St. Catherine's School

Year 10AB Mathematics
Yearly Examination
October 2007

Time allowed: *2 hours + 5 minutes reading time*

INSTRUCTIONS

- There are 27 questions in this paper.
- Marks for each part of a question are indicated.
- All questions should be attempted.
- All necessary working should be shown
- Approved scientific calculators and drawing templates may be used

Algebra	Q1-5			/
Solids	Q6	Q12		/
Co-ordinate geometry & graphs	Q7,8	Q27		/
Consumer arithmetic	Q9-11			/
Trigonometry	Q13-20			/
Surds	Q21-23			/
Probability	Q24-26			/

1. Simplify each of the following algebraic expressions (answers without negative indices):

(a) $6x^4 \times 3x^3$ **2**

(b) $(8y^3)^{\frac{1}{3}}$ **2**

(c) $(-2m^3)^{-2}$ **2**

2. Solve the following equations: **2**

a) $\frac{14x-8}{3} = 5x+7$

b) $m^2 = 25$ **2**

c) $(x-4)^2 = 9$ **2**

3. Solve the following quadratic equations by **any** suitable method:

(a) $x^2+5x-6=0$ **2**

(b) $6x^2-x-1=0$ **2**

(c) $7x^2-4x-1=0$ **2**

4. Factor the following quadratic expressions:

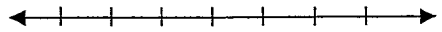
(a) $4r^2-100$ **2**

(b) $x^2+4x-32$ **2**

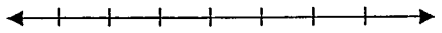
(c) $3x^2+2x-1$ **2**

5. Solve and graph on the number line provided:

a) $6x-2<3x+10$ **2**



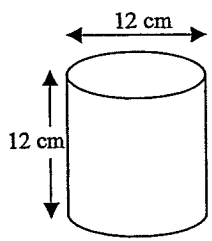
b) $5-2x\geq -35$ **2**



c) Find the simultaneous solution of this pair of equations: **3**

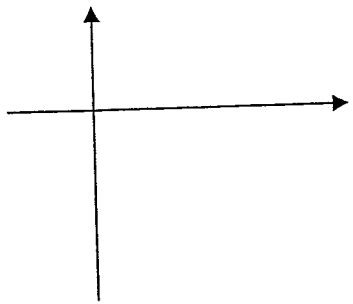
$x+2y=6$
$-2x+y=-2$

6. Calculate the surface area of the cylinder below (correct to 3 significant figures).

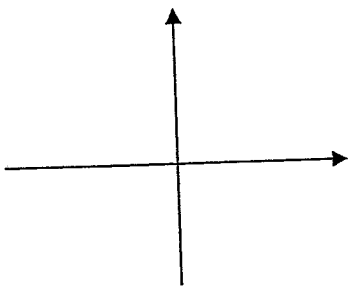


3

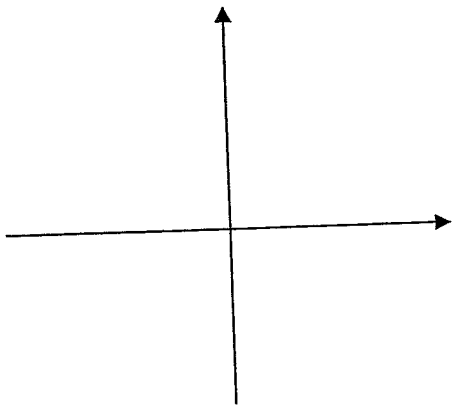
7. **a)** Sketch the parabola $y = x^2 - 8x - 20$ showing its x and y intercepts and its vertex. **3**



b) Sketch the curve $y = -x^3 + 1$ showing its x and y intercepts. **3**



c) Using a sketch diagram of $y = x^2$ and $y = 2 - x$, show that $x^2 = 2 - x$ has 2 solutions. **2**



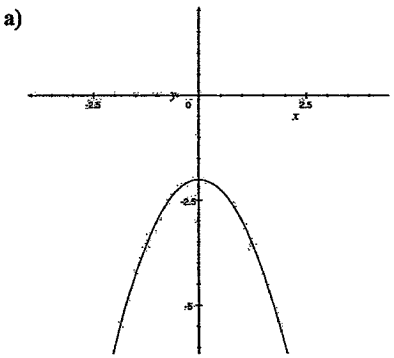
8. **4**

Choose from this set of equations the correct one for each of the four graphs shown:

