^{-2}$		
	$= -\dfrac{1}{x^2}$ ✓		
	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$		
	$= \dfrac{-1}{\sqrt{1-u^2}} \times -\dfrac{1}{x^2}$	2	
	$= \dfrac{1}{x^2\sqrt{1-(\frac{1}{x}-1)^2}}$		
	$= \dfrac{1}{x^2\sqrt{1-(\frac{1}{x^2}-\frac{2}{x}+1)}}$		
	$= \dfrac{1}{x^2\sqrt{\frac{2}{x}-\frac{1}{x^2}}}$		

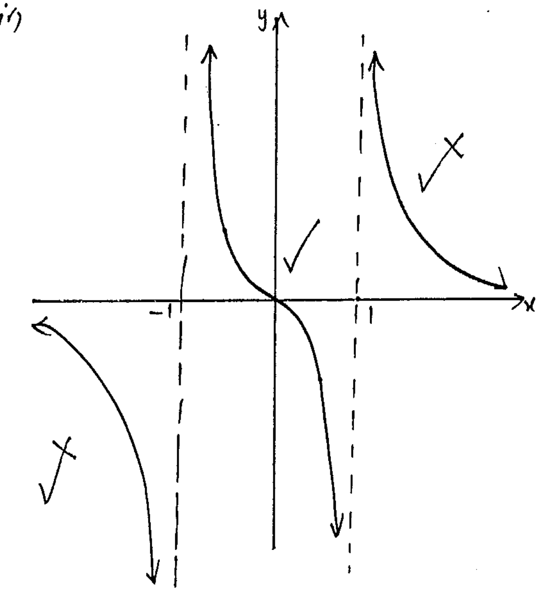
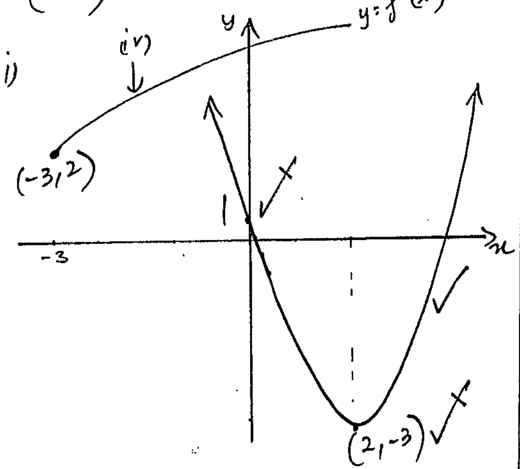
Qn	Solutions	Marks	Comments: Criteria
1(d)	$= \dfrac{1}{x^2\sqrt{\frac{2x-1}{x^2}}}$ $= \dfrac{1}{x^2 \frac{\sqrt{2x-1}}{\sqrt{x^2}}}$ $= \dfrac{1}{x^2 . \frac{\sqrt{2x-1}}{x}}$ $= \dfrac{1}{x . \sqrt{2x-1}}$ ✓		
	(ii) $y = \tan^{-1}\sqrt{x^2-1}$ let $u = \sqrt{x^2-1}$ $\frac{du}{dx} = \frac{1}{2}(x^2-1)^{-\frac{1}{2}} \times 2x$ $= \frac{x}{\sqrt{x^2-1}}$ ✓ $y = \tan^{-1}u$ $\frac{dy}{du} = \frac{1}{1+u^2}$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $= \frac{1}{1+(\sqrt{x^2-1})^2} \cdot \frac{x}{\sqrt{x^2-1}}$ $= \frac{x}{x^2 . \sqrt{x^2-1}}$ $= \frac{1}{x . \sqrt{x^2-1}}$ ✓	2	1 off for no $f'(x)$

