Name:____________________

Mathematics

St. Catherine's School Waverley

2013

PRELIMINARY ASSESSMENT TASK 3

(15%)

Working time 57 mins
Reading time 3 minutes

- Marks for each question are indicated

General Instructions

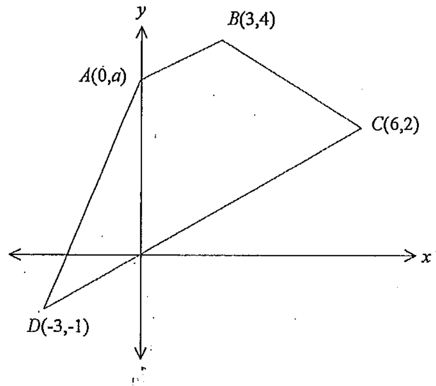
- Start each question in a **new booklet**.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary **working** must be shown.
- Marks may be deducted for careless or badly arranged work.

TEACHER'S USE ONLY

Linear Functions & Lines	Q1, Q4	/16
Quadratic Polynomial	Q2, Q3, Q5	/22
Locus & Parabola	Q6	/12
TOTAL		/50

Circle the correct answer: Questions 1, 2 and 3 are worth 1 mark each.

1. The points A, B, C and D have coordinates $(0,a)$, $(3,4)$, $(6,2)$ and $(-3,-1)$ respectively.



What is the value of a if AB is parallel to DC?

(A) $a = 2.5$
(B) $a = 3$
(C) $a = 3.5$
(D) $a = 5$

2. Which of the following is true for the equation $7x^2 - 5x + 2 = 0$?

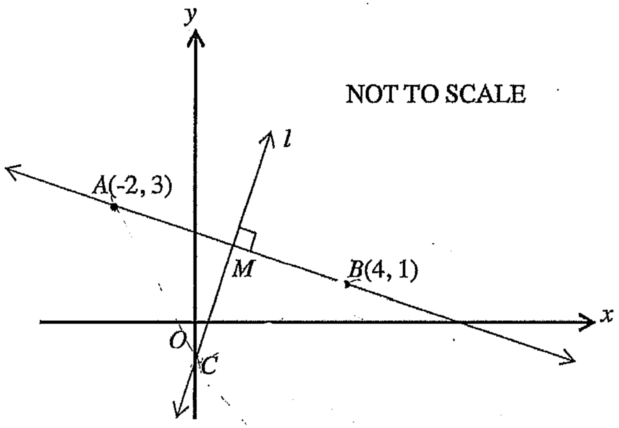
(A) No real roots
(B) One real root
(C) Two real distinct roots
(D) Three real roots

3. What is the value of k if the sum of the roots of $x^2 - (k-1)x + 2k = 0$ is equal to the product of the roots?

(A) -3
(B) -2
(C) -1
(D) 1

Question 4. **(15 Marks)**

(a)



The diagram shows the points $A(-2, 3)$ and $B(4, 1)$. M is the midpoint of the interval AB. l is the line through M perpendicular to AB.

(i)	Find the gradient of the line AB.	1
(ii)	Find the coordinates of M.	1
(iii)	Show that the equation of AB is $x+3y-7=0$.	1
(iv)	Show that the equation of l is $3x-y-1=0$.	1
(v)	Find the coordinates of C, the point of intersection of l and the y-axis.	1
(vi)	Show that C is equidistant from A and B	2
(vii)	Find the perpendicular distance from C to the line AB.	2
(viii)	Hence find the area of the triangle ABC.	2
(ix)	State the inequality that defines the region of the half-plane *under* the line AB.	1

(b) Find the equation of the line that passes through the point of intersection of $2x-y-3=0$ and $5x-y=0$ and passes through the point (1,2) **3**

Question 5 (22 Marks) **Marks**

(a) Find the coordinates of the points of intersection of the line $2x-y-2=0$ and the hyperbola $xy=4$ **3**

(b) Solve the equation for x: $3^{2x}-8\times 3^x+15=0$. **3**

(c) For what values of k does the equation $x^2+(k+6)x-2k=0$ have two real and different roots? **2**

(d) α and β are the roots of the quadratic equation $2x^2-5x-8=0$. Without finding the roots, find the value of:

(i)	$\alpha+\beta$	1
(ii)	$\alpha\beta$	1
(iii)	$\alpha^2+\beta^2$	2
(iv)	$\alpha^3+\beta^3$	2

(e) Find the value of k for which the equation $x^2-(k-4)x+(k-9)=0$ has

(i)	roots which are equal in magnitude, but opposite in sign.	1
(ii)	roots which are reciprocals of each other	1

(f)

(i)	By finding the discriminant, how many roots does $f(x)$ have, if $f(x)=2x^2-5x+6$.	2
(ii)	Explain the graphical meaning of the result in part (i).	1

(g) Show that the equation $a^2x^2+x^2+ax+2=0$ has no real roots for all values of a. **3**

Question 6 (12 Marks) **Marks**

(a) $A(0,4)$ and $B(5,0)$ are two fixed points. $P(x,y)$ is a variable point which moves such that $\angle APB$ is a right angle.