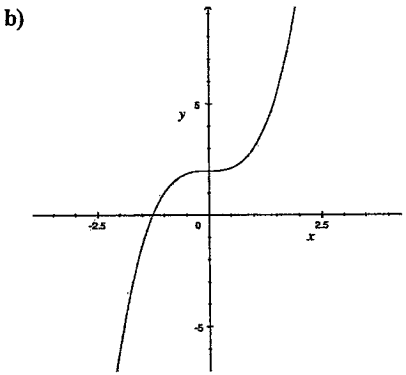
A $y = -(x-2)^2$ B $y = -(x+2)^2$ C $y = (x+2)^3$ D $y = x^3 + 2$

E $y = -x^2 - 2$ F $y = -x^2 + 2$ G $y = (x-2)^3 + 8$ H $y = (x-2)^3$

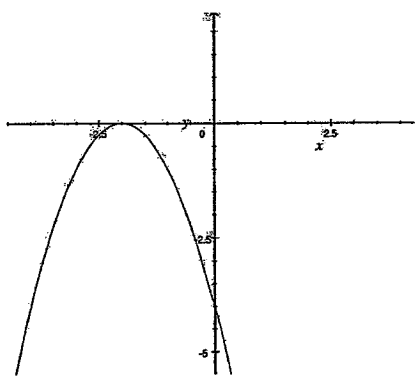
a)



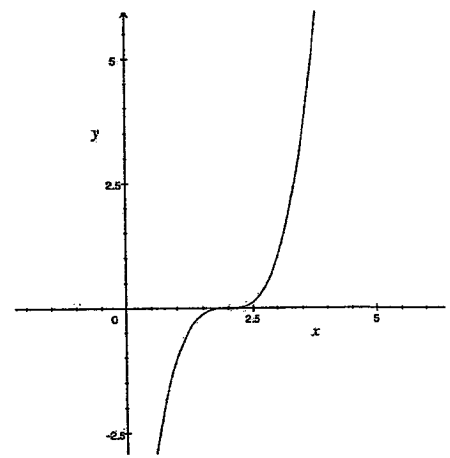
b)



c)



d)



9. The tax laws allow depreciation of 30% p.a. on computer equipment. Michael's computer was worth $8000 at the beginning of the tax year in 2007. What will it be worth in 3 years' time? (to nearest dollar) 3

10. Melissa works at Colesworths, and as a staff member gets 20% off all purchases, including sale items. She buys a dress in a "30% off" sale. The original price of the dress was $120. How much does Melissa pay. 2

11. a) Cathy buys a pair of shoes marked "30% off" in a sale, and pays $66.50 for them. Find the original price of the shoes. 2

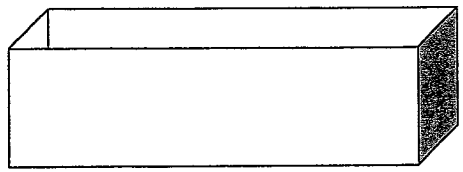
b) Alex buys a bicycle valued at $320 on terms of 10% deposit and monthly instalments of $20 for 1.5 years. 2

How much does she pay for the bicycle on these terms?

How much interest does she effectively pay on these terms?

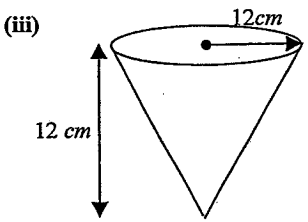
12. The diagram shows an open metal box which is used to carry sand.

It is 2 m long and the ends are 80 cm square.



(i) Find the exact volume of the box in cubic centimetres 2

(ii) Find the exact area of sheet metal required to make the box in square metres 2

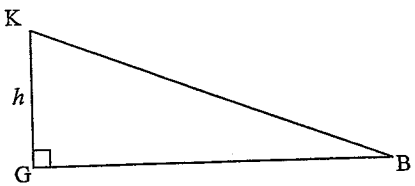
(iii) 

This conical vessel is used to fill the box above.