Qn	Solutions	Marks	Comments: Criteria
2	(a) $2x^3-5x-1=0$ $\alpha^{-1}\beta^{-1}\gamma^{-1} = \frac{1}{\alpha}\cdot\frac{1}{\beta}\cdot\frac{1}{\gamma}$ $= \frac{1}{\alpha\beta\gamma}$ where $\alpha\beta\gamma = -\frac{d}{a}$ $= -\frac{-1}{2}$ $= \frac{1}{2}$ ✓ $\therefore \alpha^{-1}\beta^{-1}\gamma^{-1} = \frac{1}{\frac{1}{2}}$ $= 2$ ✓	2	
	(b) $P(-4) = (-4)^3 + 2(-4)^2 - 23(-4) - 60$ $= -64 + 32 + 92 - 60$ $= 0$ $\therefore x+4$ is a factor. ✓ $x+4 \overline{) x^3+2x^2-23x-60}$ quotient $x^2-2x-15$ x^3+4x^2 $-2x^2-23x-60$ $-2x^2-8x$ $-15x-60$ ✓ $-15x-60$ 0 $\therefore P(x) = (x+4)(x^2-2x-15)$ $=(x+4)(x-5)(x+3)$ ✓	3	

Qn	Solutions	Marks	Comments: Criteria
(c) (i)	$\int (x^2+1)^3 \cdot 2x\,dx$ let $u = x^2+1$, $\frac{du}{dx} = 2x$ ✓, $du = 2x\,dx$ $= \int u^3 \cdot du$ $= \frac{u^4}{4} + C$ $= \frac{(x^2+1)^4}{4} + C$ ✓	2	
(ii)	$\int \sqrt{1-x^2}\,dx$ let $x = \sin\theta$, $\frac{dx}{d\theta} = \cos\theta$ ✓, $dx = \cos\theta\,d\theta$ $= \int \sqrt{1-\sin^2\theta}\,dx$ $= \int \sqrt{\cos^2\theta} \cdot \cos\theta\,d\theta$ $= \int \cos^2\theta \cdot d\theta$ $= \frac{1}{2}\int 1+\cos 2\theta\,d\theta$ ✓ $= \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]$ $= \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + C$ ✓ $= \frac{1}{2}\sin^{-1}x + \frac{1}{4} \cdot 2x\sqrt{1-x^2} + C$ $= \frac{1}{2}\sin^{-1}x + \frac{x}{2}\sqrt{1-x^2} + C.$	3	Should give answer in terms of x. From this point forward this <u>must</u> be done

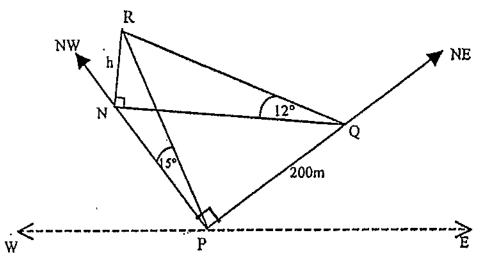
Qn	Solutions	Marks	Comments: Criteria
(d)	$\int_{-1}^{0} x\sqrt{1+x}\,dx$ let $u = 1+x$, $\therefore x = u-1$, $\frac{dx}{du} = 1$ ✓, $dx = du$ $= \int_{-1}^{0} (u-1)\sqrt{u}\,du$ ✓ $= \int_{-1}^{0} (u-1) \cdot u^{1/2}\,du$ $= \int_{-1}^{0} u^{3/2} - u^{1/2}\,du$ ✓ $= \left[\frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3}\right]_0^1$ ✓ when $x=0, u=1$; $x=-1, u=0$ $= \left[\frac{2}{5}(1)^{5/2} - \frac{2}{3}(1)^{3/2}\right] - [0]$ $= \frac{2}{5} - \frac{2}{3}$ $= -\frac{4}{15}$ ✓	4	

Qn	Solutions	Marks	Comments: Criteria
3	(a) $f(x) = \dfrac{x}{x^2-1}$		
	(i) $x^2-1 \neq 0$ $(x-1)(x+1) \neq 0$ $x \neq \pm 1$ ✓	1	
	(ii) $y = f(x) = \dfrac{x}{x^2-1}$ $f(-x) = \dfrac{-x}{(-x)^2-1}$ $= \dfrac{-x}{x^2-1}$ $= -\left(\dfrac{x}{x^2-1}\right)$ ✓ $= -f(x)$ $\therefore$ function is odd	1	
	(iii) $f'(x) = \dfrac{u'v - v'u}{v^2}$ let $u = x$, $v = x^2-1$ $u' = 1$, $v' = 2x$ $= \dfrac{1(x^2-1) - 2x \cdot x}{(x^2-1)^2}$ $= \dfrac{x^2-1-2x^2}{(x^2-1)^2}$ ✓ $= \dfrac{-1-x^2}{(x^2-1)^2}$ ✓ $= \dfrac{-(x^2+1)}{(x^2-1)^2} < 0$ since $x^2+1>0$ and $(x^2-1)^2>0$ $\therefore f'(x) < 0$, $x \neq \pm 1$	2	

