



St. Catherine's School  
 Waverley  
 PRELIMINARY ASSESSMENT TASK 4  
 September  
**2006**

# Mathematics

**General Instructions**

- Working time – 120 minutes (+ 5 min reading time)
- Write using black or blue pen
- Board-approved calculators may be used.

**Total marks – 84**

- Attempt all questions
- All questions are of equal value
- All necessary working should be shown

Question 1	/12
Question 2	/12
Question 3	/12
Question 4	/12
Question 5	/12
Question 6	/12
Question 7	/12

**QUESTION 1 (12 Marks)**

*Start a new page*

**Marks**

a) Evaluate  $\sqrt{\frac{3.5 \times 7.62}{\pi}}$ , giving answer correct to:

- (i) 3 decimal places 1
- (ii) 3 significant figures 1

b) Simplify  $\sqrt{2} + \sqrt{18}$  1

c) Solve  $3(x-4) - (x+2) = 12$  2

- d) Factorise:
- i)  $am + an - bm - bn$  2
  - ii)  $9x^2 - y^4$  2

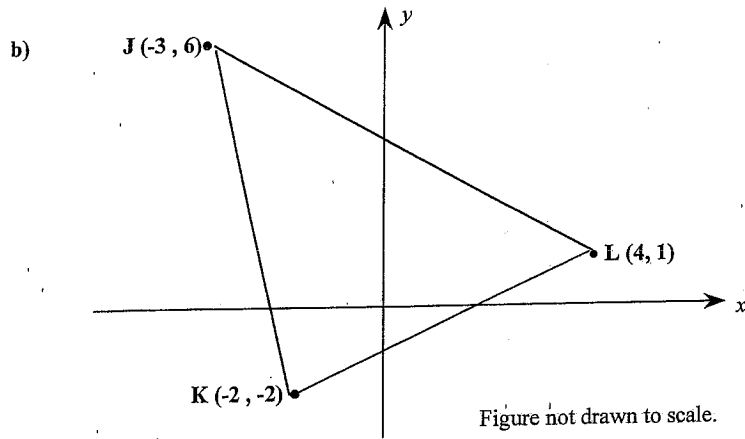
e) Solve:  $|3x + 2| \leq 10$  and graph the results on a number line. 3

QUESTION 2 (12 Marks)

Start a new page

Marks

- a) Evaluate  $\frac{4}{3}\pi r^3$  to 1 decimal place given  $r = 4.2$  1



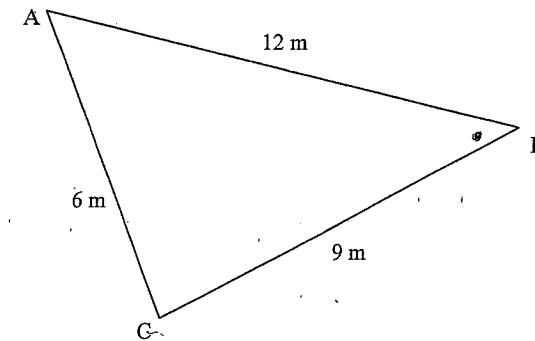
- i) Find the length of KL (leave answer in simplified surd form) 1
- ii) Show that the gradient of KL is  $\frac{1}{2}$  1
- iii) Find the equation of the line KL in general form 2
- iv) What is the perpendicular distance of the point J from KL (leave answer in surd form) 2
- v) Hence or otherwise, find the area of  $\Delta JKL$  2
- c) i) Solve the simultaneous equations 2
- $$\begin{aligned} 5x - y &= 4 \\ 3x + y &= 12 \end{aligned}$$
- ii) Hence state the point of intersection of the lines 1
- $$5x - y = 4 \text{ and } 3x + y = 12$$

QUESTION 3 (12 Marks)

Start a new page

Marks

- a) i) Express  $\frac{7}{4-\sqrt{2}}$  with a rational denominator 2
- ii) Hence find  $a$  and  $b$  such that  $\frac{7}{4-\sqrt{2}} = a + b\sqrt{2}$  1
- b) Use the cosine rule to find angle ABC in this triangle (to nearest minute): 3



