Student Number: ____________

St. Catherine's School Waverley

April 2008

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 2– 30%
Half-Yearly Examination

Mathematics

General Instructions

- **Reading Time - 5 minutes**
- **Working time – 180 minutes**
- Start each question on a new page in your answer booklet.
  Section A – Questions 1 – 3
  Section B – Questions 4 – 6
  Section C – Questions 7 -10
  Start a new booklet for each section.
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Write using black or blue pen only.
- Standard integrals are printed at the back the paper. Board-approved calculators may be used.
- All necessary **working** must be shown.
- Marks may be deducted for careless or

Total marks – 120

- Attempt all questions
- All questions are of equal value

SECTION A – (Questions 1 – 3)

Question 1 (12 Marks)

a) Write the value of $\sqrt[3]{45.6+6.05}$ to 2 decimal places. 1

b) Write in simplest form: $\frac{x+5}{3}-\frac{2x-4}{5}$ 2

c) Simplify $2\sqrt{8}+\sqrt{50}-\sqrt{2}$ 2

d) Solve for x: $|x-5|=2$ 2

e) Factorise completely $3x^3-5x^2+2x$ 2

f) Show all working to express $1\cdot0\dot{7}\dot{4}$ as a fraction in its lowest terms. 3

Question 2 START A NEW PAGE **(12 Marks)**

a) The quadratic equation $2x^2 + 9x - 3 = 0$ has roots α and β

Find the value of

i) $\alpha+\beta$ **1**

ii) $\alpha\beta$ **1**

iii) $\frac{3}{\alpha}+\frac{3}{\beta}$ **2**

iv) $\alpha^2+\beta^2$ **2**

b) Prove that, for all θ

$\tan\theta + \cot\theta = \sec\theta\operatorname{cosec}\theta$ **3**

c) For the arithmetic sequence

$8+3\sqrt{2},\ 10+4\sqrt{2},\ 12+5\sqrt{2}.......$ find

i) The common difference d **1**

ii) The sum of the first ten terms **2**

Question 3 START A NEW PAGE **(12 Marks)**

a) Find $\frac{dy}{dx}$ if

i) $y=4x^3-2$ **1**

ii) $y=(3x-5)^4$ **2**

iii) $y=\frac{1}{\sqrt[4]{x}}$ **2**

iv) $y=\frac{3x^2-5}{x+1}$ **2**

b) Show that the tangent to the curve $y=x^2+\frac{1}{x}$ at the point where $x=-1$, has equation $3x+y+3=0$ **2**

c) Find the point(s) on the curve $y=12+5x-x^3$ where the gradient of the tangent equals 2 **3**

SECTION B – (Questions 4 – 6)

Question 4 START A NEW BOOKLET (12 Marks)

a) The points A (2,5), B (-1, 5) and C(8,2) lie on the number plane.

i) Draw a neat sketch of the number plane, showing ΔABC — 1

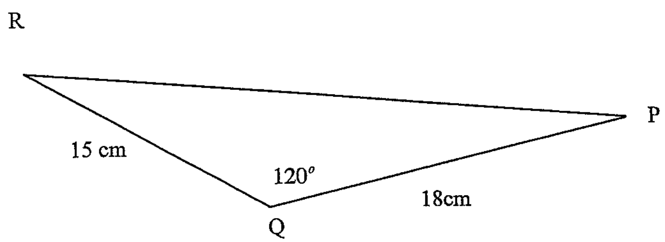
ii) Find the length of side AC in simplest exact form. — 1

iii) Show that the equation of BC is

$$x + 3y - 14 = 0$$ — 2

iv) Find the area of ΔABC — 2

b) The triangle PQR has PQ = 18 cm, QR = 15 cm, and $\angle PQR = 120^0$



i) Find the length of RP in exact form — 2

ii) Find the area of ΔPQR in exact form — 2

c) Given that $\cos\alpha = \dfrac{2}{13}$ and $\tan\alpha < 0$,

find the exact value of $\sin\alpha$ — 2

Question 5 START A NEW PAGE (12 Marks)

