Student Number:

St. Catherine's School
Waverley

2010

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 3 – 15%
11th May

Mathematics

Q1	/12
Q2	/13
Q3	/6
Q4	/8
TOTAL	/39

General Instructions

- Working time – 55 minutes
- Start each question on a new page in your answer booklet.
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary **working** must be shown.
- Marks may be deducted for careless or badly arranged work.

- Attempt Questions 1–4
- Questions are not of equal value

Question 1 (12 Marks) **Marks**

a. Find the following indefinite integrals:

i. $\int (x - e^{3x})\ dx$ 2

ii. $\int \frac{x^2 - 4}{x^2} dx$ 2

b. Evaluate the following definite integral:

$\int_0^1 (e^x + e^{-x})^2 dx$ 3

c. **i.** Complete this table for the function $y = e^{(\pi x)}$ giving answers to 2 decimal places 5

x	0.1	0.2	0.3
$f(x)$			

ii. Use these values and Simpson's rule to estimate $\int_{0.1}^{0.3} e^{(\pi x)} dx$ to 2 decimal places

iii. Use calculus to evaluate correct to 2 decimal places $\int_{0.1}^{0.3} e^{(\pi x)} dx$

Question 2 (13 Marks) Start a new page

Marks

a. Differentiate the following functions with respect to x:

i. $y = \dfrac{1}{4e^{4x}}$ **2**

ii. $y = (e^{2x} + 1)^{-2}$ **2**

b. Find the equation of the normal to the curve $y = e^{2x} + x$ when $x = 0$ **3**

c. Find the equation of a curve which passes through the point $(0,4)$ if the gradient function of the curve is given by $\dfrac{dy}{dx} = e^{2x} + x - 1$ **3**

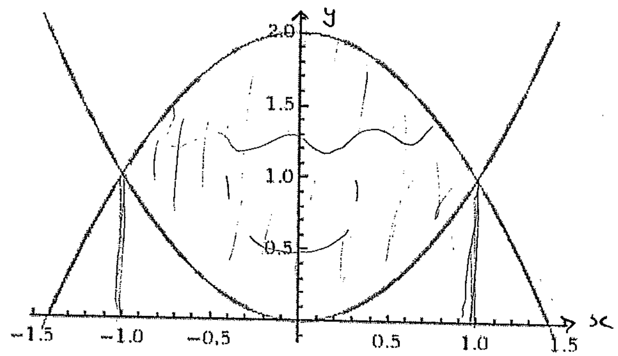
d. **i.** Differentiate $y = xe^{x}$ **1**

ii. Hence find $\displaystyle\int_0^1 xe^{x}\,dx$ **2**

Question 3 (6 Marks) Start a new page

Marks

a. These graphs represent the functions $y = x^2$ and $y = 2 - x^2$ which meet at $(1,1)$ and $(-1,1)$

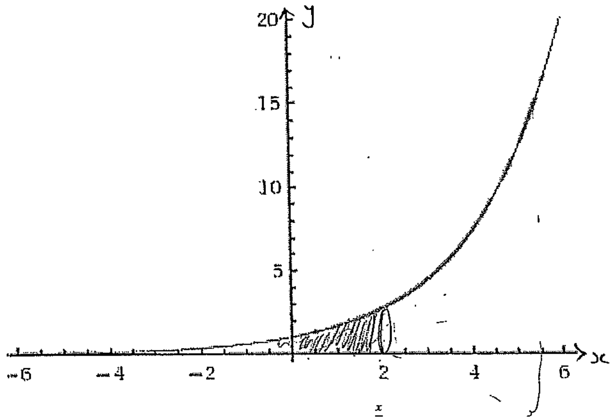


Find to two decimal places the area between the two curves **3**

b. Find the x value of the stationary point on the curve $y = \dfrac{e^{-x}}{x}$ **3**

Question 4 (8 Marks) **Start a new page** **Marks**

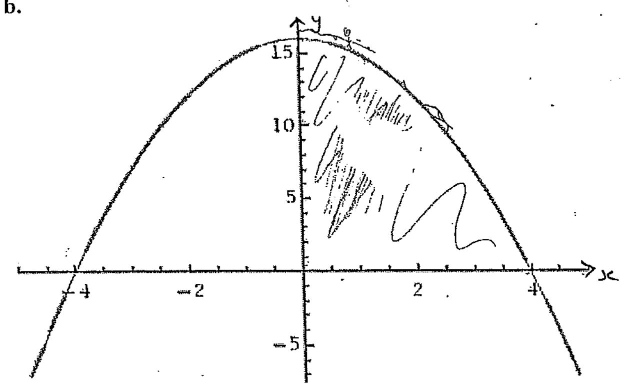
a. 4



This graph represents the function $y = e^{\frac{x}{2}}$

Find the **exact** volume of the solid formed if the area under the curve and above the x axis between $x = 0$ and $x = 2$ is rotated about the x axis through 360^0

b.



This graph represents the curve $y = 16 - x^2$. The area between the curve and the x and y axes in the first quadrant is rotated through 360^0 about the y axis. Find the **exact** volume of the solid formed.