α) What is the volume of the cone? 2

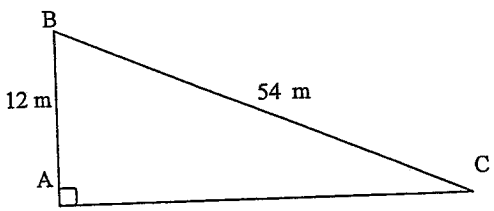
β) How many times must it be filled and emptied into the box, to fill up the box? 1

13. 2

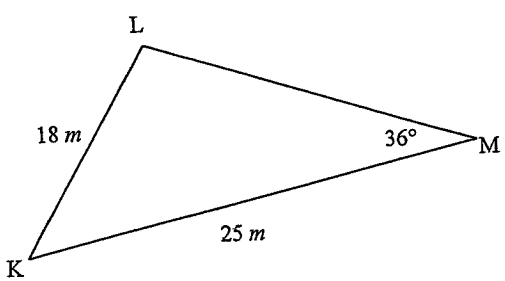


A boy is flying a kite on a 200 m length of string.
The string makes an angle of 20^0 with the ground.
Label the diagram above and use it to calculate
the height h of the kite above the ground.

14. Find the size of $\angle ABC$ to nearest degree. 2



15. Find the size of $\angle KLM$ to nearest minute, using the sine rule 3



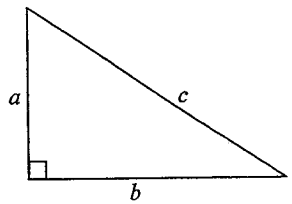
16. True or false?.
For **all** x 1

(a) $\cos x = \sin x$ _____ 1

(b) $\cos x = \sin(90^\circ - x)$ _____ 1

(c) $\sin x = \sin(90^\circ - x)$ _____ 1

(d) $\tan x = \dfrac{\sin x}{\cos x}$ _____ 1



17. If $\cos\theta = -\dfrac{4}{11}$ and $\tan\theta < 0$, find the exact value of $\sin\theta$. **3**

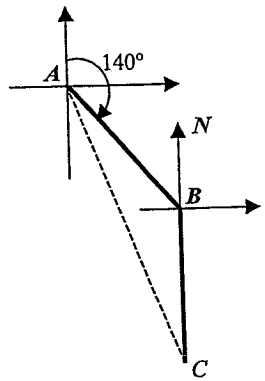
18. Find the exact value of:

(a) $\sin 120^\circ$ 1

(b) $\tan 135^o$ 1

19. A pirate has buried his treasure. From the beach at point A, he walked 90 m on a bearing of 140^0 to B and then 100 m on a bearing of 180^0 to bury the treasure at C.

(i) Complete this information on the diagram below, clearly marking the length of AB, BC and $\angle ABN$ and $\angle ABC$. **2**



(ii) Calculate the distance AC in a straight line, to the nearest metre. **3**

20. Find all possible values of θ, for $0^\circ \le \theta \le 180^\circ$, to the nearest degree if $\sin\theta = \frac{1}{5}$. **2**

21. Expand and simplify the following:

a) $\left(3+\sqrt{3}\right)^2$ **2**

b) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$ **2**

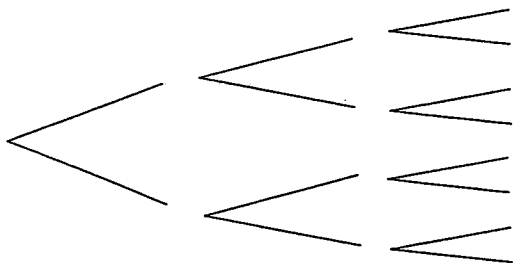
22. Express with a rational denominator in simplest form: **2**

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}}$$

23. Show that $\frac{6\sqrt{2}+5}{\sqrt{3}} - \frac{12+5\sqrt{2}-3\sqrt{6}}{\sqrt{6}}$ is a rational number **3**

24. **a)** Fill in this tree diagram to show the possible outcomes of tossing a fair coin three times **2**

OUTCOMES



(b) Use the tree diagram to calculate the following probabilities

i) P(tossing 3 heads) _____ **1**

ii) P(tossing exactly 2 heads) _____ **1**

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25. A bag contains 3 red and 9 green sweets. Meg chooses two of the sweets at random (without replacement).