Qn	Solutions	Marks	Comments: Criteria
3	a (iv) 	2	
	(b) $y = (x-2)^2 - 3$ (i) 	2	
	(ii) The required domain for which $f(x)$ has an inverse is $x \geq 2$. ✓	1	

Qn	Solutions	Marks	Comments: Criteria
(iii)	$y=(x-2)^2-3$ ✓✗ $(x-2)^2 = y+3$ $x-2=\pm\sqrt{y+3}$ $x=\pm\sqrt{y+3}+2$ ✓ $\therefore f^{-1}(x)=\sqrt{x+3}+2$ ✓ Domain: $x \geq -3$ ✓✗	2	When calculating inverse should show $\pm\sqrt{\ }$ then for inverse function choose either $\sqrt{\ }$ or $-\sqrt{\ }$
(iv)	see diagram on previous page. ✓	1	

✓ means 1 mark
✓✗ means 0.5 mark

Qn	Solutions	Marks	Comments: Criteria
4(a)	(i) $\cos^{-1}\left(-\frac{1}{2}\right)-\sin^{-1}\left(-\frac{1}{2}\right)$ $=\pi-\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)$ ✓✗ note: $\cos^{-1}(-x)=\pi-\cos^{-1}x$ $\sin^{-1}(-x)=-\sin^{-1}x$ $=\pi-\frac{\pi}{3}+\frac{\pi}{6}$ ✓ $=\frac{6\pi-2\pi+\pi}{6}$ $=\frac{5\pi}{6}$ ✓✗	2	
	(ii) $2\tan^{-1}(1)+\tan^{-1}(-\sqrt{3})$ $=2\tan^{-1}(1)-\tan^{-1}(\sqrt{3})$ ✓✗ $=2.\frac{\pi}{4}-\frac{\pi}{3}$ ✓✗ note: $\tan^{-1}(-x)=-\tan^{-1}x$ $=\frac{\pi}{2}-\frac{\pi}{3}$ ✓✗ $=\frac{\pi}{6}$ ✓✗	2	
b)	 In ΔNPR, $\angle RNP=90^\circ$ $\frac{NP}{h}=\cot 15^\circ \Rightarrow NP=h\cot 15^\circ$ ✓ In ΔNQR $\frac{h}{NQ}=\tan 12^\circ$ $\therefore \frac{NQ}{h}=\cot 12^\circ \Rightarrow NQ=h\cot 12^\circ$ ✓	4	

Qn	Solutions	Marks	Comments: Criteria
4(b)	and $\angle NPQ = 90^\circ$		
	$NP^2 + PQ^2 = NQ^2$		
	$h^2\cot^2 15^\circ + 200^2 = h^2\cot^2 12^\circ$ ✓		
	$h^2(\cot^2 12^\circ - \cot^2 15^\circ) = 40000$		
	$h = \sqrt{\dfrac{40000}{\cot^2 12^\circ - \cot^2 15^\circ}}$ ✓		
	$\therefore h = \dfrac{200}{\sqrt{\cot^2 12^\circ - \cot^2 15^\circ}}$ ✓		
	$= 70\text{ m}.$ ✓		
(c)	$\dfrac{1}{2\cos^2\alpha} - \tan^2\alpha$		
	$= \dfrac{1}{2\cos^2\alpha} - \dfrac{\sin^2\alpha}{\cos^2\alpha}$ ✓		
	$= \dfrac{1 - 2\sin^2\alpha}{2\cos^2\alpha}$ ✓ note: $\cos 2\alpha = 1 - 2\sin^2\alpha$	2	
	$= \dfrac{\cos 2\alpha}{2\cos^2\alpha}$ ✓		