a) Show that $f(x) = \dfrac{x}{x^2+1}$ is an odd function — 2

b) For the parabola $(x-5)^2 = -8(y+2)$, write — 4

i) the co-ordinates of the vertex
ii) The focal length
iii) The equation of the directrix
iv) The co-ordinates of the focus

c) The point $P(x,y)$ is equidistant from points A(2,5) and B(-1,-4) — 4

Find the locus of P as an equation and describe the locus geometrically

d) Find all solutions, $0^o \le \theta \le 360^o$, of — 2

$$\sin^2\theta = \frac{1}{4}$$

Question 6 START A NEW PAGE (12 Marks)

a) The sum to infinity of the series **3**

$(x+1)+(x+1)^2+(x+1)^3+\ldots\ldots$ equals 5

Find the value of x

b) Shade the region on the number plane for which **3**

$x^2+y^2 \le 16$ and $y \le 2$

c) For what values of k does $x^2-(k-2)x+1=0$ have real roots? **3**

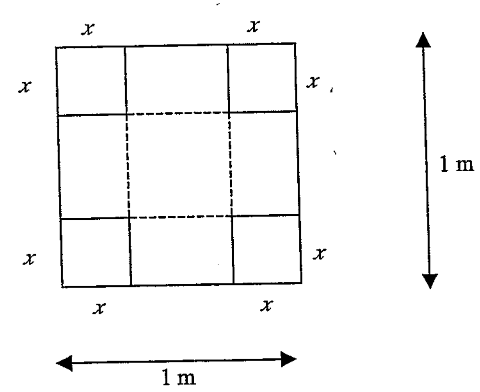
d) The first term of a geometric series is 16 and the fourth term is $\frac{1}{4}$. **3**

Find the sum of the first 8 terms.

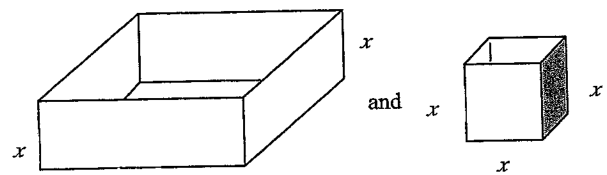
SECTION C – (Questions 7 -10)

Question 7 START A NEW BOOKLET (12 Marks)

a) A piece of card 1m by 1m has 4 squares cut from the corners, as shown.



The card is folded along the dotted lines to form an open box, and the four cut-out squares are joined to form a cube with no top or base.



box 1 box 2

i) What is the length of the base of box 1? **1**

ii) Show that the combined volume of the two boxes is

$$V = 5x^3 - 4x^2 + x$$ **2**

(iii) Find the two values of x that will give $V' = 0$ **2**

(iv) By examining the second derivative of V, or otherwise, decide which of these two values of x gives a local maximum value for V. **2**

Question 7 - CONTINUED

b) Two towns, Walterton and Pinkerton, each had a population of 10 000 in 1960. Since then, the population of Walterton has declined steadily to 1000, while the population of Pinkerton has increased at an increasing rate, to 20 000.

Sketch a graph with Time on the horizontal axis, and Population on the vertical axis, to illustrate this information. **2**

c) Find any stationary points on the curve **3**

$$y = 2x^3 - 6x + 4$$

and determine their nature.

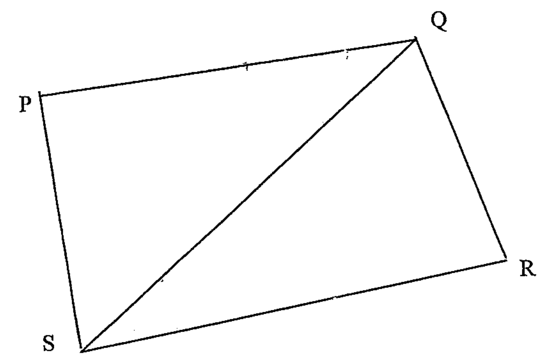
Question 8 START A NEW PAGE **(12 Marks)**

a) Solve $3^{2x} + 2.3^x - 15 = 0$ **3**

b) In the diagram below, PQRS is a quadrilateral with PQ=SR and $\angle PQS = \angle RSQ$.
Copy your diagram onto your booklet showing the given information.

