Student Number:________________

St. Catherine's School
Waverley

2009
Yearly Examination
ASSESSMENT TASK 4
(Weighting 45%)

Mathematics Preliminary
Year 11

General Instructions

- Reading time – 5 minutes
  Working time – 2 hours
- Questions are to be answered in 5 separate booklets. Booklet 1 – Questions 1 and 2
  Booklet 2 – Questions 3 and 4
  Booklet 3 – Question 5
  Booklet 4 – Question 6
  Booklet 5 – Question 7
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work.

Total marks – 84

- Attempt all questions.

Question 1 (12 marks) **Start a new writing booklet** **Marks**

a) Evaluate $\dfrac{\pi}{\sqrt{2.6+1.91}}$ to one decimal place. **2**

b) If $\sqrt{18}+\sqrt{8}=\sqrt{a}$ find the value of a. **2**

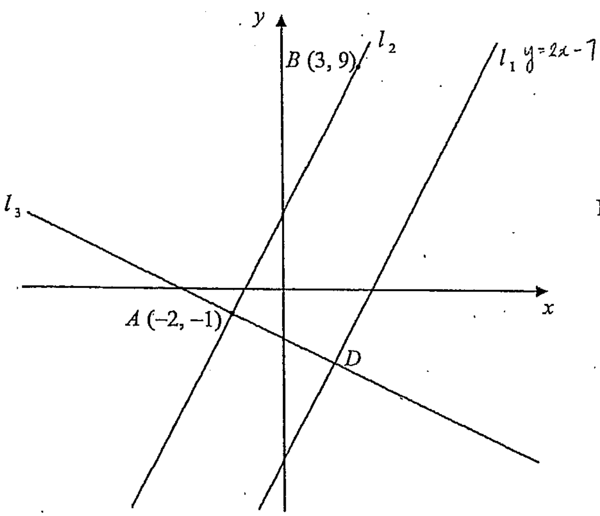
c) Express $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ with a rational denominator in simplest form **2**

d) As she has an excellent driving record Alice receives a 60% no claim discount on her car insurance. If she pays $260 with the discount what would Alice have paid without a discount? **2**

e) Express $\dfrac{a+2b}{3}-\dfrac{2a-b}{4}$ as a fraction in its simplest form. **2**

f) Factorise ab^2-b+a^2b-a. **2**

Question 2 (12 marks) **Start a new page** **Marks**



NOT TO SCALE

The line l_1 has equation $y=2x-7$.
The line l_2 passes through the points A (−2, −1) and B (3, 9).

Copy the diagram into your answer booklet.

(i) Find the gradient of the line l_2. **1**

(ii) Show that the line l_3, which is perpendicular to AB and passes through the point A (−2, −1), has equation $x+2y+4=0$. **2**

(iii) Find the coordinates of D, the point of intersection of line l_1, $y=2x-7$ and line l_3, $x+2y+4=0$. **2**

(iv) Find the length of the interval AB, leaving the answer in simplest exact form. **2**

(v) Find the exact perpendicular distance of B (3, 9), to the line l_1 $y=2x-7$. **2**

(vi) Show that the point (5, 3) lies on the line $y=2x-7$. **1**

(vii) On your diagram, shade the region satisfied by **2**

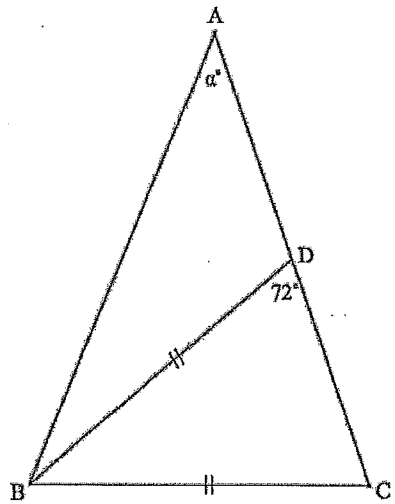
$$\begin{cases} y \geq 2x-7 \\ x+2y+4>0 \\ y<0 \end{cases}$$

Question 3 (12 marks) **Start a new booklet** **Marks**

a) If $|x+3|>2$ find all possible values of x **2**

b) Given $\cos\theta = \frac{3}{4}$ and $0^0 \leq \theta \leq 360^0$ find the two exact values for $\sin\theta$ **2**

c) In the diagram below AB=AC and BD=BC and $\angle BDC = 72^0$ **3**
Copy this diagram onto your answer page



Find the size of α giving full reasons

d) It is known that $x^2-(k-6)x+4=0$ has only one real solution. Find all possible value(s) of k **2**

e) Solve $y=x^2+5x-1$ and $y=2x^2+3$ simultaneously **3**

Question 4 (12 marks) **Start a new page** **Marks**

a) Find the coordinates of the focus of $y^2 = 6x - 12$ **2**

b) i) Prove that $\sec A \sin A = \tan A$. **1**

ii) Hence or otherwise solve
$\sec A \sin A + 1 = 0$ if $0^0 \le A \le 720^0$ **2**