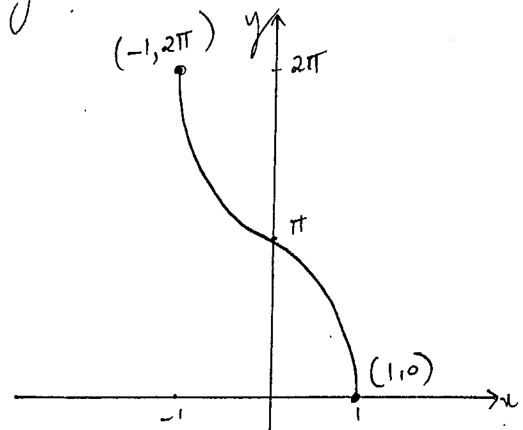
Qn	Solutions	Marks	Comments: Criteria
(d)	$\int \cos^2 4x \, dx$		
	$= \frac{1}{2}\int (1 + \cos 8x)\, dx$ ✓		
	$= \frac{1}{2}\left[x + \frac{1}{8}\sin 8x\right]$ ✓		
	$= \frac{x}{2} + \frac{1}{16}\sin 8x + C$		
	OR $\int \cos^2 4x \, dx$ let $u = 4x$	2	
	$\frac{du}{dx} = 4$ ✓		
	$= \frac{1}{4}\int \cos^2 u \, du$ ✓ $\frac{du}{4} = dx$		
	$= \frac{1}{4}\int \frac{1}{2}(1 + \cos 2u)\, du$		
	$= \frac{1}{8}\left[u + \frac{1}{2}\sin 2u\right]$ ✓		
	$= \frac{1}{8}\left[4x + \frac{1}{2}\sin 8x\right]$ ✓		
	$= \frac{x}{2} + \frac{1}{16}\sin 8x + C$		

Qn	Solutions	Marks	Comments: Criteria
5	(a) $P(2ap, ap^2)$ $Q(2aq, aq^2)$ $x^2 = 4ay$		
	(i) Tangent at P: $y = px - ap^2$ — ①		
	Tangent at Q: $y = qx - aq^2$ — ②		
	At R, the pt of intersection		
	$px - ap^2 = qx - aq^2$		
	$px - qx = ap^2 - aq^2$		
	$x(p-q) = a(p^2 - q^2)$ ✓		
	$x = \dfrac{a(p^2-q^2)}{p-q}$ ✓	2	
	$x = \dfrac{a\cancel{(p-q)}(p+q)}{\cancel{(p-q)}}$ ✓		
	$x = a(p+q)$		
	$y = p[a(p+q)] - ap^2$		
	$= ap^2 + apq - ap^2$ ✓		
	$= apq$		
	$\therefore R\,[a(p+q), apq]$		

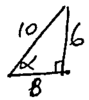
Qn	Solutions	Marks	Comments: Criteria
5(a) (ii)	If $\angle POQ$ is a right angle		
	$\therefore m_{OQ} \times m_{OP} = -1$		
	$m_{OQ} = \dfrac{ap^2 - 0}{2ap - 0}$ $\quad m_{OP} = \dfrac{q}{2}$		
	$= \dfrac{ap^2}{2ap}$		
	$= \dfrac{p}{2}$	2	
	$\therefore \dfrac{p}{2} \times \dfrac{q}{2} = -1$ ✓		
	$\dfrac{pq}{4} = -1$		
	$pq = -4$ ✓		
	Now for R:		
	$x = a(p+q)$ $\quad y = apq$		
	but $pq = -4$ $\therefore y = -4a$ ✓ (where a is a constant)		
	Since $y = -4a$ is always true this must be the locus of R.		

(15)

Qn	Solutions	Marks	Comments: Criteria
5 (b)	$P(x) = x^3 + 2x - 4$		
	$P'(x) = 3x^2 + 2 > 0$ for all real x,		
	$\therefore$ the curve $y = P(x)$ has no turning point. ✓		
	The graph of $P(x)$ intersects the x-axis only once; $\therefore P(x) = 0$ has only one real root! ✓	2	
	$P(1) = 1 + 2 - 4 = -1 < 0$ ✓		
	$P(1.5) = 3.375 + 3 - 4 = 2.375 > 0$ ✓		
	Since $P(1)$ & $P(1.5)$ are of opposite signs, the root lies between 1 and 1.5.		
(ii)	$P(1.2) = 1.2^3 - 2(1.2) - 4$		
	$= 0.128$ ✓		-0.5 if using $x_1 = 1.5$
	$P'(1.2) = 3(1.2)^2 + 2$ ✓		
	$= 6.32$	2	
	$x_2 = x_1 - \dfrac{P(1.2)}{P'(1.2)}$ ✓		
	$= 1.2 - \dfrac{0.128}{6.32}$ ✓		
	$= 1.1797\ldots$		
	$= 1.18$ (to 2 dp)		