(i) Show that the locus of P is the equation of the circle **2**

$$x^2+y^2-5x-4y=0$$

(ii) Find the centre and the radius of the circle in part (i) **2**

(b) A parabola has its vertex at the point $(3,1)$ and its directrix has equation $y=-1$.

(i) What is the focal length? **1**

(ii) State the co-ordinates of the focus. **1**

(iii) Find the equation of the parabola. **1**

(c) Consider the parabola $x^2+6x-8y+1=0$.

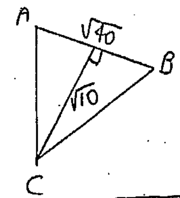
(i) Find the coordinates of the vertex **2**

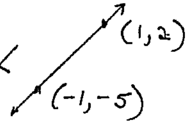
(ii) Write down the coordinates of the focus. **2**

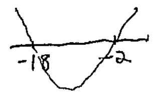
(iii) Write down the equation of the directrix. **1**

End of Task

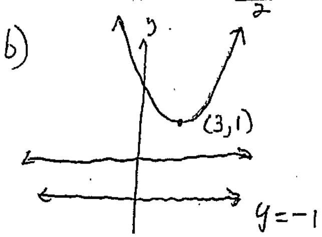
Qn	Solutions	Marks	Comments: Criteria
1	$\frac{4-a}{3} = \frac{3}{9}$ $a = 3$ (B)	1	
2	$\Delta = b^2 - 4ac$ $= 25 - 4(7)(2) < 0$ NO REAL ROOTS (A)	1	
3	$\alpha+\beta = -\frac{b}{a}$ $\quad \alpha\beta = \frac{c}{a}$ $= k-1$ $\quad = 2k$ $\therefore k-1 = 2k$ $k = -1$ (C)	1	
4 a)	i) $m = \frac{3-1}{-2-4}$ $= -\frac{1}{3}$	1	
	ii) $M = \left(\frac{-2+4}{2}, \frac{3+1}{2}\right)$ $= (1, 2)$	1	
	iii) $y - y_1 = m(x - x_1)$ $y - 2 = -\frac{1}{3}(x-1)$ $3y - 6 = -x + 1$ $x + 3y - 7 = 0$	1	
	iv) $m_\perp = 3$ $\quad (1, 2)$ $y - 2 = 3(x-1)$ $y - 2 = 3x - 3$ $3x - y - 1 = 0$	1	

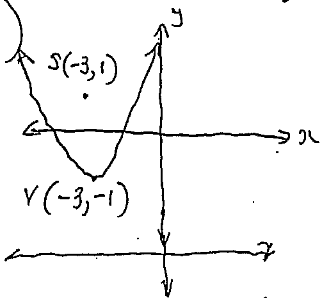
Qn	Solutions	Marks	Comments: Criteria
v)	y-INT, $x = 0$ $-y - 1 = 0$ $y = -1$ $\therefore$ C IS $(0, -1)$	1	
vi)	$d_{CA} = \sqrt{(0+2)^2 + (-1-3)^2}$ $= \sqrt{20}$ $d_{CB} = \sqrt{(4-0)^2 + (1+1)^2}$ $= \sqrt{20}$ $\therefore d_{CA} = d_{CB}$	2	
vii)	$d = \sqrt{(1-0)^2 + (2+1)^2}$ OR $d = \frac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2+b^2}}$ $= \sqrt{10}$ UNITS $\quad = \frac{\lvert 1(0) + 3(-1) - 7 \rvert}{\sqrt{1^2+3^2}}$ $= \frac{\lvert -10 \rvert}{\sqrt{10}} = \frac{10}{\sqrt{10}}$ $= \sqrt{10}$ units	2	
viii)	 $d_{AB} = \sqrt{(4+2)^2 + (1-3)^2}$ $= \sqrt{40}$ $\therefore A = \frac{1}{2}bh$ $= \frac{1}{2} \times \sqrt{40} \times \sqrt{10}$ $= 10\ u^2$	2	
ix)	$x + 3y - 7 < 0$? TEST $(0,0)$ $0 + 0 - 7 < 0$ TRUE $\therefore$ REGION IS $x + 3y - 7 < 0$	1	$\frac{1}{2}$ for $x+3y-7 \le 0$

Qn	Solutions	Marks	Comments: Criteria
b)	$2x-y-3=0$ ① $5x-y \quad\quad = 0$ ② $3x \quad +3 = 0$ ②−① ✓ $3x=-3$ $x=-1$ ✓ $\therefore -5-y=0$ $y=-5$ ✓ PT OF INTERSECTION IS $(-1,-5)$ $m=\frac{2+5}{1+1}$ $=\frac{7}{2}$ ✓  $y-2=\frac{7}{2}(x-1)$ ✓ $2y-4=7x-7$ $7x-2y-3=0$ ✓ OR $2x-y-3+k(5x-y)=0$ ✓ SUB $(1,2)$ $2-2-3+k(5-2)=0$ ✓ $3k=3$ $k=1$ ✓ $\therefore$ LINE IS $2x-y-3+5x-y=0$ ✓ $7x-2y-3=0$ ✓	3	

