St Catherine's School

Year: 12
Subject: Mathematics
Time allowed: 55 minutes

Assessment Task No: 1
Date: February 2005

Student Number: ____________________

Directions to candidates:

- All questions are to be attempted.
- Marks may be deducted for careless or badly arranged work
- All necessary working must be shown in every question.
- Approved calculators are required.
- Staple question paper to the back of Section 1.
- Put your identification on ALL three bundles.

Section 1	Total Q1+Q2+Q3+Q4
Section 2	Total Q5, Q6, Q7
Section 3	Total Q8, Q9
	TOTAL

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1. Consider the arithmetic sequence 98, 95, 92, ...
   (a) Find the value of the common difference *d*. (1m)
   (b) Find the value of the seventeenth term. (2m)
   (c) Find the value of the first negative term, giving reasons. (3m)

6 marks

2. Insert four geometric terms between 3 and 96.

3 marks

3. For the curve $y = x^2 + 3x - 2$,
(a) find the gradient of the tangent to the curve at the point where $x = -1$, (2m)
(b) find the point on the curve where the tangent is parallel to the line $y=5x-2$. (2m)
4 marks

4. Consider the curve with equation

$$y = x^3 - 6x^2 + 9x - 4$$

(a) Find any stationary points and their nature. (5m)
(b) Find any points of inflexion. (2m)
(c) Find the y– intercept. (1m)
(d) Sketch the curve for $-1 \le x \le 5$, showing all features. (2m)
(e) State the absolute maximum and minimum points in this interval. (2m)
12 marks

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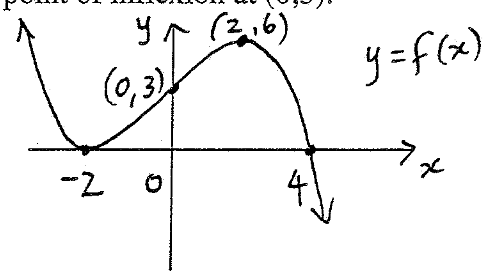
5. Consider the sketch of the curve $y = f(x)$, with point of inflexion at (0,3).
State the values of x for which
(a) $f(x) > 0$ (1m)
(b) $f'(x) = 0$ (1m)
(c) $f''(x) < 0$. (1m)



3 marks

6. (a) Find the first derivative of $f(x) = \dfrac{1}{2x+1}$. (2m)

(b) Hence explain why the curve $f(x) = \dfrac{1}{2x+1}$ is decreasing for all x except $x = -0.5$.
(2m)

4 marks

7. Determine whether or not the curve $f(x)=(x+2)^4$ has a point of inflexion. Explain your reasoning.

3 marks

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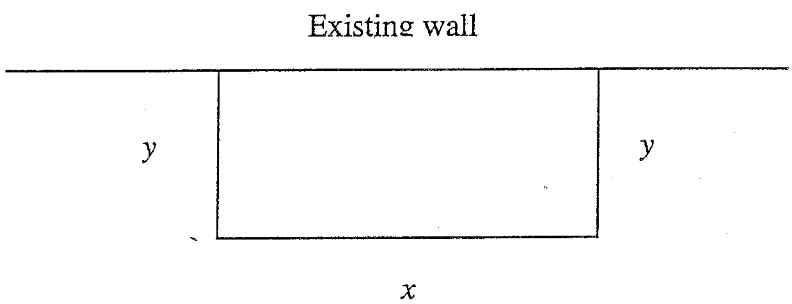
8. A local council allows a rectangular floor plan area of 200 square metres for a new house. One wall length is already existing as it is on the boundary of the neighbouring property. A builder must build the other three walls and wants to use dimensions such that the sum, S metres, of the lengths of the three walls is a minimum. Let the length of the house be x metres and the width be y metres, as shown in the diagram.



(a) Show that an equation for S is given by

$$S = x + \frac{400}{x}.$$

(2m)

(b) Hence find the dimensions of the rectangular floor plan such that the sum of the three walls is a minimum. (4m)

6 marks

9. (a) State the condition on r for a geometric series $a + ar + ar^2 + ar^3 + \ldots$ to have a limiting sum. (1m)

(b) Find the value of x if the series

$$(x+2)+(x+2)^2+(x+2)^3+\ldots$$

has a limiting sum. (3m)

4 marks

END

Yr 12-MATHS ASSESS TEST 1, FEB 2005 SOL : ST. CATHERINE'S

1. a) $d = 95 - 98$
$= -3$ (1) $T_n = a + (n-1)d$

b) $T_{17} = 98 + (17-1)(-3)$
$= 50$ (2)

c) $T_n < 0$
$98 + (n-1)(-3) < 0$
$98 - 3n + 3 < 0$
$101 - 3n < 0$
$101 < 3n$ (3)
$n > \frac{101}{3}$
$\therefore n = 34$
$\therefore T_{34} = 98 + (34-1)(-3)$
$= -1$