Qn	Solutions	Marks	Comments: Criteria
5 (c)	If $t = \tan\frac{\theta}{2}$, $\sin\theta = \dfrac{2t}{1+t^2}$		
	$\cos\theta = \dfrac{1-t^2}{1+t^2}$		
	$\dfrac{1-\cos\theta}{\sin\theta}$		
	$= \dfrac{1 - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}}$ ✓	2	
	$= \dfrac{\frac{1+t^2-1+t^2}{1+t^2}}{\frac{2t}{1+t^2}}$ ✓		
	$= \dfrac{2t^2}{2t}$ ✓		
	$= t$		
	$= \tan\frac{\theta}{2}$		
(d)	$y = 2\cos^{-1}x$		
		2	½ mark for each endpoint 1 mark for correct shape + y-int.

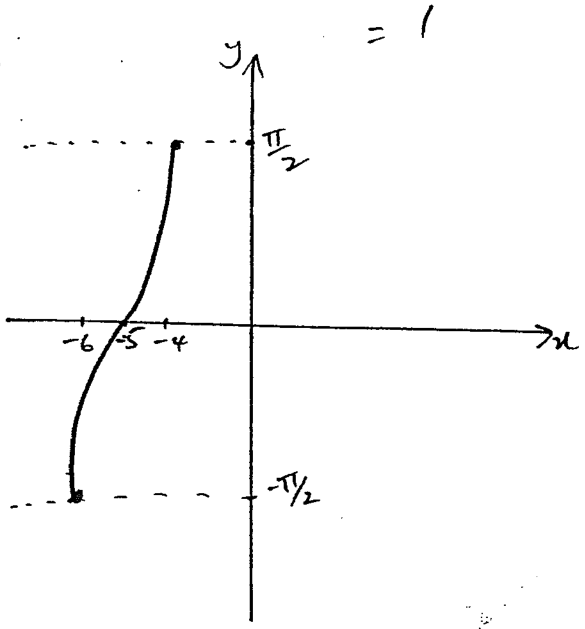
Qn	Solutions	Marks	Comments: Criteria
6(a)	$\sin^{-1}x + \frac{1}{2}\cos^{-1}y = \frac{\pi}{3}$ —— (1)		
	$3\sin^{-1}x - \frac{1}{2}\cos^{-1}y = \frac{2\pi}{3}$ —— (2)		
	(1) + (2)		
	$4\sin^{-1}x = \frac{\pi}{3} + \frac{2\pi}{3}$		
	$4\sin^{-1}x = \pi$		
	$\sin^{-1}x = \frac{\pi}{4}$		
	$x = \frac{1}{\sqrt{2}}$	3	
	sub. x in (1) to solve for y: $\frac{\pi}{4} + \frac{1}{2}\cos^{-1}y = \frac{\pi}{3}$		
	$\frac{1}{2}\cos^{-1}y = \frac{\pi}{3} - \frac{\pi}{4}$		
	$\cos^{-1}y = 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$		
	$\cos^{-1}y = \frac{\pi}{6}$		
	$\therefore y = \frac{\sqrt{3}}{2}$		