i) Using congruent triangles, prove that PS=QR. **3**

ii) What kind of quadrilateral is PQSR?
(Give reasons for your answer) **1**



Not to scale

c) Find **i)** $\int \sqrt{x}\, dx$ **2**

ii) $\int (3x-5)^4 dx$ **3**

Question 9 START A NEW PAGE **(12 Marks)**

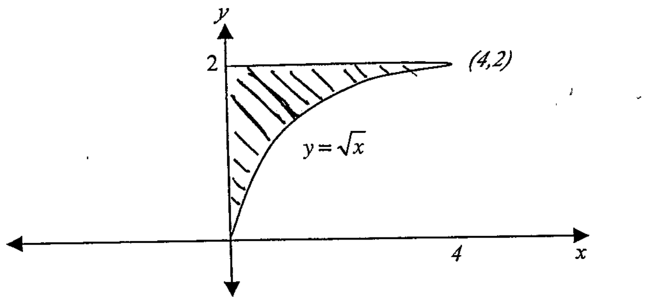
a) i) Copy and complete the table for $F(x)=\dfrac{1}{x^2+1}$ **1**

Give values to 2 decimal places.

x	1.0	1.5	2.0	2.5	3.0
F(x)	0.50				

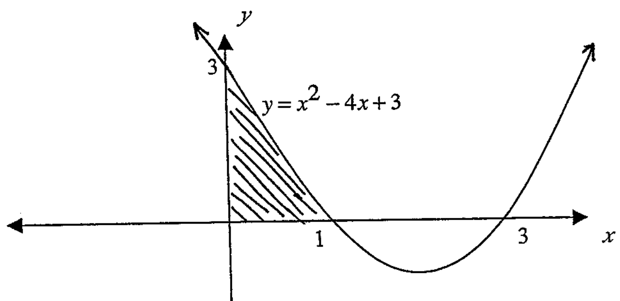
ii) Use these values to estimate $\int_1^3 \frac{1}{x^2+1}\,dx$ using Simpson's Rule **4**

iii) Find the shaded area. **3**



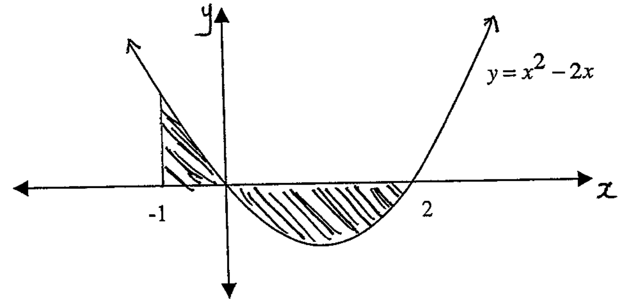
b) i) Show that $(x^2-4x+3)^2 = x^4-8x^3+22x^2-24x+9$ **1**

ii) Find the volume enclosed when the shaded region is rotated about the x-axis. **3**



Question 10 START A NEW PAGE **(12 Marks)**

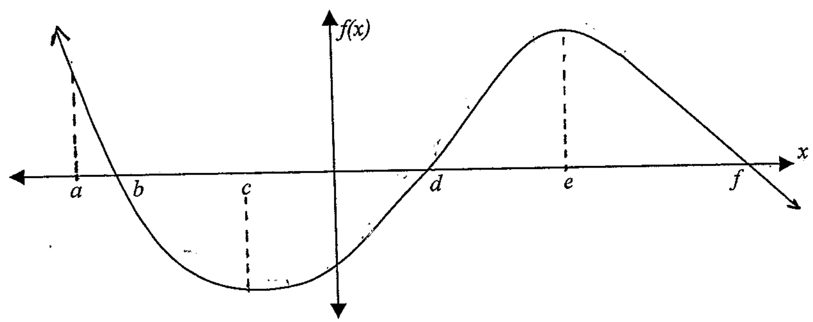
a) i) Find the shaded area: **4**



ii) Show that $\int_{-1}^{2} (x^2-2x)dx = 0$ **2**

iii) Explain why the answers to parts i) and ii) above are not equal. **1**

b) Given this graph of $y=f(x)$, graph $y=f'(x)$ for $a \le x \le f$ **2**



Question 10 - CONTINUED

c) i) A continuous curve $y = f(x)$ has exactly two stationary points. Using the data below, identify the x-values of both stationary points and determine their nature. **2**

x	-6	-5	-4	0	1	2
$f'(x)$	-5	0	-6	-2	0	5

ii) What is the value of the second derivative at $x = -5$
Explain your answer. **1**