2. Let $T_1 = 3 = a$ $T_6 = 96 = ar^5$
3 _ _ _ _ 96
$\therefore 3r^5 = 96$
$r^5 = 32$
$r = 2$ (3)
$\therefore 3, 6, 12, 24, 48, 96$

3. $y = x^2 + 3x - 2$
$y' = 2x + 3$
a) $x = -1$ $y' = 2(-1) + 3$
$\therefore y' = 1$ (2)

b) parallel to $y = 5x - 2 \therefore y' = 5$
$5 = 2x + 3$ (2)
$2 = 2x$
$x = 1$, $y = 1^2 + 3(1) - 2$
$\therefore$ Point is $(1, 2)$

4. $y = x^3 - 6x^2 + 9x - 4$
a) $y' = 3x^2 - 12x + 9$
$= 3(x^2 - 4x + 3)$

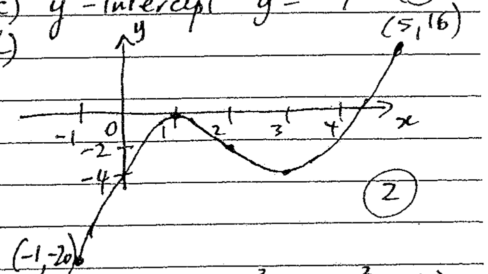
Stat pts $x = 3, 1$ $y'' = 6x - 12$
$x = 3$ $y'' = 6(3) - 12$
$> 0 \therefore$ Min $x = 3$
$x = 1$ $y'' = 6(1) - 12$ (5)
$= -6 < 0 \therefore$ Max $x = 1$
$\therefore x = 3$ $y = 3^3 - 6(3)^2 + 9(3) - 4$
$= -4 \therefore$ Min $(3, -4)$
$x = 1$ $y = 1^3 - 6(1)^2 + 9(1) - 4$
$= 0 \therefore$ Max $(1, 0)$.

b) Pts of inflexion occur when $y'' = 0$ AND change in concavity.
$\therefore y'' = 0 = 6x - 12$ (2)
$12 = 6x$
$x = 2$ Test

x	1	2	3
y''	$-$	0	$+$

$\therefore x = 2$ $y = 2^3 - 6(2)^2 + 9(2) - 4$
$= -2$
$\therefore$ Pt. of inflexion $(2, -2)$.

c) y-intercept $y = -4$ (1)

d)


(2)

e) $x = -1$ $y = (-1)^3 - 6(-1)^2 + 9(-1) - 4$
$= -20$ Abs. min $(-1, -20)$
$x = 5$ $y = 5^3 - 6(5)^2 + 9(5) - 4$
$= 16$ Abs. Max $(5, 16)$
(2)

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5. a) $f(x) > 0$ for $x < -2$, $-2 < x < 4$ (1)
b) $f'(x) = 0$ for $x = -2$, $x = 2$ (1)
c) $f''(x) < 0$ for $x > 0$. (1)

6. a) $f(x) = (2x+1)^{-1}$
$f'(x) = -(2x+1)^{-2} \cdot (+2)$
$= \frac{-2}{(2x+1)^2}$ (2)

b) $f'(x) < 0$ for all $x \neq -0.5$
since $(2x+1)^2 \geq 0$.
$\therefore f(x)$ is decreasing for all $x \neq -0.5$. (2)

7. $f(x) = (x+2)^4$
$f'(x) = 4(x+2)^3$
$f''(x) = 12(x+2)^2$
Point of inflexion will occur when $f'' = 0$ and there is a change in concavity. (3)
$\therefore 0 = 12(x+2)^2$
$\therefore x = -2$ Test

x	-3	-2	0
f''	$+$	0	$+$

No change in Concavity
$\therefore f(x)$ has no pt. of inflexion.

8. a) $S = 2y + x$ $A = xy$
$\therefore 200 = xy$
$\therefore S = 2\left(\frac{200}{x}\right) + x$ $\therefore y = \frac{200}{x}$
$\therefore S = x + \frac{400}{x}$ (2)
$\therefore S = x + 400x^{-1}$

b) $S' = 1 - 400x^{-2}$
$= 1 - \frac{400}{x^2}$
Min occurs when $S' = 0$ and $S'' > 0$.
$0 = 1 - \frac{400}{x^2}$
$\frac{400}{x^2} = 1$ $S'' = 800x^{-3}$
$x^2 = 400$ $= \frac{800}{x^3}$
$x = \pm 20$
but $x > 0$ since length $\therefore x = 20$
Test $x = 20$ $S'' = \frac{800}{20^3} > 0$
$\therefore$ Min S when $x = 20$
$\therefore y = \frac{200}{20}$ (4)
$= 10$
$\therefore$ length is 20 m, width 10 m

9a) limit exists if $|r| < 1$ (1)
b) $(x+2) + (x+2)^2 + \ldots$
$r = \frac{(x+2)^2}{x+2}$ $(x \neq -2)$
$= x + 2$
$\therefore |r| < 1$ (3)
$-1 < x + 2 < 1$
$-3 < x < -1$.