Qn	Solutions	Marks	Comments: Criteria
5	a) $2x-y-2=0$ ① $xy=4$ ② $y=2x-2$ FROM ① $x(2x-2)=4$ INTO ② $2x^2-2x-4=0$ $x^2-x-2=0$ $(x-2)(x+1)=0$ $x=2,-1$ $y=2,-4$ $\therefore$ Pts of INTERSECTION ARE $(2,2)$ AND $(-1,-4)$	3	
	b) $3^{2x}-8(3^x)+15=0$ LET $m=3^x$ $m^2-8m+15=0$ $(m-3)(m-5)=0$ $m=3,5$ $\therefore 3^x=3 \quad 3^x=5$ $x=1 \quad\quad = \log_3 5$	3	
	c) FOR REAL AND DIFFERENT ROOTS $\Delta>0$ $\Delta=b^2-4ac$ $(k+6)^2-4(-2k)>0$ $k^2+12k+36+8k>0$ $k^2+20k+36>0$ $(k+18)(k+2)>0$  $k<-18,\ k>-2$	2	
	d) i) $\alpha+\beta=-\frac{b}{a}$ $=\frac{5}{2}$ ii) $\alpha\beta=\frac{c}{a}$ $=\frac{-8}{2}$ $=-4$	i) 1 ii) 1	

Qn	Solutions	Marks	Comments: Criteria
	iii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{5}{2}\right)^2-2(-4)$ $=14\frac{1}{4}$	2	
	iv) $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=\left(\frac{5}{2}\right)\left(14\frac{1}{4}+4\right)$ $=\frac{365}{8}$ OR $45\frac{5}{8}$, $45{\cdot}625$	2	
e)	i) $x^2-(k-4)x+(k-9)=0$ Let roots be $x=\alpha,-\alpha$ $\alpha+\beta=-\frac{b}{a}$ $\alpha-\alpha=k-4$ $0=k-4$ $\therefore k=4$	1	
	ii) LET ROOTS BE $x=\alpha,\frac{1}{\alpha}$ $\alpha\beta=\frac{c}{a}$ $\alpha\times\frac{1}{\alpha}=k-9$ $1=k-9$ $\therefore k=10$	1	
f)	$f(x)=2x^2-5x+6$ $\Delta=b^2-4ac$ $=25-4(2)(6)$ $=-23<0$	2	
	ii) NO ROOTS, ie THE PARABOLA DOES NOT CROSS/TOUCH THE x-AXIS.	1	

Q	Solutions	Marks	Comments
	g) $a^2x^2+x^2+ax+2=0$ $x^2(a^2+1)+ax+2=0$ $\Delta=b^2-4ac$ $=a^2-4(a^2+1)2$ $=a^2-8a^2-8$ $=-7a^2-8$ Since $a^2\geq 0$ FOR ALL a $-7a^2\leq 0$ $-7a^2-8\leq -8$ $\therefore -7a^2-8<0$ FOR ALL a $\therefore$ NO REAL ROOTS SINCE $\Delta<0$ FOR ALL a.	3	
6	a) i) $m_{PA}\times m_{PB}=-1$ $\frac{y-4}{x}\times\frac{y}{x-5}=-1$ ✓ $y^2-4y=-(x^2-5x)$ ✓ $x^2-5x+y^2-4y=0$ (or Pythagoras' Method using distances)	2	
	ii) $\left(x-\frac{5}{2}\right)^2+(y-2)^2=\frac{25}{4}+4$ $\left(x-\frac{5}{2}\right)^2+(y-2)^2=\frac{41}{4}$ $\therefore$ CENTRE $=\left(\frac{5}{2},2\right)$ ✓ RADIUS $=\frac{\sqrt{41}}{2}$ ✓	2	
	b) 		
	i) $a=2$ ✓	1	
	ii) FOCUS $=(3,3)$ ✓	1	
	iii) $(x-h)^2=4a(y-k)$ ✓ $(x-3)^2=8(y-1)$ ✓	1	

Q	Solutions	Marks	Comments
	c) i) $x^2+6x-8y+1=0$ $x^2+6x=8y-1$ $(x+3)^2=8y-1+9$ $(x+3)^2=8(y+1)$ ✓ $\therefore$ VERTEX IS $(-3,-1)$ ✓	2	
	ii)  $(x-h)^2=4a(y-k)$ $(x+3)^2=8(y+1)$ ✓ NOTE $4a=8$ $a=2$ $\therefore$ FOCUS IS $(-3,1)$ ✓	2	$\frac{1}{2}$ for $a=2$
	iii) $y=-3$	1	