Draw up a tree diagram to show all possible outcomes **2**

What is the probability;

a) Both sweets are green? _____ **1**

b) At least one sweet is red? _____ **1**

c) Meg takes one of each kind? _____ **1**

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26 The 100 *m* race at the Sports Carnival has 8 entrants. They are allocated lanes at random. Julie is one of the entrants.

(a) Write True or False: **1**

"Julie has one chance in eight of winning." _____ **1**

Explain your answer:

_____

_____ **1**

(b) Write True or False:

"Julie has one chance in eight of running in lane 3." _____

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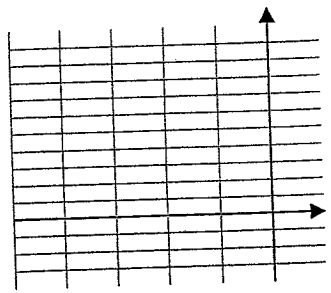
27. (i) On the number plane below plot the points A (0,1), B (-4 , 1) and C (-2 , 7) **1**



(ii) Find the midpoint E of AC. **1**

(iii) Find the gradient of AC. **1**

(iv) Write down the gradient of any line perpendicular to AC. **1**

(v) Find the equation of the line through E perpendicular to AC **2**

(vi) Find the length of side BC. **2**

(vii) Find the area of the triangle ABC. **2**

End of Examination

SOLUTIONS

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Year 10AB Mathematics
Yearly Examination
October 2007

Time allowed: *2 hours + 5 minutes reading time*

INSTRUCTIONS

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- Marks for each part of a question are indicated.
- All questions should be attempted.
- All necessary working should be shown
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Algebra	Q1-5			/
Solids	Q6	Q12		/
Co-ordinate geometry & graphs	Q7,8	Q27		/
Consumer arithmetic	Q9-11			/
Trigonometry	Q13-20			/
Surds	Q21-23			/
Probability	Q24-26			/

1. Simplify each of the following algebraic expressions (answers without negative indices):

(a) $6x^4 \times 3x^3$ = $18x^7$ **2**

(b) $(8y^3)^{\frac{1}{3}}$ = $2y$ **2**

(c) $(-2m^3)^{-2}$ = $\frac{1}{4m^6}$ **2**

2. Solve the following equations: **2**

a) $\frac{14x-8}{3} = 5x+7$

$14x - 8 = 15x + 21$

$-29 = x$ $\quad x = -29$

b) $m^2 = 25$ **2**

$m = \pm 5$

c) $(x-4)^2 = 9$ **2**

$x - 4 = \pm 3$ $\quad x = 1,\ x = 7$

3. Solve the following quadratic equations by **any** suitable method:

(a) $x^2+5x-6=0$ **2**

$(x-1)(x+6)=0$

$x=1$ or $x=-6$

(b) $6x^2-x-1=0$ **2**

$(3x+1)(2x-1)=0$

$x=-\frac{1}{3}, \quad x=\frac{1}{2}$

(c) $7x^2-4x-1=0$ **2**

$x=\frac{4\pm\sqrt{16-4\times7\times-1}}{14}$

$=\frac{4\pm\sqrt{44}}{14}=\frac{2\pm\sqrt{11}}{7}$

4. Factor the following quadratic expressions:

(a) $4r^2-100$ $=4(r-5)(r+5)$ **2**

(b) $x^2+4x-32$ $=(x+8)(x-4)$ **2**

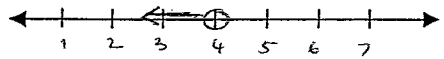
(c) $3x^2+2x-1$ $(3x-1)(x+1)$ **2**

5. Solve and graph on the number line provided:

a) $6x-2<3x+10$ **2**

$3x<12$

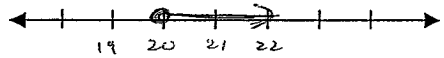
$x<4$



b) $5-2x\geq-35$ **2**

$-2x\geq-40$

$x\leq20$



c) Find the simultaneous solution of this pair of equations: **3**

$x+2y=6$

$-2x+y=-2$

$\therefore x=6-2y$

$-2(6-2y)+y=-2$

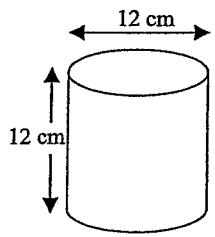
$-12+4y+y=-2$

$5y=10$

$y=2$

$x=2$

6. Calculate the surface area of the cylinder below (correct to 3 significant figures).



3

$SA=2\pi r^2+2\pi rh$

$=2\pi\times36+2\pi\times72$

$=216\pi \quad u^2$

$\doteq 679 \quad u^2$ (3 sig fig)

