



St. Catherine's School  
Waverley

2008  
ASSESSMENT TASK 2  
(20%)

# Mathematics Year 11

## General Instructions

- Working time – 55 minutes
- Attempt Questions 1–4
- Start each question on a new page in your answer booklet.
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary **working** must be shown.
- Marks may be deducted for careless or badly arranged work

Student Number: \_\_\_\_\_

Total marks – 48

## QUESTION 1

(15 marks)

- (a) Simplify  $(2 + 3\sqrt{3})(4\sqrt{5} - \sqrt{15})$  /2
- (b) Given that  $\frac{\sqrt{5}-1}{\sqrt{5}+1} = a - b\sqrt{5}$ , find the value of  $a$  and  $b$  /3
- (c) Solve for  $x$ :  
(i)  $-2 < 4 - 3x \leq 7$  /2  
(ii)  $|x+7| = 2x-1$  /3  
(iii)  $|5x-1| \geq 2$  /2
- (d) Find the value of  $x$  if  $f(x) = 7$  where  $f(x) = 2^{x+1} - 1$  /3

## QUESTION 2

(12 marks)

- (a) If  $f(x) = x^2 - 3x + 5$  find  
(i)  $f(1) + f(0)$  /1  
(ii)  $f(x-1)$  /2
- (b) State the natural domain and range of the following functions:  
(i)  $y = x^2 - 1$  /2  
(ii)  $y = \sqrt{9 - x^2}$  /2  
(iii)  $y = \frac{3}{2x+1}$  /2
- (c) Write down the equation of the vertical asymptote of  $y = \frac{4x}{x-2}$  /1
- (d) Find  $\lim_{x \rightarrow -2} \frac{x+2}{x^2+5x+6}$  /2

**QUESTION 3****(11 marks)**

- (a) Show that  $f(x) = x^3 + x$  is an odd function /2
- (b) Sketch the following graphs showing their essential features
- (i)  $y = \frac{1}{x-2} - 1$  /2
- (ii)  $y = -2^x$  /2
- (iii)  $y = x^2 - 3x - 4$  /2
- (c) The function  $f(x)$  is defined as follows:

$$f(x) = \begin{cases} mx + 4 & \text{when } x > 2 \\ 1 & \text{when } -2 \leq x \leq 2 \\ x^2 & \text{when } x < -2 \end{cases}$$

If  $f(-3) = f(3)$ , find the value of  $m$  /3

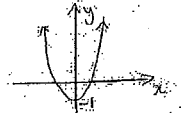
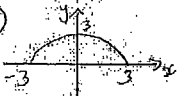
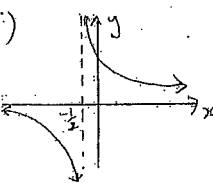
**QUESTION 4****(10 marks)**

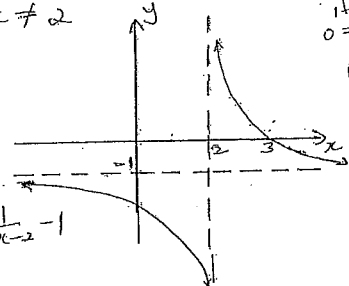
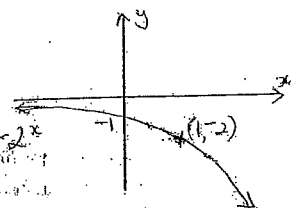
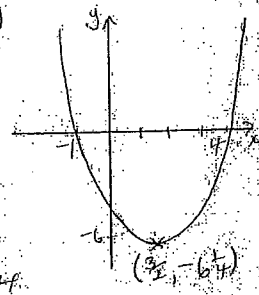
- (a) Shade the region defined simultaneously by  $y \geq |x + 3|$  and  $y < 2$  /4
- (b) Write down the equation of the circle with centre  $(6, -8)$  with radius 10 units, and find the coordinates of the points where this circle cuts the x-axis /3
- (c) Sketch  $f(x) = \frac{x^2 - 4x}{x}$ , showing any points of discontinuity /3

**END OF TEST**

Qn	Solutions	Marks	Comments+Criteria
1(a)	$(2+3\sqrt{3})(4\sqrt{5}-\sqrt{5})$ $= 8\sqrt{5} - 2\sqrt{5} + 12\sqrt{15} - 3\sqrt{15}$ ✓ $= 6\sqrt{5} + 9\sqrt{15}$ ✗ $= 10\sqrt{5} - \sqrt{5}$ ✗	2	✗ for $\sqrt{15}$
(b)	$\frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{\sqrt{5}-1}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$ ✗ $= \frac{5 - 2\sqrt{5} + 1}{5-1}$ ✗ $= \frac{6 - 2\sqrt{5}}{4}$ $= \frac{6}{4} - \frac{2\sqrt{5}}{4}$ $= \frac{3}{2} - \frac{\sqrt{5}}{2}$ ✓ $a - b\sqrt{5} = \frac{3}{2} - \frac{\sqrt{5}}{2}$ $\therefore a = \frac{3}{2}$ ✗ $b = \frac{1}{2}$ ✗	3	
(c) (i)	$-2 < 4 - 3x \leq 7$ $-2 - 4 < -3x \leq 7 - 4$ $-6 < -3x \leq 3$ ✓ $\frac{-6}{-3} > x > \frac{3}{-3}$ $2 > x > -1$ ✓ $\therefore -1 < x < 2$	2	