- c) Sketch the curves showing all essential features. Use a separate set of axes for each sketch. Each sketch should take about one quarter of a page
- i)  $x^2 + y^2 = 9$  2
- ii)  $y = |x + 2|$  2
- iii)  $xy = 4$  2

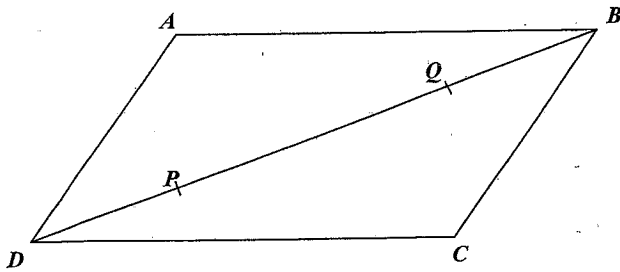
## QUESTION 4 (12 Marks)

Start a new page

Marks

- a) i) Sketch the parabola  $y = -x^2 - 4x + 5$  showing its intercepts and vertex. 4
- ii) What is the range for the parabola  $y = -x^2 - 4x + 5$ ?
- iii) State the values of  $x$  for which  $-x^2 - 4x + 5 > 0$

- b)  $ABCD$  is a parallelogram. Points  $P$  and  $Q$  lie on the diagonal  $DB$  such that  $DP = BQ$ . 4



Copy the diagram into your answer booklet. Join  $AQ$  and  $PC$

Prove that  $AQ = PC$ .

- c) Write in simplest form: 4

i)  $\log_2 \frac{1}{8}$

ii)  $\log_2 \sqrt{8}$

iii)  $\log_2 9.5 + \log_2 \left(\frac{1}{19}\right)$

## QUESTION 5 (12 Marks)

## START BOOKLET 2

- a) For the Arithmetic sequence 100,94,88.....
- i) Write down the first term  $a$  and common difference  $d$  2
- ii) Calculate  $T_{12}$  2
- iii) Write down the formula for  $S_n$  1
- iv) Find the smallest value of  $n$  for which  $S_n < 0$  3
- b) For the Geometric Series with  $a = 4$  and  $r = -0.8$
- i) Find the value of  $T_4$  2
- ii) Find the sum to infinity for this series 2

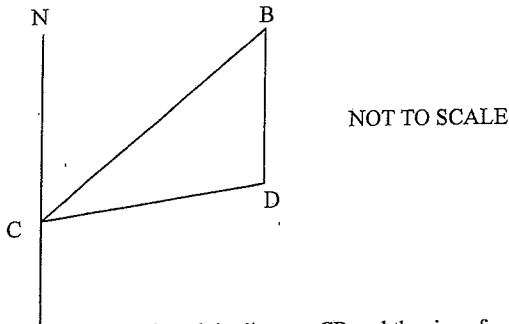
**QUESTION 6 (12 Marks)**

*Start a new page*

**Marks**

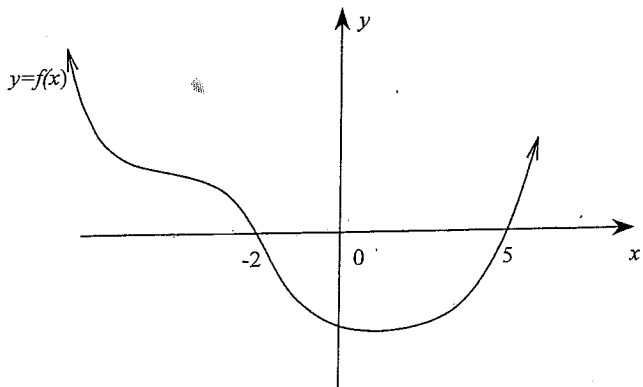
- a) Francine stands on point C on Coogee beach and observes a ship B 10 km out to sea on a bearing of  $040^\circ T$ .