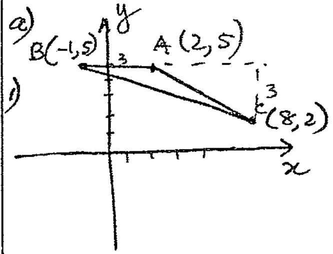
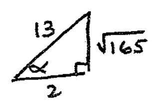
End of paper

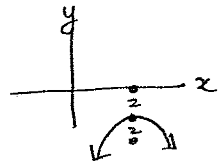
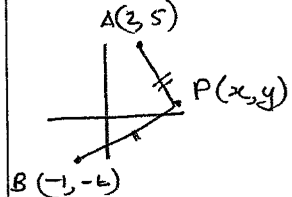
Year 12 Assessment Task 2 [illegible]

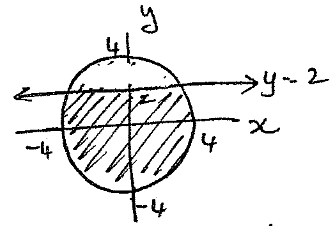
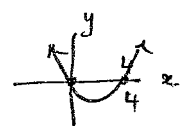
Qn	Solutions	Marks	Comments+Criteria
Q1	a) 3.72	1	
	b) $\frac{x+5}{3} - \frac{2x-4}{5} = \frac{5x+25-6x+12}{15}$	1	
	$= \frac{-x+37}{15}$	1	−1 mark for −12
	c) $2\times 2\sqrt{2} + 5\sqrt{2} - \sqrt{2} = 8\sqrt{2}$	2	
	d) $\lvert x-5 \rvert = 2$ $x-5=2$ or $x-5=-2$ $x=7$ $\quad x=3$	2	
	e) $3x^3 - 5x^2 + 2x$ $= x(3x^2-5x+2)$ $= x(3x-2)(x-1)$	2	
	f) $x = 1.0747474\ldots$ $100x = 107.4747474\ldots$ $99x = 106.4$	(3)	
	$x = \frac{106.4}{99} = \frac{1064}{990}$	$(2\frac{1}{2})$	
	$= \frac{532}{495}$	$(\frac{1}{2})$	

Qn	Solutions	Marks	Comments+Criteria
2	a) $2x^2+9x-3=0$		
	i) $\alpha+\beta = \frac{-9}{2}$	1	
	ii) $\alpha\beta = \frac{-3}{2}$	1	
	iii) $\frac{3}{\alpha} + \frac{3}{\beta} = \frac{3\alpha+3\beta}{\alpha\beta} = \frac{3(\alpha+\beta)}{\alpha\beta}$	1	
	$= 3\times\frac{-9}{2} \div \frac{-3}{2}$ $= 9$	1	
	iv) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	1	
	$= \left(\frac{-9}{2}\right)^2 - 2\times\frac{-3}{2}$ $= \frac{81}{4} + 3 = \frac{93}{4}$ or $23\frac{1}{4}$	1	
	b) $\tan\theta + \cot\theta = \sec\theta\ \text{cosec}\,\theta$ LHS $= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ $= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$ $= \frac{1}{\cos\theta\sin\theta}$ RHS $= \frac{1}{\sin\theta}\times\frac{1}{\cos\theta}$ $= \frac{1}{\sin\theta\cos\theta}$ $=$ LHS ☺	3	
	c) i) $d = 2+\sqrt{2}$	1	
	ii) $S_{10} = \frac{10}{2}(2a+9d)$	1	
	$= 5(16+6\sqrt{2}+18+9\sqrt{2})$ $= 170 + 75\sqrt{2}$	1	