7. a) Sketch the parabola $y = x^2 - 8x - 20$ showing its x and y intercepts and its vertex. 3

$x = 0 \quad y = -20$

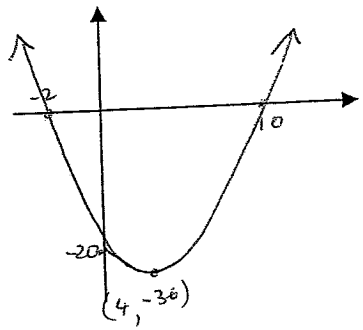
$y = 0 \quad x^2 - 8x - 20 = 0$

$(x - 10)(x + 2) = 0$

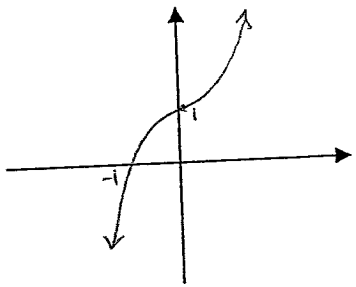
$x = 10, \; x = -2$

V: $x = \frac{-b}{2a} = \frac{8}{2} = 4$

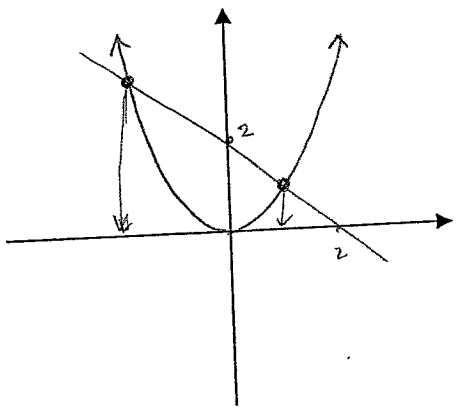
$V(4, -36)$



b) Sketch the curve $y = -x^3 + 1$ showing its x and y intercepts. 3



c) Using a sketch diagram of ~~$y = x^2$~~ and ~~$y = 2 - x$~~, show that $x^2 = 2 - x$ has 2 solutions. 2



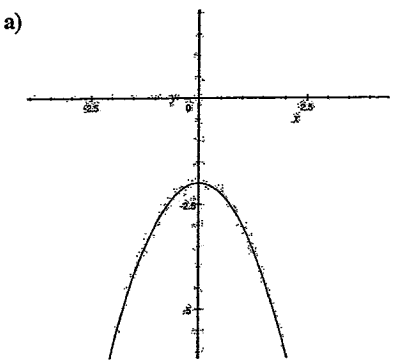
8. 4

Choose from this set of equations the correct one for each of the four graphs shown:

A $y = -(x-2)^2$ B $y = -(x+2)^2$ C $y = (x+2)^3$ D $y = x^3 + 2$

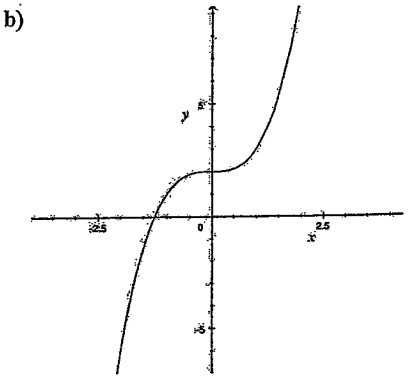
E $y = -x^2 - 2$ F $y = -x^2 + 2$ G $y = (x-2)^3 + 8$ H $y = (x-2)^3$

a)



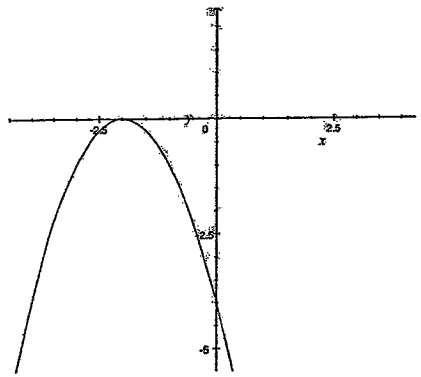
E

b)



