Student Number:____________________

St. Catherine's School
Waverley

2009

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 1 – 15%
CLASS TEST: 16th February

Mathematics

Q1	/11
Q2	/17
Q3	/17
TOTAL	/45

General Instructions

- Working time – 55 minutes
- Start each question on a new page in your answer booklet.
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary **working** must be shown.
- Marks may be deducted for careless or badly arranged work.

Total marks –

- Attempt Questions 1–3
- Questions are not of equal value

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Question 1 Start a new page (11 marks)

(a) Find the length of the radius of the circle $x^2+y^2-4x+2y-4=0$. /2

(b) Given the equation of the parabola $(x-1)^2=12(y+2)$ find:
- (i) the coordinates of the vertex /1
- (ii) the coordinates of the focus /1
- (iii) the equation of the directrix /1

(c) Find the equation of the parabola with vertex (-1, 2), the axis of symmetry parallel to the y-axis and passing through the point (-4, -1) /3

(d) A point P (x,y) moves so that its distance from the y-axis is equal to its distance from the point (3,4). Find the equation of the locus of P. /3

Question 2 Start a new page (17 marks)

(a) Differentiate the following with respect to x

(i) $y=4x^3+3x-2$ /1

(ii) $y=\dfrac{1}{(4x-1)^5}$ /2

(iii) $y=\dfrac{x^2+3}{5x-2}$ /2

(iv) $y=\sqrt{x}\,(2x+1)$ /2

(b) (i) Find the gradient of the tangent to the curve $y=4x+\dfrac{1}{x}$ at $x=1$ /2

(ii) Find the equation of this tangent at $x=1$ /2

(iii) What are the coordinates of the point(s) on the curve at which the tangent(s) are parallel to the x axis? /3

(c) The curve $y=ax+\dfrac{b}{x^2}$ cuts the x-axis at $x=1$, and the gradient of the tangent to the curve at $x=1$ is 3. Find the values of a and b. /3

Question 3 Start a new page (17 marks)

(a) The first three terms of an arithmetic sequence are 5,9,13
- (i) Find the twentieth term /1
- (ii) How many terms of the sequence are needed to make the sum of the terms equal to 230? /3

(b) Find 4+9+14+........+74 /2

(c) Evaluate $\sum_{r=2}^{8} 2^{r-3}$ /3

(d) In a geometric series the fifth term is $\dfrac{81}{8}$ and the second term is 3. Find the first term and the common ratio. /2

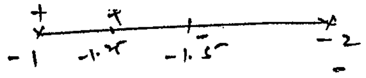
(e) (i) For what set of values of x will a limiting sum exist for the infinite geometric series $1-2x+4x^2+.....$? /1

(ii) Find the value of x for which this limiting sum is $\dfrac{3}{2}$ /2

(f) Three numbers whose product is 216 form the terms of a geometric sequence. If 1,4 and 8 are subtracted from them respectively, the terms form an arithmetic sequence. Find the numbers. /3

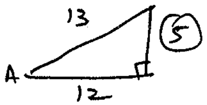
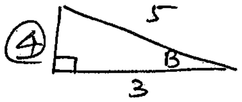
End of Paper

Qn	Solutions	Marks	Comments+Criteria
1. a.	$2x^3-3x^2+4x-6=0$		
	$\alpha+\beta+\gamma = \frac{3}{2}$	0.5	
	$\alpha\beta+\beta\gamma+\alpha\gamma = \frac{4}{2}=2$	1	
	$\alpha\beta\gamma = \frac{6}{2} = 3.$	0.5	no penalty for subsequent error.
	$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2\Sigma\alpha\beta$	1	
	$= \left(\frac{3}{2}\right)^2 - 2(2)$ $= -\frac{7}{4}.$	1	
	$(\alpha+1)(\beta+1)(\gamma+1)$ $= \alpha\beta\gamma + \Sigma\alpha\beta + \Sigma\alpha + 1$	1	
	$= 3 + 2 + \frac{3}{2} + 1$	1	
	$= \frac{15}{2}$		
1b	$P(x) = x^3-7x-6$ $P(-1) = -1+7-6$ $= 0$		
	$(x+1)$ is a factor.	1M	
	$\begin{array}{r} x^2-x-6 \\ x+1\,\overline{)\,x^3-7x-6} \end{array}$	1M	
	$\therefore P(x) = (x+1)(x^2-x-6)$ $= (x+1)(x-3)(x+2)$	1M.	

Qn	Solutions	Marks	Comments+Criteria
c)	$x^3+6x^2+3x-10=0$		
	Let the roots be $a-d,\quad a,\quad a+d.$		If 1st information is not used to find the roots; $\frac{2}{3}$
	$a-d+a+a+d=-6$ $3a=-6$ $a=-2$	1M	
	$(a-d)a(a+d)=10$ $a(a^2-d^2)=10$ $-2(4-d^2)=10$ $4-d^2=-5$ $d^2=9$ $d=\pm3$	1M	
	The roots are $-5, -2, 1$	1M.	
d)	Let $P(x):\ 2x^3-x+3=0$ $P(-2) = -16+4+3$ < 0	$\frac{1}{2}$M	
	$P(-1) = -2+1+3$ > 0	$\frac{1}{2}$M	
	$P(x)$ is continuous ... $\therefore$ There is a root between -2 and -1	$\frac{1}{2}$M.	
			