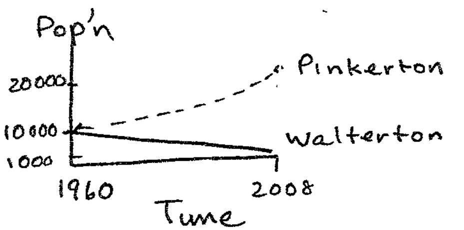
Qn	Solutions	Marks	Comments+Criteria
3	a) i) $\frac{dy}{dx} = 12x^2$	1	
	ii) $\frac{dy}{dx} = 4(3x-5)^3 \times 3$	1	
	$= 12(3x-5)^3$	1	
	iii) $\frac{dy}{dx} = -\frac{1}{4}x^{-1\frac{1}{4}}$ as $\left(y = x^{-\frac{1}{4}}\right)$		
	$= \frac{-1}{4x\sqrt[4]{x}}$	2	
	iv) $\frac{dy}{dx} = \frac{(x+1)(6x) - (3x^2-5)\times 1}{(x+1)^2}$		
	$= \frac{6x^2+6x-3x^2+5}{(x+1)^2}$		
	$= \frac{3x^2+6x+5}{(x+1)^2}$	2	
	b) $y = x^2 + x^{-1}$ $(-1, 0)$		
	$y' = 2x - x^{-2}$		
	at $x=-1$, $y' = -2-1 = -3$ $\therefore m = -3$	1	
	$y - 0 = -3(x - -1)$		
	$y = -3x - 3$	1	
	$\therefore 3x + y + 3 = 0$ (2)		
	c) $y = 12 + 5x - x^3$		
	$y' = 5 - 3x^2$	1	
	$y' = 2$ when $5 - 3x^2 = 2$		
	$-3x^2 = -3$		
	$x^2 = 1$		
	$x = \pm 1$	1	
	$\therefore$ at $(1, 16)$ and $(-1, 8)$	1	

Qn	Solutions	Marks	Comments+Criteria
4	a) i) 	1	
	ii) $AC = \sqrt{(8-2)^2+(2-5)^2}$		
	$= \sqrt{6^2+(-3)^2}$		
	$= \sqrt{45} = 3\sqrt{5}$	1	
	iii) $grad_{BC}$: $m = \frac{2-5}{8--1} = \frac{-3}{9} = \frac{-1}{3}$		
	eq_{BC}: $y - 5 = \frac{-1}{3}(x - -1)$		
	$3y - 15 = -x - 1$		
	$\therefore x + 3y - 14 = 0$	2	
	iv) $A = \frac{1}{2}b \times h$	2	Many other ways.
	$= \frac{1}{2} \times 3 \times 3$		(NB – AB is base of ΔABC with height 3 units. see diagram.)
	$= 4.5 \ u^2$		
	b) cos rule:		
	i) $q^2 = 15^2 + 18^2 - 2 \times 15 \times 18 \times \cos 120°$		
	$= 819$		
	$q = \sqrt{819} = 3\sqrt{91}$	2	accept $\sqrt{819}$
	ii) $A = \frac{1}{2}ab\sin C$		
	$= \frac{1}{2} \times 18 \times 15 \times \sin 120°$		
	$= \frac{1}{2} \times 18 \times 15 \times \frac{\sqrt{3}}{2} = \frac{135\sqrt{3}}{2}$	2	
	c) 		
	$\cos > 0$, $\tan < 0$ $\therefore$ Q4	2	$\frac{1}{2}$ for $\sqrt{165}$
	$\sin\alpha = -\frac{\sqrt{165}}{13}$		$\frac{1}{2}$ for Q4
			1 for answer

Qn	Solutions	Marks	Comments+Criteria
5	a) For an odd fn, $f(-a) = -f(a)$ $f(-a) = \frac{-a}{(-a)^2+1}$ $\quad -f(a) = -\left(\frac{a}{a^2+1}\right)$ $= \frac{-a}{a^2+1}$ $\quad = \frac{-a}{a^2+1}$	1	
	$f(-a) = f(a)$ ∴ odd fn.	1	$-\frac{1}{2}$ for focal length -2
	b) i) V $(5,-2)$	1	
	ii) focal length = 2	1	
	iii) directrix : $y=0$	1	
	iv) focus : $(5,-4)$	1	
	c) AP = BP i) $\sqrt{(x-2)^2+(y-5)^2} = \sqrt{(x+1)^2+(y+4)^2}$	1	
	$x^2-4x+4+y^2-10y+25$ $= x^2+2x+1+y^2+8y+16$	1	
	$0 = 6x+18y-12$ ∴ $0 = x+3y-2$	1	
	ii) which is a straight line (perp bisector of AB)	1	
	d) $\sin^2\theta = \frac{1}{4}$		
	$\sin\theta = \pm\frac{1}{2}$	1	-1 if $-$ve not considered.
	∴ $\theta = 30°, 150°, 210°, 330°$	1	