The ship is sailing due South. After 10 minutes its bearing is  $085^\circ T$  and it is at point D



Copy the diagram and mark in distance CB and the size of angles NCB and NCD 1

- i) Angle  $CBD = 40^\circ$ . Explain why. 1
  - ii) Angle  $BCD = 45^\circ$ . Explain why. 1
  - iii) Angle  $BDC = 95^\circ$ . Explain why. 1
  - iv) Find the length of BD to 1 decimal place 3
  - v) Find the speed of the ship in km/hr 1
- b) If  $\sin \theta = -\frac{2}{3}$  and  $\cos \theta > 0$ , find the value of  $\tan \theta$  2  
 (express your answer in surd form)
- c) The graph shows the graph of  $y = f(x)$  2  
 State the values of  $x$  for which  $f(x) > 0$



**QUESTION 7 (12 Marks)**

*Start a new page*

**Marks**

- a) Simplify  $(1 - \sin^2 x)(1 + \tan^2 x)$  3
- b) A series is given by:  
 $(1 + r) + (1 + r)^2 + (1 + r)^3 + \dots + (1 + r)^n + \dots$
- i) Show the series is Geometric. 1
  - ii) For which values of  $r$  does a sum to infinity exist? 2
- c) i) Given that  $PQ \parallel BC$  and  $QR \parallel AB$ , prove that the triangles APQ and ABC are similar. 2

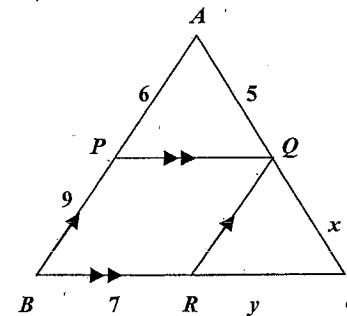


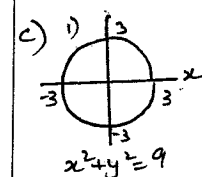
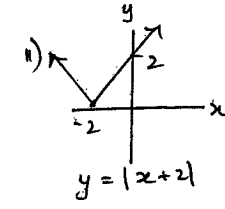
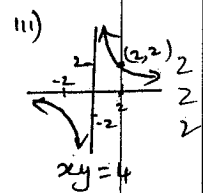
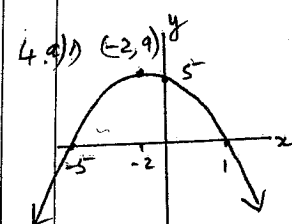
Figure not to scale

- ii) Hence find the values of  $x$  and  $y$  (Give full reasons) 2
- d) Simplify  $\frac{4 - x^2}{6 - x - x^2} \times \frac{3x + 9}{4x^2 + 16x + 16}$  2