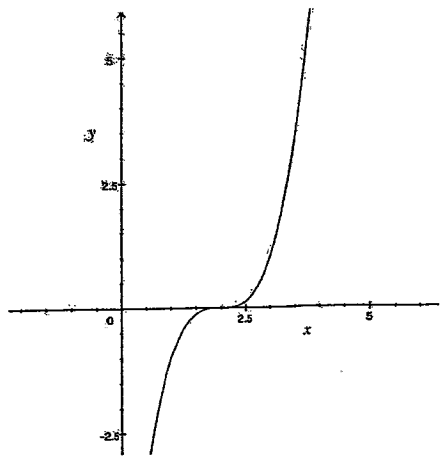
D

c)



B

d)



H

9. The tax laws allow depreciation of 30% p.a. on computer equipment. Michael's computer was worth $8000 at the beginning of the tax year in 2007. What will it be worth in 3 years' time? (to nearest dollar) 3

$A = P(1-r)^3$

$= \$8000 \times (0.7)^3$

$= \$2744$

10. Melissa works at Colesworths, and as a staff member gets 20% off all purchases, including sale items. She buys a dress in a "30% off" sale. The original price of the dress was $120. How much does Melissa pay. 2

70% of 80% of $120

$\frac{7}{10} \times \frac{8}{10} \times 120 = \$67.20$

11. a) Cathy buys a pair of shoes marked "30% off" in a sale, and pays $66.50 for them. Find the original price of the shoes. 2

70% of $x = \$66.50$

100% of $x = \frac{66.5 \times 100}{70}$

$= \$95$

b) Alex buys a bicycle valued at $320 on terms of 10% deposit and monthly instalments of $20 for 1.5 years. 2

How much does she pay for the bicycle on these terms?

Pays $\$32 + 18 \times \$20$

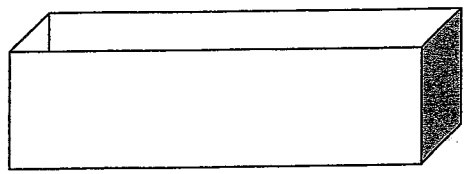
$= \$392$

How much interest does she effectively pay on these terms?

Interest $= \$392 - \$320 = \$70$

12. The diagram shows an open metal box which is used to carry sand.

It is 2 m long and the ends are 80 cm square.



(i) Find the exact volume of the box in cubic centimetres 2

$V = 200 \times 80 \times 80$

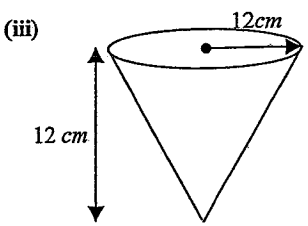
$= 1280\,000 \text{ cm}^3$

(ii) Find the exact area of sheet metal required to make the box in square metres 2

$A = 3 \times 8 \times 2 + 2 \times (8)^2$

$= 6.08 \text{ m}^2$

(iii)



This conical vessel is used to fill the box above.

α) What is the volume of the cone? 2

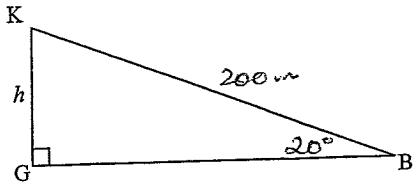
$V = \frac{1}{3} \times \pi \times 12^2 \times 12$

$\doteq 1809.6 \text{ cm}^3$

β) How many times must it be filled and emptied into the box, to fill up the box? 1

707.3 times

13. 2

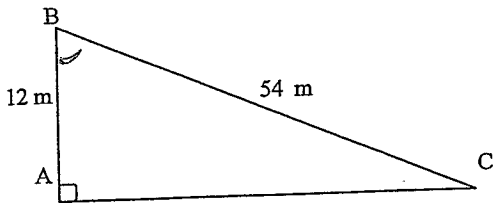


$\sin 20^\circ = \frac{h}{200}$

$h \doteq 68.4$ m

A boy is flying a kite on a 200 m length of string.
The string makes an angle of 20° with the ground.
Label the diagram above and use it to calculate
the height h of the kite above the ground.