Qn	Solutions	Marks	Comments+Criteria
6.	a $\quad S_\infty = \frac{a}{1-r}$	1	
	∴ $5 = \frac{x+1}{1-(x+1)}$		
	$5 = \frac{x+1}{-x}$	1	
	$-5x = x+1$ $-6x = 1$ ∴ $x = \frac{-1}{6}$	1	
	b) 	3	1 mark for circle 1 mark for line 1 mark for region
	c) real roots if $\Delta \geq 0$	3	1 for $\Delta \geq 0$
	$b^2-4ac \geq 0$ $(k-2)^2-4(1\times1) \geq 0$ $k^2-4k+4-4 \geq 0$		1 for $k^2-4k \geq 0$
	$k^2-4k \geq 0$ $k(k-4) \geq 0$ ∴ $k \leq 0$ or $k \geq 4$ 		1 for answer
	d) $a = 16 \quad T_4 = ar^3 = \frac{1}{4}$ ∴ $16r^3 = \frac{1}{4}$ $r^3 = \frac{1}{64}$ $r = \frac{1}{4}$		1 for $r = \frac{1}{4}$
	$S_8 = \frac{a(1-r^n)}{1-r} = \frac{16\times(1-(\frac{1}{4})^8)}{1-\frac{1}{4}}$	1	
	$\doteqdot 21\frac{341}{1024}$	1	accept 21.33

Qn	Solutions	Marks	Comments+Criteria
7.	i) $1-2x$	1	
	ii) $V=(1-2x)^2\times x \quad + x^3$	1	
	$=(1-4x+4x^2)x \quad + x^3$		
	$= x-4x^2+4x^3+x^3$		
	$V = 5x^3-4x^2+x$ ☺	1	
	iii) $\dot{V} = 15x^2-8x+1$	$\frac{1}{2}$	
	$=(5x-1)(3x-1)$	$\frac{1}{2}$	
	$\dot{V}=0$ when $x=\frac{1}{5}$ or $x=\frac{1}{3}$	1	
	iv) $\ddot{V}=30x-8$	$\frac{1}{2}$	
	at $x=\frac{1}{5}$, $\ddot{V}=6-8=-2<0$ ∴ max	$\frac{1}{2}$	
	at $x=\frac{1}{3}$ $\ddot{V}=10-8=2>0$ ∴ min	$\frac{1}{2}$	
	$x=\frac{1}{5}$ gives a local max.	$\frac{1}{2}$	
b)	(graph below)	1	Pinkerton concave up
		1	Walterton straight
			Scale not indicated $-\frac{1}{2}$



Qn	Solutions	Marks	Comments+Criteria
c)	$y=2x^3-6x+4$		
	$y'=6x^2-6$		
	$y'=0$ when $6x^2-6=0$ ∴ $x=\pm 1$	1	
	∴ Pts are $(1,0)$ and $(-1,8)$	1	
	$y''=12x$ ∴ $(1,0)$ is min tp, $(-1,8)$ is max tp	1	

Qn	Solutions	Marks	Comments+Criteria
8a)	i) let $m=3^x$		
	then $m^2+2m-15=0$		
	$(m-3)(m+5)=0$		
	∴ $m=3$ or $m=-5$		
	now $3^x=-5$ has no soln		
	$3^x=3 \Rightarrow x=1$	3	
	∴ $x=1$ is only soln		
b)	In ΔPQS, RSQ		
	SQ is common		
	$PQ=RS$ (given)		
	$\angle PQS=\angle RSQ$ (given)		
	∴ $\Delta PQS \equiv \Delta RSQ$ (SAS)		
	∴ $PS=QR$ (corresp sides of cong Δs)	3	
	ii) parallelogram (eq. opp sides)	1	
c)	i) $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx$		
	$=\frac{x^{3/2}}{3/2}+c$	2	
	$=\frac{2x^{3/2}}{3}$ or $\frac{2x\sqrt{x}}{3}+c$		
	ii) $\int (3x-5)^4\,dx$		
	$=\frac{(3x-5)^5}{5\times 3}+c$	3	
	$=\frac{(3x-5)^5}{15}+c$		

