Student Number:__________________

St. Catherine's School
Waverley

2007

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 3 – 20%
CLASS TEST: 4th June

Mathematics

General Instructions

- Working time – 55 minutes
- Start each question on a new page in your answer booklet.
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary **working** must be shown.
- Marks may be deducted for careless or badly arranged work.

Total marks – 43

- Attempt Questions 1–3
- Questions are not of equal value.

TEACHER'S USE ONLY

Question 1	/13
Question 2	/15
Question 3	/15
TOTAL	/43

Question 1 **Marks**

a) Find $\frac{dy}{dx}$ for

(i) $y = e^{3x+5}$ 1

(ii) $y = xe^{x}$ 2

b) Evaluate $\int_0^2 e^{2x}\,dx$ correct to 1 decimal place 2

c) Find the gradient of the tangent to $f(x) = \frac{e^{-x}}{x}$ at $x=1$ 3

d) Find $\frac{d}{dx}e^{x^2}$ and hence find the exact value of $\int_0^1 xe^{x^2}\,dx$ 4

Question 2 **Marks**

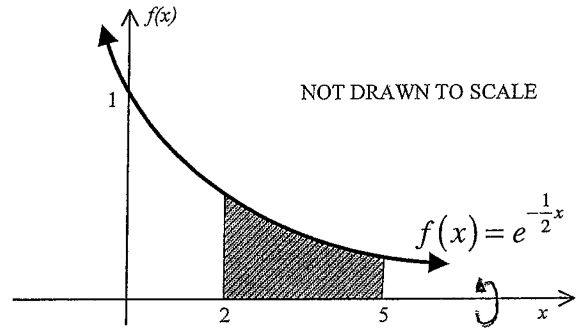
a) Find $\frac{dy}{dx}$ for

(i) $y = x^2 \log_e x$ 2

(ii) $y = \log_e \sqrt{3x-5}$ 3

b) Evaluate $\int_e^{e^2} \frac{6}{x}\,dx$ as an exact value 3

c) 4



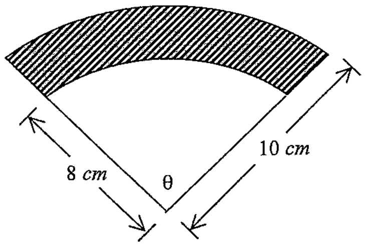
The section of the curve $f(x) = e^{-\frac{1}{2}x}$ between $x = 2$ *and* $x = 5$ shown above is rotated about the x-axis. Find the volume enclosed.

d) Find the equation of the tangent to the curve $y = 4 + 3\log_e x$ at the point where $x = 1$ 3

Question 3 | **Marks**

a) What is the exact value of $\sin\frac{2\pi}{3}$ **1**

b) Find the area shaded in the diagram below if $\theta = 2$ **3**



c) (i) Sketch the curve $y = 2\sin 2x$ for $0 \le x \le 2\pi$ **3**

showing a clear scale on both axes.

(ii) By graphing the line $y = 1$ on your diagram, show that the equation

$$2\sin 2x = 1$$

has four solutions for $0 \le x \le 2\pi$ **1**

d) Differentiate:

(i) $y = e^{2\tan x}$ **2**

(ii) $y = \sin^3 x \cos x$ **3**

(iii) $y = \log_e(\cos x)$ **2**

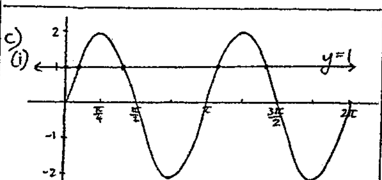
End of Paper

2007 MATHEMATICS ASSESSMENT TASK 3 SOLUTIONS

Qn	Solutions	Marks	Comments+Criteria
1.	A) (i) $y = e^{3x+5}$		
	$y' = f'(x)e^{f(x)}$		
	$= 3e^{3x+5}$	1	for correct answer
	(ii) $y = xe^x$		
	$u = x \quad v = e^x$ $u' = 1 \quad v' = e^x$	1	for correct use of product rule
	$y' = vu' + uv'$ $= e^x + xe^x$		
	$= e^x(1+x)$	1	for correct answer
	B) $\int_0^2 e^{2x}dx = \frac{1}{2}\int_0^2 2e^{2x}dx$	$\frac{1}{2}$	for re-writing integral
	$= \frac{1}{2}\left[e^{2x}\right]_0^2$	$\frac{1}{2}$	for correct integration
	$= \frac{1}{2}\left[e^4 - e^0\right]$	$\frac{1}{2}$	for substitution
	$= \frac{e^4}{2} - \frac{1}{2}$		
	$= 26.8$ (to 1 d.p.)	$\frac{1}{2}$	for answer to 1 d.p.
	c) $f(x) = \frac{e^{-x}}{x} = (xe^x)^{-1}$		
	$f'(x) = -(xe^x)^{-2} \times e^x(1+x)$	1	for correct use of chain or quotient rule
	$= \frac{-e^x(1+x)}{x^2e^{2x}}$		
	$= \frac{-(1+x)}{x^2e^x}$	1	for correct simplification
	$\therefore$ when $x=1$, $m_T = \frac{-2}{e}$	1	
	$= \ldots$		