Qn	Solutions	Marks	Comments+Criteria
1(c) (ii)	$ x+7  = 2x-1$ $x+7 = 2x-1$ or $-(x+7) = 2x-1$ $x = 8$ ✓ $-x-7 = 2x-1$ $-6 = 3x$ $x = -2$ ✓	3	
	check $ 8+7  = 2 \times 8 - 1$ $  -2+7  = 2 \times 2 - 1$ $ 15  = 15$ true ✓ $ 5  = 5$ false ✗ $\therefore x = 8$ is only solution	3	$\frac{2}{3}$ if didn't check answers
(iii)	$ 5x-1  \geq 2$ $5x-1 \geq 2$ or $-(5x-1) \geq 2$ $5x \geq 3$ $5x-1 \leq -2$ $x \geq \frac{3}{5}$ ✓ $5x \leq -1$ $x \leq -\frac{1}{5}$ ✓	2	
(d)	$f(x) = 2^{x+1} - 1$ $7 = 2^{x+1} - 1$ ✓ $8 = 2^{x+1}$ $2^3 = 2^{x+1}$ ✓ $x+1 = 3$ $\therefore x = 2$ ✓	3	

Qn	Solutions	Marks	Comments+Criteria
2(a)	$f(x) = x^2 - 3x + 5$ $f(1) + f(0) = (1)^2 - 3(1) + 5 + 0^2 - 3(0) + 5$ $= 8$	1	
(b)	$f(x-1) = (x-1)^2 - 3(x-1) + 5$ $= x^2 - 2x + 1 - 3x + 3 + 5$ $= x^2 - 5x + 9$	2	2.5 for (a) ① 1 for (b) ② 0.5 for (b) ③
(b)	$D = \{x : x \in \mathbb{R}\}$ $R = \{y : y \geq -1\}$ 	2	
(ii)	 $D = \{x : -3 \leq x \leq 3\}$ $R = \{y : 0 \leq y \leq 3\}$	2	
(iii)	$2x + 1 \neq 0$ $x \neq -\frac{1}{2}$ $D: \{x : x \in \mathbb{R}, x \neq -\frac{1}{2}\}$ $R: \{y : y \in \mathbb{R}, y \neq 0\}$ 	2	
(c)	$y = \frac{4x}{x-2}$ $x-2 = 0$ vert. asym. $x=2$	1	no marks are awarded for just a '2'
(d)	$\lim_{x \rightarrow -2} \frac{x+3}{x^2+5x+6} = \lim_{x \rightarrow -2} \frac{x+3}{(x+3)(x+2)}$ $= \lim_{x \rightarrow -2} \frac{1}{x+2}$ $= \frac{1}{-2+3}$ $= 1$	2	

Qn	Solutions	Marks	Comments+Criteria
3(a)	odd: $f(-x) = -f(x)$ $f(-x) = (-x)^3 + (-x)$ $= -x^3 - x$ ✓ $-f(x) = -(x^3 + x)$ $= -x^3 - x$ ✓ $\therefore f(-x) = -f(x)$ $\therefore$ an odd function	2	
(b)	(i) $x \neq 2$  $y = \frac{1}{x-2} - 1$ if $y=0$ $0 = \frac{1}{x-2} - 1$ $1 = \frac{1}{x-2}$ $x-2 = 1$ $\therefore x=3$	2	asymptote $x=2$ hyperbola $y=-1$ point(s) $(3, 0)$
(ii)	 $y = -2^x$	2	right location and shape $y=0$ (0, -1) $y=0$
(iii)	$0 = (x-4)(x+1)$ $\therefore x = 4, -1$ axis of symmetry $x = \frac{4+(-1)}{2}$ $x = \frac{3}{2}$ min. pt. when $x = \frac{3}{2}$ $y = (\frac{3}{2})^2 - 3(\frac{3}{2}) - 4$ $= -6\frac{1}{4}$ 	2	one more point $y=0$ Parabolic $y=0$ axis of symmetry $y=0$ vertex $y=0$ or x-axis intercept $y=0$ vertex $y=0$

Qn	Solutions	Marks	Comments+Criteria
3(e)	$f(-3) = (-3)^2 = 9$ ✓ $f(3) = 3m + 4$ ✓ if $f(-3) = f(3)$ $9 = 3m + 4$ $5 = 3m$ $\frac{5}{3} = m$ ✓	3	
4(a)	<p>test (0,0) for  <math>y \geq  x+3 </math>  <math>0 \geq  0+3 </math>                      false</p>	4	not simultaneous not dotted vs solid correct $y=2$ ✓ " $y= x+3 $ ✓ not showing testing
(b)	$(x-6)^2 + (y+8)^2 = 100$ when $y=0$ $(x-6)^2 + (8)^2 = 100$ $(x-6)^2 = 36$ $x-6 = \pm\sqrt{36}$ $x-6 = \pm 6$ $x = \pm 6 + 6$ $\therefore x = 12, 0$ $\therefore$ cuts $x$ -axis at $(12, 0)$ and $(0, 0)$	1/2 M for $y=0$ 1 3	-1/2 M for not stating coordinates

Qn	Solutions	Marks	Comments+Criteria
4(c)	$f(x) = \frac{x^2 - 4x}{x}$ $x \neq 0$		1/2 M for $x \neq 0$
	$\lim_{x \rightarrow 0} \frac{x^2 - 4x}{x}$ $= \lim_{x \rightarrow 0} \frac{x(x-4)}{x}$ $= \lim_{x \rightarrow 0} (x-4)$ $= -4$ $\therefore f(x) = \frac{x^2 - 4x}{x}$ $= x-4$ undefined at $(0, -4)$		
		3	