End Of Test

Qn	Solutions	Marks	Comments+Criteria
1 a	i. $\int (x - e^{3x})\,dx = \frac{x^2}{2} - \frac{e^{3x}}{3} + c$	2	
	ii. $\int \frac{x^2-4}{x^2}\,dx = \int (1 - 4x^{-2})\,dx$	1	
	$= x + \frac{4}{x} + c$	1	
b	$\int_0^1 (e^x + e^{-x})^2\,dx = \int_0^1 (e^{2x} + 2 + e^{-2x})\,dx$	1	
	$= \left[\frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2}\right]_0^1$	1	
	$= \left(\frac{e^2}{2} + 2 - \frac{1}{2e^2}\right) - \left(\frac{1}{2} + 0 - \frac{1}{2}\right)$		
	$= \frac{e^2}{2} + 2 - \frac{1}{2e^2}$	1	
c	i. x: 0·1, 0·2, 0·3; $f(x)$: 1·37, 1·87, 2·57	1	
	ii. $A = \frac{0 \cdot 1}{3}\left[1 \cdot 37 + 4 \times 1 \cdot 87 + 2 \cdot 57\right]$		
	$= 0 \cdot 38$ (2 d.p.)	2	
	iii. $\int_{0.1}^{0.3} e^{(\pi x)}\,dx = \frac{1}{\pi}\left[e^{\pi x}\right]_{0.1}^{0.3}$		
	$= \frac{1}{\pi}\left[e^{.3\pi} - e^{.1\pi}\right]$		
	$= 0 \cdot 38$ (2 d.p.)	2	

Qn	Solutions	Marks	Comments+Criteria
2 a	i. $y = \frac{e^{-4x}}{4}$		
	$\therefore \frac{dy}{dx} = \frac{-4e^{-4x}}{4} = -e^{-4x}$ or $\frac{-1}{e^{4x}}$	2	
	ii. $y = (e^{2x}+1)^{-2}$		
	$\frac{dy}{dx} = -2\left[e^{2x}+1\right]^{-3} \times 2e^{2x}$		
	$= \frac{-4e^{2x}}{(e^{2x}+1)^3}$	2	
b	If $y = e^{2x} + x$ then $\frac{dy}{dx} = 2e^{2x} + 1$		
	$\therefore f'(0) = 2e^0 + 1 = 3$		
	giving gradient normal $= -\frac{1}{3}$		
	Now when $x = 0$ $y = e^0 + 0 = 1$		
	$\therefore y - 1 = -\frac{1}{3}(x - 0)$		
	$\Rightarrow x + 3y - 3 = 0$	3	
c	If $\frac{dy}{dx} = e^{2x} + x - 1$		
	then $y = \frac{e^{2x}}{2} + \frac{x^2}{2} - x + c$		
	when $x = 0$, $y = 4$		
	$\therefore 4 = \frac{e^0}{2} + 0 - 0 + c$		
	$\Rightarrow c = 3\frac{1}{2}$		
	Hence equation is		
	$y = \frac{e^{2x}}{2} + \frac{x^2}{2} - x + \frac{7}{2}$	3	

Qn	Solutions	Marks	Comments+Criteria
2 d	i) $y = xe^{x}$ using product rule $\frac{dy}{dx} = xe^{x} + e^{x}$	1	
	ii) Now $\int_0^1 xe^{x}\,dx = \int_0^1 (xe^{x} + e^{x}) - \int_0^1 e^{x}\,dx$ $= \left[xe^{x} - e^{x}\right]_0^1$ $= (e^{1} - e^{1}) - (0 - e^{0})$ $= 1.$	2	

Qn	Solutions	Marks	Comments+Criteria
3 a	Area $= 2\int_0^1 (2-x^2) - x^2\,dx$ $= 2\int_0^1 (2-2x^2)\,dx$ $= 2\left[2x - \frac{2x^3}{3}\right]_0^1$ $= 2\left((2-\frac{2}{3}) - 0\right)$ $= 2 \times \frac{4}{3} = \frac{8}{3} = \underline{2.67\,u^2}$	3	
b.	$y = \frac{e^{-x}}{x}$ Stationary when $\frac{dy}{dx} = 0$ Using quotient rule $\frac{dy}{dx} = \frac{x \times -e^{-x} - e^{-x} \times 1}{x^2}$ $= \frac{-e^{-x}[x+1]}{x^2}$ $e^{-x} \neq 0 \quad \therefore \quad x+1 = 0$ $\Rightarrow x = -1$	3	

Qn	Solutions	Marks	Comments+Criteria
4 a	Volume $= \pi \int_0^2 \left(e^{\frac{x}{2}}\right)^2 dx$ $= \pi \int_0^2 e^x \, dx$ $= \pi \left[e^x\right]_0^2 = \pi\left[e^2 - 1\right] u^3$	4	
b.	$y = 16 - x^2 \Rightarrow x^2 = 16 - y$ $V = \pi \int_0^{16} (16-y) \, dy$ $= \pi \left[16y - \frac{y^2}{2}\right]_0^{16}$ $= \pi \left[(256 - 128) - 0\right]$ $= 128\pi \ u^3$		