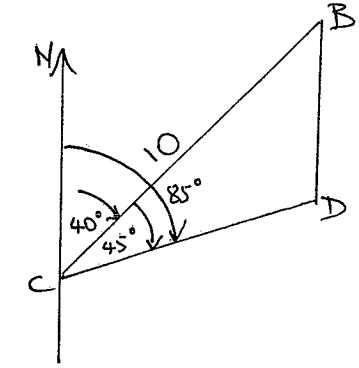
*End of Paper*

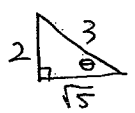
# Maths Yr 11 Prelim '06 Solutions

Qn	Solutions	Marks	Comments+Criteria
1	a) i) 2.914 ii) 2.91	1 1	
	b) $\sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$	1	
	c) $3x - 12 - x - 2 = 12$ $2x - 14 = 12$ $2x = 26$ $x = 13$	2	1 for simplifying 1 for solution
	d) i) $a(m+n) - b(m+n)$ $(a-b)(m+n)$	2	
	ii) $9x^2 - y^4 = (3x - y^2)(3x + y^2)$	2	
	e) $-10 \leq 3x + 2 \leq 10$ $-12 \leq 3x \leq 8$ $-4 \leq x \leq 2\frac{2}{3}$	3	1 for each solution of x 1 for correct graph of solution.
2	b) i) $d = \sqrt{(4-2)^2 + (1-2)^2}$ $= \sqrt{4+1} = \sqrt{5} \therefore KL = 3\sqrt{5}$	2	Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
	ii) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{4-2} = \frac{-1}{2} = -\frac{1}{2}$	1	Formula $m = \frac{y_2 - y_1}{x_2 - x_1}$
	iii) $y - 1 = \frac{1}{2}(x - 4)$ $2y - 2 = x - 4$ $x - 2y - 2 = 0$ eq. of KL	2	Formula
	(iv) $d = \left  \frac{1x - 3 - 2x + 6 - 2}{\sqrt{1^2 + 2^2}} \right  = \left  \frac{-17}{\sqrt{5}} \right  = \frac{17}{\sqrt{5}}$	2	Formula $d = \left  \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right $
	v) $A = \frac{1}{2} b \times h = \frac{1}{2} \times 3\sqrt{5} \times \frac{17}{\sqrt{5}} = 25.5 \text{ u}^2$	2	
	c) $8x = 16 \therefore x = 2$ $y = 6$ $\therefore P(2, 6)$	1	

Qn	Solutions	Marks	Comments+Criteria
3	a) $\frac{7}{4-\sqrt{2}} \times \frac{4+\sqrt{2}}{4+\sqrt{2}} = \frac{28+7\sqrt{2}}{16-2} = \frac{4+\sqrt{2}}{2}$	2	
	i) $\therefore a = 2$ $b = \frac{1}{2}$	1	
	b) $\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{12^2 + 9^2 - 6^2}{2 \times 12 \times 9}$ $= 0.875$ $B = 28^\circ 27'$ (nearest min)	3	
	c) i) 		
	ii) 		
	iii) 		
	d) i) 		
	ii) Range $y \leq 9$ $y \in \mathbb{R}$ ①		
	iii) $-5 < x < 1$ ①		
	b) In $\Delta$ s ABQ, CDP $\angle ABQ = \angle CDP$ (alt $\angle$ s AB // DC) $QB = PD$ (given) $AB = DC$ (opp side    gram) $\therefore \Delta ABQ \cong \Delta CDP$ (SAS) $\therefore AQ = CP$ (corresp sides of cong. $\Delta$ s)	2 1 1	<u>NB</u> Sequence is important.
	c) $\log_2 \frac{1}{8} = \log_2 2^{-3} = -3$	1	
	$\log_2 \sqrt{8} = \log_2 2^{3/2} = \frac{3}{2}$	1	
	$\log_2 9.5 + \log_2 \frac{1}{19} = \log_2 (9.5 \times \frac{1}{19})$ $= \log_2 \frac{1}{2} = -1$	2	

Qn	Solutions	Marks	Comments+Criteria
5a)	100, 94, 88, ...		
	(i) $a = 100$ $d = -6$	✓ ✓	
	(ii) $T_{12} = a + 11d$ $= 100 - 66$ $= 34$	✓ ✓	
	(iii) $S_n = \frac{n}{2}(2a + (n-1)d)$ $\left[ \begin{aligned} &= \frac{n}{2}(200 + (n-1)(-6)) \\ &= \frac{200n - 3n(n-1)}{2} \\ &= 103n - 3n^2 \end{aligned} \right]$	✓	accept either formula $\frac{1}{2}$ for both with one incorrect.
	(iv) $S_n < 0$ means $103n - 3n^2 < 0$ $n(103 - 3n) < 0$ $n > \frac{103}{3}$ $n < 0$ as $n > 0$ $\therefore n > 34\frac{1}{3}$ ie $n = 35$ $[S_{35} = -70]$	✓ ✓ ✓	-1 for dividing by $x$ without explanation that $n > 0$ $\frac{1}{2}$ for not stating $n = 35$
(b)	$a = 4$ $r = -0.8$		
	(i) $T_4 = ar^3$ $= 4 \cdot (-0.8)^3$ $= -2.048$ $\left[ \begin{aligned} &= \frac{-256}{125} \\ &= -2\frac{6}{125} \end{aligned} \right]$	✓ ✓	