Qn	Solutions	Marks	Comments: Criteria
6(b)	$P(x) = x^3 + rx^2 + sx + t$		
	the zeros are: $1, \alpha, -\alpha$		
	(i) Sum of roots $= 1 + \alpha - \alpha = -\frac{b}{a}$		
	$-\frac{b}{a} = 1$		
	$-r = 1$	1	
	$\therefore r = -1$		
	(ii) If 1 is a zero and $r = -1$ then $P(1) = 0$		
	i.e. $1^3 + r(1)^2 + 1s + t = 0$		
	$1 + r + s + t = 0$	2	
	$1 - 1 + s + t = 0$		
	$\therefore s + t = 0$		
(c)	(i) $8\cos x + 6\sin x$		
	$= 10\cos(x - 36°52')$	2	
	$= 10\cos(x - 0.643)$		
	 $\therefore \alpha = 36°52'$		
	(ii) $8\cos x + 6\sin x = 5$		
	$\therefore 10\cos(x - 36°52') = 5$		
	$\cos(x - 36°52') = \frac{1}{2}$		
	$\therefore x - 36°52' = \cos^{-1}\frac{1}{2}$		
	$x - 36°52' = 60°, 300°$	2	
	$\therefore x = 96°52', 336°52'$		
	$x = 1.691, 5.879.$		−0.5 if answers are not in radians.

(19)

Qn	Solutions	Marks	Comments: Criteria
6(d)	$x^3-2x^2+a \equiv (x+2)\,Q(x)+3$ Since $(x+2)$ is a factor $\therefore (-2)^3-2(-2)^2+a \equiv (-2+2)Q(x)+3$ ✓ $-8-8+a \equiv 0.Q(x)+3$ ✓ $-16+a \equiv 0+3$ $\therefore a = 19$ ✓	2	
7(a)	Step 1: Prove true for $n=1$ $LHS = n\times 2^{n-1}$ $= 1\times 2^{1-1}$ $= 1\times 2^0$ $= 1\times 1$ $= 1$ $RHS = 1+(n-1)2^n$ $= 1+(1-1)2^1$ $= 1+0$ $= 1$ $LHS = RHS$ $\therefore$ true for $n=1$ ✓ Step 2: Assume true for $n=k$ ✓ ie. $1\times2^0+2\times2^1+3\times2^2+\dots+k\times2^{k-1} = 1+(k-1)2^k$ ✓ Step 3: Aim to prove true for $n=k+1$ ie. $\underline{1\times2^0+2\times2^1+3\times2^2+\dots+k.2^{k-1}}+(k+1).2^k = 1+k.2^{k+1}$ $LHS = \boxed{1+(k-1)2^k}+(k+1).2^k$ ✓ $= 1+k.2^k-\cancel{2^k}+k.2^k+\cancel{2^k}$ $= 1+2k.2^k$ ✓ $= 1+k.2^1.2^k$ ✓ $= 1+k.2^{k+1} = RHS$	4	

Qn	Solutions	Marks	Comments: Criteria
7 (c)	$5x = 90^\circ$ $2x+3x = 90^\circ$ $2x = 90^\circ-3x$ ✓ $\therefore \sin 2x = \sin(90^\circ-3x)$ ✓ $= \cos 3x$ ✓ (note: complementary angles)	2	
	OR $\sin 2x$ $= \sin(5x-3x)$ $= \sin 5x\cos 3x - \cos 5x \sin 3x$ $= \sin 90^\circ . \cos 3x - \cos 90^\circ \sin 3x$ $= 1.\cos 3x - 0.\sin 3x$ $= \cos 3x$ 21449040	2	

Qn	Solutions	Marks	Comments: Criteria
	Step 4: Therefore true for $n=k+1$ if true for $n=k$. Since true for $n=1$ then true for $n=2,3,4,----$ ✗ $\therefore$ Statement is true for all $n\geq 1$ by mathematical induction.		
b)	$f(x)=\sin^{-1}(x+5)$ (i) Domain: $-1\leq x+5\leq 1$ $-6\leq x\leq -4$ ✓ Range: $-\frac{\pi}{2}\leq f(x)\leq\frac{\pi}{2}$ ✓	2	
	(ii) $f'(x)=\frac{1}{\sqrt{1-u^2}}\cdot 1$ let $u=x+5$, $\frac{du}{dx}=1$ $=\frac{1}{\sqrt{1-(x+5)^2}}$ ✓ $y=\sin^{-1}u$, $\frac{dy}{du}=\frac{1}{\sqrt{1-u^2}}$ when $x=5$, $f'(x)=\frac{1}{\sqrt{1-(\cancel{5}+5)^2}}$ $=\frac{1}{\sqrt{1-0}}$ ✓ $=1$	2	
	(iii) 	2	1 mark for shape of curve 1 mark for x-intercept and showing the range