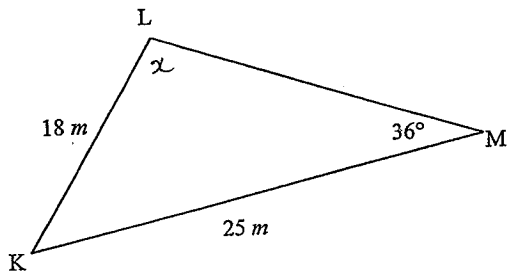
14. Find the size of $\angle ABC$ to nearest degree.



$\cos B = \frac{12}{54}$

$B \doteq 77^\circ$ (nearest $^\circ$) 2

15. Find the size of $\angle KLM$ to nearest minute, using the sine rule 3



$\frac{\sin x}{25} = \frac{\sin 36}{18}$

$\sin x = 0.8163$

$x \doteq 54^\circ 43'$

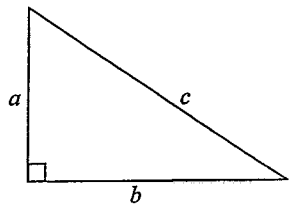
16. True or false?. For **all** x 1

(a) $\cos x = \sin x$ F 1

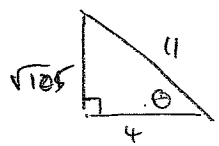
(b) $\cos x = \sin(90^\circ - x)$ T 1

(c) $\sin x = \sin(90^\circ - x)$ F 1

(d) $\tan x = \frac{\sin x}{\cos x}$ T 1



17. If $\cos\theta = -\frac{4}{11}$ and $\tan\theta < 0$, find the exact value of $\sin\theta$. 2nd Q $\sin > 0$ 3



$\sin\theta = \frac{\sqrt{105}}{11}$

18. Find the exact value of:

(a) $\sin 120^\circ = \sin(180-60)$ 1

$= \sin 60$

$= \frac{\sqrt{3}}{2}$

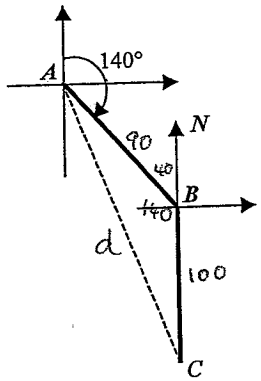
(b) $\tan 135^\circ = \tan(180-45^\circ)$ 1

$= -\tan 45^\circ$

$= -1$

19. A pirate has buried his treasure. From the beach at point A, he walked 90 m on a bearing of 140^0 to B and then 100 m on a bearing of 180^0 to bury the treasure at C.

(i) Complete this information on the diagram below, clearly marking the length of AB, BC and $\angle ABN$ and $\angle ABC$. 2



(ii) Calculate the distance AC in a straight line, to the nearest metre. 3

$d^2 = 90^2 + 100^2 - 2 \times 90 \times 100 \cos 140$

$= 31888.8$

$d \doteq 178.6$ m (1dp)

$d \doteq 179$ m (nearest m)

20. Find all possible values of θ, for $0^\circ \le \theta \le 180^\circ$, to the nearest degree if $\sin\theta = \frac{1}{5}$. 2

$\sin\theta = \frac{1}{5}$

$\theta = 12^\circ, \ 180 - 12^\circ$

$= 12^\circ$ or 168°

21. Expand and simplify the following:

a) $(3+\sqrt{3})^2$ 2

$9 + 6\sqrt{3} + 3$

$= 12 + 6\sqrt{3}$

b) $(3+\sqrt{3})(3-\sqrt{3})$ 2

$9 - 3 = 6$

22. Express with a rational denominator in simplest form: 2

$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{18} - \sqrt{12}}{6}$

$= \frac{3\sqrt{2} - 2\sqrt{3}}{6}$

23. Show that $\frac{6\sqrt{2}+5}{\sqrt{3}} - \frac{12+5\sqrt{2}-3\sqrt{6}}{\sqrt{6}}$ is a rational number 3

$\frac{6\sqrt{2}+5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{12+5\sqrt{2}-3\sqrt{6}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