Qn	Solutions	Marks	Comments+Criteria
5b)	(ii) $S_\infty = \frac{a}{1-r}$ $= \frac{4}{1+0.8}$ $= \frac{20}{9} = 2.\bar{2}$	✓ ✓	
6(a)		✓	
	(i) $\angle CBD = 40^\circ$ (alt $\angle$ s on $\parallel$ lines $BD, NC$ )	✓	accept (alt $\angle$ s) only 1
	(ii) $\angle BCD = 45^\circ$ (Adj angle to $\angle NCB$ and $\angle NCD = 85^\circ$ )	✓	accept subtraction of $\angle$ s if explained
	(iii) $\angle BDC = 95^\circ$ ( $\angle$ s sum $\triangle BCD$ )	✓	accept ( $\angle$ s sum $\triangle$ )
	(iv) $\frac{BD}{\sin 45^\circ} = \frac{10}{\sin 95^\circ}$ $\therefore BD = \frac{10 \sin 45^\circ}{\sin 95^\circ}$ $= 7.0980 \dots \doteq 7.1$ (1dp)	✓	1 for $\sin 45 = \frac{BD}{10}$ if correct RO $\checkmark(\checkmark)$ (RO)

Qn	Solutions	Marks	Comments+Criteria
6(a)(v)	$\text{Speed} = \frac{D}{T}$ $= \frac{7.0980\dots}{\frac{1}{6}}$ $= 42.5884\dots$ <p style="text-align: right;">Km/hr</p>	✓	Ignore ROE
(b)	$\sin \theta = -\frac{2}{3} \quad \cos \theta > 0$ $\therefore \tan \theta = -\frac{2}{\sqrt{5}}$ $= -\frac{2\sqrt{5}}{5}$	✓	
	 $\therefore \tan \theta = -\frac{2}{\sqrt{5}}$ $= -\frac{2\sqrt{5}}{5}$	✓	
(c)	$f(x) > 0$ for $x < -2$ $x > 5$	✓ ✓	$-1$ for $f(x) < -2$ $f(x) > 5$ $-\frac{1}{2}$ for $-2 < x > 5$ $-1$ for $x \leq -2$ $x \geq 5$

Qn	Solutions	Marks	Comments+Criteria
7. a)	$(1 - \sin^2 x)(1 + \tan^2 x)$ $= \cos^2 x \times \sec^2 x$ $= \cos^2 x \times \frac{1}{\cos^2 x} = 1$	2 1	
b)	$\frac{T_n}{T_{n-1}} = \frac{T_3}{T_2} = \frac{T_L}{T_1} = 1+r$ <p><math>S_{\infty}</math> exists when <math> 1+r  &lt; 1</math></p> $\therefore -1 < 1+r < 1$ $-2 < r < 0$	1 2	
c)	<p>In <math>\Delta</math>s APQ, ABC</p> <p><math>\angle A</math> is common</p> <p><math>\angle APQ = \angle ABC</math> (corresp, <math>PQ \parallel BC</math>)</p> <p>(<math>\angle AQP = \angle ACB</math>, " " )</p> <p><math>\therefore \Delta APQ \sim \Delta ABC</math></p> <p><math>\therefore \frac{6}{15} = \frac{5}{5+x}</math> corresp sides of sim <math>\Delta</math>s</p> <p><math>x = 7.5</math></p> <p>also <math>PQ = 7</math> (opp sides of p'gram)</p> <p><math>\frac{6}{7} = \frac{15}{7+y}</math> (same)</p> <p><math>42 + 6y = 105</math></p> <p><math>6y = 63 \quad y = 10.5</math></p>	2 1 1	
d)	$\frac{4-x^2}{6-x-x^2} \times \frac{3x+9}{4x^2+16x+16}$ $= \frac{(2-x)(2+x)}{(2-x)(3+x)} \times \frac{3(x+3)}{4(x+2)(x+2)}$ $= \frac{3}{4(x+2)}$	2	