Qn	Solutions	Marks	Comments+Criteria
9	a) i)		

x	1	1·5	2	2·5	3
$F(x)$	·50	·31	·20	0·14	0·10

Marks: 1

ii)

fx	w	$w \times fx$
·50	1	·50
·31	4	1·24
·20	2	·40
·14	4	·56
·1	1	·10
		$\Sigma =$ 2·80

$A = \frac{h}{3} \times \Sigma$

$\frac{\cdot 5}{3} \times 2\cdot 80$

$= \frac{7}{15}$

Marks: 4

b) $A = \int_0^2 x\,dy = \int_0^2 y^2\,dy$

$= \left[\frac{1}{3}y^3\right]_0^2 = \frac{8}{3}$

Marks: 3

Comments+Criteria: or $2 \square 4 - \int_0^4 \sqrt{x}\,dx$

c) i) $(x^2 - 4x + 3)(x^2 - 4x + 3) = x^4 - 4x^3 + 3x^2 - 4x^3 + 16x^2 -$
$+ 3x^2 - 12x + 9$

$= x^4 - 8x^3 + 22x^2 - 24x + 9.$

Marks: 1

c) ii) $V = \pi \int y^2\,dx$

$= \pi \int_0^1 (x^2 - 4x + 3)^2\,dx$

$= \pi \int_0^1 x^4 - 8x^3 + 22x^2 - 24x + 9\,dx$

Marks: 3

$= \pi \left[\frac{x^5}{5} - \frac{8x^4}{4} + \frac{22x^3}{3} - \frac{24x^2}{2} + 9x\right]_0^1$

$= \pi\left(\frac{1}{5} - 2 + \frac{22}{3} - 12 + 9\right) - 0$

$= \frac{38\pi}{15}\ u^3$

Qn	Solutions	Marks	Comments+Criteria

10. a) i) $A_1 = \int_{-1}^{0} x^2 - 2x = \left[\frac{1}{3}x^3 - x^2\right]_{-1}^{0}$

$= 0 - \left(-\frac{1}{3} - 1\right)$

$= +\frac{4}{3}$

Marks: $1\frac{1}{2}$

$A_2 = \int_0^2 x^2 - 2x$

$= \left[\frac{1}{3}x^3 - x^2\right]_0^2 = \left(\frac{8}{3} - 4\right) - 0$

$= -\frac{4}{3}.$

Marks: $1\frac{1}{2}$

$A_1 + |A_2| = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$

Marks: 1

ii) $\int_{-1}^{2} x^2 - 2x\,dx = \left[\frac{x^3}{3} - x^2\right]_{-1}^{2}$

$= \left(\frac{8}{3} - 4\right) - \left(\frac{-1}{3} - 1\right)$

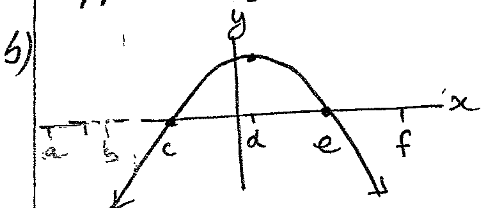
$= \frac{4}{3} - \frac{4}{3} = 0.$

Marks: 2

iii) The shaded portion under the x-axis and the portion above the x-axis have equal areas of opposite sign.

Marks: 1

b)



Marks: 2

c) at $x = -5$, $f'(x) = 0$
$\therefore$ hpi at $x = -5$

Marks: 1

at $x = 2$, $f'(x) = 0$
$\therefore$ min tp at $x = 2$

Marks: 1

at $x = -5$, $f''(x) = 0$ (change in concavity) 1