Qn	Solutions	Marks	Comments+Criteria
1	d) $\frac{d}{dx}e^{x^2} = 2xe^{x^2}$	1	for differentiation
	$\therefore e^{x^2} = \int 2xe^{x^2}dx$		
	$e^{x^2} = 2\int xe^{x^2}dx$	1	for appropriate use of derivative to form integral
	$\Rightarrow \frac{e^{x^2}}{2} = \int xe^{x^2}dx$		
	$\therefore \int_0^1 xe^{x^2}dx = \left[\frac{e^{x^2}}{2}\right]_0^1$	1	for integration
	$= \frac{e^1}{2} - \frac{e^0}{2}$		
	$= \frac{1}{2}(e-1)$	1	for solution after substitution
2	A) (i) $y = x^2\log_e x$		
	$u = x^2 \quad v = \ln x$ $u' = 2x \quad v' = \frac{1}{x}$	1	for correct use of product rule
	$y' = vu' + uv'$ $= 2x\ln x + x^2 \times \frac{1}{x}$		
	$= 2x\ln x + x$	1	for correct answer
	$= x(2\ln x + 1)$		
	(ii) $y = \log\sqrt{3x-5}$ $y = \log(3x-5)^{\frac{1}{2}}$		
	$y = \frac{1}{2}\log(3x-5)$	$1\frac{1}{2}$	for use of log law
	$y' = \frac{1}{2} \times \frac{3}{(3x-5)}$		
	$= \frac{3}{6x-10}$	$1\frac{1}{2}$	for correct answers

Qn	Solutions	Marks	Comments+Criteria
2	b) $\int_e^{e^2} \frac{6}{x}dx = 6\int_e^{e^2} \frac{1}{x}dx$		
	$= 6\left[\ln x\right]_e^{e^2}$	1	for integration
	$= 6(\ln e^2 - \ln e)$		
	$= 6(2\ln e - \ln e)$	1	for substitution
	$= 6(2-1)$		
	$= 6$	1	for correct answer
	c) $y = e^{-\frac{1}{2}x}$		
	$\therefore y^2 = (e^{-\frac{1}{2}x})^2 = e^{-x}$		
	$\therefore V = \pi\int_2^5 e^{-x}dx$	$1\frac{1}{2}$	
	$= -\pi\int_2^5 -e^{-x}dx$	1	
	$= -\pi\left[e^{-x}\right]_2^5$	1	for correct integration
	$= -\pi(e^{-5} - e^{-2})$	$\frac{1}{2}$	for substitution
	D) $y = 4 + 3\log_e x$		
	$dy/dx = \frac{3}{x}$	1	for derivative
	when $x = 1$, $y = 4 + 3\ln 1 = 4$ $(1,4)$	$\frac{1}{2}$	for correct y coord.
	$x = 1$, $m_T = \frac{3}{1} = 3$	$\frac{1}{2}$	for correct m_T
	$\therefore y - y_1 = m(x - x_1)$ $y - 4 = 3(x-1)$ $y - 4 = 3x - 3$ $3x - y + 1 = 0$	1	for correct equation

Qn	Solutions	Marks	Comments+Criteria
3	A) $\sin\frac{2\pi}{3} = \sin(\pi - \frac{2\pi}{3})$		
	$= \sin\frac{\pi}{3}$		
	$= \frac{\sqrt{3}}{2}$	1	for exact value
	B) Area $= \frac{1}{2}(R^2\theta - r^2\theta)$		
	$= \frac{1}{2}(10^2 \times 2 - 8^2 \times 2)$		
	$= \frac{1}{2}(200 - 128)$	2	for substitution
	$= \frac{1}{2} \times 72$		
	$= 36 cm^2$	1	for correct answer
	c) (i) 	1	for correct shape of graph starting from y=0
		1	for correct period
		1	for correct amplitude
	(ii) See above The line $y=1$ has 4 points of intersection with $y = 2\sin 2x$ over $0 \le x \le 2\pi$	$\frac{1}{2}$	for graph of $y=1$
	$\therefore$ there are 4 solutions	$\frac{1}{2}$	for showing 4 points of intersection/solutions
	D) (i) $y = e^{2\tan x}$	1	for correct derivative of $\tan x$
	$y' = 2\sec^2 x\, e^{2\tan x}$	1	for correct answer

Qn	Solutions	Marks	Comments+Criteria
3)	(ii) $y = \underbrace{\sin^3 x}_{u}\ \underbrace{\cos x}_{v}$		
	$u = \sin^3 x \qquad v = \cos x$ ($\frac{1}{2}$)		
	($1\frac{1}{2}$) $u' = 3\cos x \sin^2 x \qquad v' = -\sin x$	$\frac{1}{2}$ each	
	$y' = vu' + uv'$		
	$= 3\cos^2 x \sin^2 x - \sin^4 x$	1	
	$= \sin^2 x(3\cos^2 x - \sin^2 x)$		
	* using identities to simplify		
	$y' = \sin^2 x(3(1-\sin^2 x) - \sin^2 x)$ $= \sin^2 x(3 - 4\sin^2 x)$ | $y' = \sin^2 x(3\cos^2 x - (1-\cos^2 x))$ $= \sin^2 x(4\cos^2 x - 1)$		
	(iii) $y = \log_e(\cos x)$		
	$y = \ln(\cos x)$		
	$y' = \frac{-\sin x}{\cos x}$	$1\frac{1}{2}$	for correct derivative of $\cos x$
	$y' = -\tan x$	$\frac{1}{2}$	for simplifying to $-\tan x$.