	Consider $P(-1.5) = -6.75+1.5+3$ $< 0.$		

Qn	Solutions	Marks	Comments+Criteria
	∴ There is a root between −1 and −1.5	$\frac{1M}{2}$	
	Consider $p(-1.25)$ $= 0.3475$ > 0	$\frac{1}{2}$	
	∴ There is a root between −1.25 and −1.5	$\frac{1}{2}$	✓
	Approximation after 2 iteration is −1.375		
e)	$f(x) = x^2 - 17$ $f'(x) = 2x$ $f(4) < 0$; $f(5) > 0$; $f(x)$ is continuous. (1M)		Accept. Any value of x_1 between 4 and 5 no need to penalise for not writing $f(x)$ is continuous.
	Take $x_1 = 4.5$ $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ (1M) $= 4.5 - \frac{4.5^2-17}{2(4.5)}$ $= 4.14$ 2.d.p	(1M)	Some have used calc. to conclude that there is a root between 4 & 5
			Students who have worked with $\sqrt{17}$, max. marks is 2.

Qn	Solutions	Marks	Comments+Criteria
	Let $P(n)$: $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(r^{n}-1)}{r-1}$		
	Consider $P(1)$ LHS: a ; RHS: $\frac{a(r^1-1)}{r-1}$ $= a$ ∴ $P(1)$ is true	(1M)	
	Let $P(k)$ be true. i.e. $a + ar + \dots + ar^{k-1} = \frac{a(r^k-1)}{r-1}$ — (A)		
	Consider $P(k+1)$: $\boxed{a + ar + \dots + ar^k = \frac{a(r^{k+1}-1)}{r-1}}$	($\frac{1}{2}$M)	
	LHS $= a + ar + \dots + ar^k$ $= \underbrace{a + ar + \dots + ar^{k-1}} + ar^k$ $= \frac{a(r^k-1)}{r-1} + ar^k$ by (A) $= \frac{a}{r-1}(r^k - 1 + r^{k+1} - r^k)$ $= \frac{a}{r-1}(r^{k+1}-1)$	(2M)	
	∴ $P(k+1)$ is true if $P(k)$ is true $P(1)$ is true; $P(k+1)$ is true if $P(k)$ is true ∴ By the principle of mathematical induction $P(n)$ is true for all $n \geq 1$.	($\frac{1}{2}$M)	

Qn	Solutions	Marks	Comments+Criteria
2 a	$105^0 = 60+45$		
	$\tan 105^\circ = \tan(60+45)$		
	$= \dfrac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45}$	1M	
	$= \dfrac{\sqrt{3}+1}{1-\sqrt{3}}$	1M.	
b)	$\cos A = \frac{12}{13}$ $\quad \cos B = \frac{3}{5}$		
	 		
	$\therefore \sin A = \frac{5}{13}$ $\quad \sin B = \frac{4}{5}$	1M	
	$\sin(A+B) = \sin A \cos B + \cos A \sin B$	1M	
	$= \frac{5}{13} \times \frac{3}{5} + \frac{12}{13} \times \frac{4}{5}$		
	$= \frac{15+48}{65} = \frac{63}{65}$	1M	
c)	$\dfrac{1+\cos\theta}{\sin\theta} = \dfrac{1+2\cos^2\theta/2 - 1}{2\sin\theta/2 \cos\theta/2}$		$\cos\theta : 2\cos^2\theta - 1$ 1M $\sin\theta = 2\sin\theta/2 \cos\theta/2$ M Ans: 1M
	$= \operatorname{cor} \theta/2$		

Qn	Solutions	Marks	Comments+Criteria
d)	$\dfrac{\sin 5\theta}{\sin\theta} - \dfrac{\cos 5\theta}{\cos\theta}$		
	$= \dfrac{\sin 5\theta \cos\theta - \cos 5\theta \sin\theta}{\sin\theta \cos\theta}$	1M	
	$= \dfrac{\sin(5\theta - \theta)}{\sin\theta\cos\theta}$	1M.	
	$= \dfrac{2 \cdot 2\sin 2\theta \cos 2\theta}{2 \cdot \sin\theta\cos\theta}$	1M	
	$= \dfrac{4 \sin 2\theta \cos 2\theta}{\sin 2\theta}$	$\frac{1}{2}$M.	
	$= 4\cos 2\theta$.		
e)	$\cos x - \sqrt{3}\sin x = A\cos(x+\alpha)$		
	$= A(\cos x \cos\alpha - \sin x \sin\alpha)$		
	$\therefore 1 = A\cos\alpha$ $\quad \therefore A = \sqrt{4}$		
	$\sqrt{3} = A\sin\alpha$ $\quad = 2$	1M	
	$\tan\alpha = \sqrt{3}$		
	$\alpha = 60^0$	1M	
	$\therefore A\cos(x+\alpha) = 2\cos(x+60^\circ)$		
(ii)	$\cos x - \sqrt{3}\sin x = 1$		
	$2\cos(x+60) = 1$		
	$\cos(x+60) = \frac{1}{2}$	$\frac{1M}{2}$	

Qn	Solutions	Marks	Comments+Criteria
	$x + 60^\circ = 360n \pm 60$	1M.	Gave 1 out of 2 if not general.
	$x = 360n \, ; \, 360n - 120^\circ$	½M.	
f)	$\sin\theta = \frac{2\tan\theta/2}{1+\tan^2\theta/2}$	1M.	
	$\sin 30 = \frac{2\tan 15}{1+\tan^2 15}$	½M.	
	$\frac{1}{2} = \frac{2t}{1+t^2}$, where $t = \tan 15^\circ$		(-1) for wrong approach but if still show $\sin\theta = \frac{2t}{1+t}$
	$1+t^2 = 4t$ $t^2 - 4t + 1 = 0$ $t = \frac{4 \pm \sqrt{16-4}}{2}$ $= \frac{4 \pm 2\sqrt{3}}{2}$ $= 2 \pm \sqrt{3}$	1½	
	$\tan 15^\circ < \tan 45^\circ$ ($y = \tan x$ is an increasing curve) $\therefore \tan 15 = 2 - \sqrt{3}$ or Concluding using Calculator	1M.	