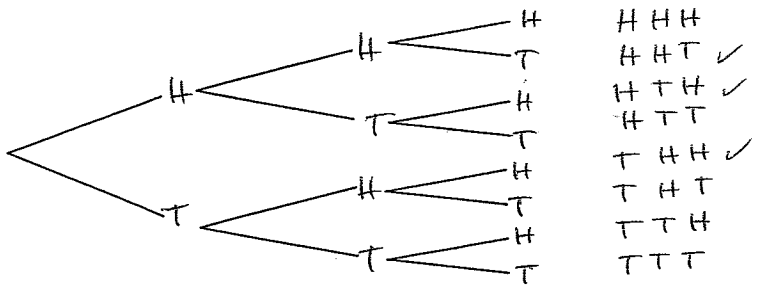
$= \frac{6\sqrt{6}+5\sqrt{3}}{3} - \frac{12\sqrt{6}+5\sqrt{12}-18}{6}$

$= 2\sqrt{6} + \frac{5}{3}\sqrt{3} - 2\sqrt{6} - \frac{5}{\cancel{6}_3} \times \frac{2\sqrt{3}}{} + \frac{18}{6}$

$= +3$

24. 2

a) Fill in this tree diagram to show the possible outcomes of tossing a fair coin three times



(b) Use the tree diagram to calculate the following probabilities

i) P(tossing 3 heads) $\frac{1}{8}$ 1

ii) P(tossing exactly 2 heads) $\frac{3}{8}$ 1

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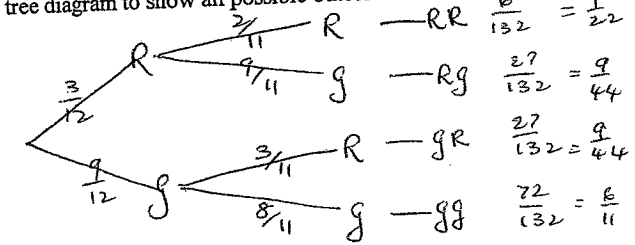
25. A bag contains 3 red and 9 green sweets. Meg chooses two of the sweets at random (without replacement).

Draw up a tree diagram to show all possible outcomes 2



What is the probability;

a) Both sweets are green? $P(gg) = \frac{9}{12} \times \frac{8}{11} = \frac{6}{11}$ 1

b) At least one sweet is red? $P(\widetilde{gg}) = 1 - \frac{6}{11} = \frac{5}{11}$ 1

c) Meg takes one of each kind? $P(Rg) + (gR) = 2 \times \frac{3}{12} \times \frac{9}{11} = \frac{9}{22}$ 1

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26 The 100 *m* race at the Sports Carnival has 8 entrants. They are allocated lanes at random. Julie is one of the entrants.

(a) Write True or False: 1

"Julie has one chance in eight of winning." F 1

Explain your answer:

Probability depends on equally likely outcomes. Julie may be fast or slow. 1

(b) Write True or False:

"Julie has one chance in eight of running in lane 3." T

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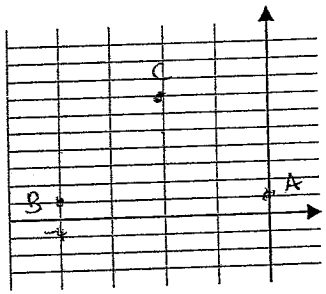
27. (i) On the number plane below plot the points A (0,1), B (-4 , 1) and C (-2 , 7) 1



(ii) Find the midpoint E of AC. 1

$E\ (-1, 4)$

(iii) Find the gradient of AC. 1

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-6}{2}$

$= -3$

(iv) Write down the gradient of any line perpendicular to AC. 1

$m = \frac{1}{3}$

(v) Find the equation of the line through E perpendicular to AC 2

$y - 4 = \frac{1}{3}(x + 1)$

$y = \frac{1}{3}x + 4\frac{1}{3}$

OR $x - 3y + 13 = 0$

(vi) Find the length of side BC. 2

$d = \sqrt{2^2 + 6^2}$

$= \sqrt{40}$

$= 2\sqrt{10}$

(vii) Find the area of the triangle ABC. 2

$A = \frac{1}{2} b \times h$

$= \frac{1}{2} \times 4 \times 6$

$= 12\ u^2$

End of Examination