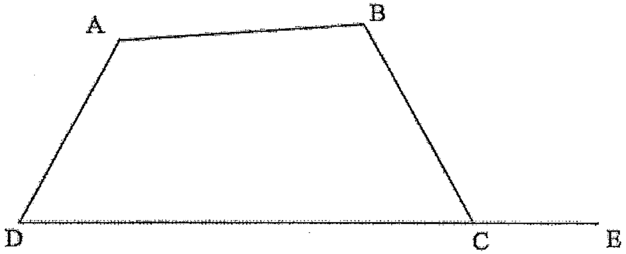
c) i) Factorise $27 + 8a^3$. **1**

ii) Factorise $-9m^2 + 64$. **1**

d) i) Sketch the graph of $y = -\sqrt{7 - x^2}$ showing where it meets the coordinate axes. **1**

ii) Find the exact area bounded by this graph and the x axis. **1**

e) **3**



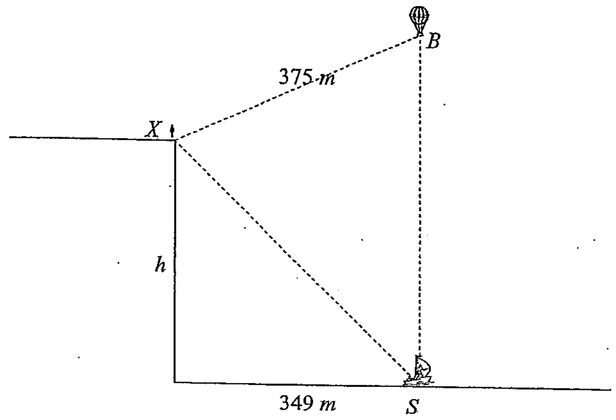
In the diagram above $\angle ABC$ and $\angle ADC$ add to 180^0

Copy the diagram onto your answer page.

Prove that $\angle BCE = \angle BAD$

Question 5 (12 marks) **Start a new writing booklet** **Marks**

a) A man is standing on a cliff top (X) and observes a hot air balloon (B) hovering directly above a sailing boat (S) as shown in the diagram below.
The boat is 349 m from the base of the cliff, and the distance of the cliff top from the balloon is 375 m. The angle of depression from the cliff top to the boat is $57°$.



(i) Copy this diagram into your answer booklet and clearly label the angle of depression. **1**

(ii) Show the height h of the cliff is 537.4 metres. **1**

(iii) Hence, or otherwise, find the height (BS) of the balloon above the sailing boat to the nearest metre. **2**

b) Differentiate the following with respect to x:

(i) $2x^2 + 3 - \dfrac{6}{x}$ **2**

(ii) $(3x^2 - x)^4$ **2**

(iii) $\dfrac{4x^3 + 5}{2x - 9}$ **2**

(iv) $2x\sqrt{4x+3}$. **2**

Question 6 (12 marks) **Start a new booklet** **Marks**

a) i) Show that the equation of the tangent to the curve $y = 3x^2 - 2x + 1$ at the point A(1,2) is given by $y = 4x - 2$ **2**

ii) If this tangent cuts the y axis at B find the coordinates of B **1**

iii) If O is the point (0,0) find the area of ΔAOB **1**

b) A point P(x,y) moves on the number plane so that the length PA equals $\frac{1}{2}$ length PB where A is (2,0) and B is (8,0). **3**
Find the equation of the locus of P.

c) From a lighthouse (L) a ship A bears 130^0T and is 14km away from the lighthouse. A ship B is 16km South West of the lighthouse.

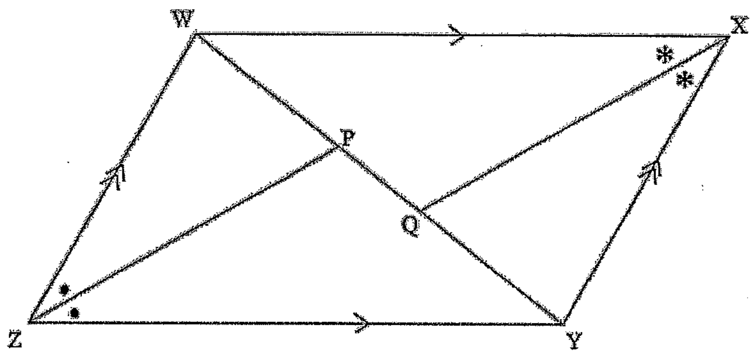
i) Draw a diagram to represent this information **1**

ii) Calculate the distance AB between the ships to the nearest metre. **2**

d) A function $f(x)$ is defined as $f(x) = 2x-1$ when $x<2$ **2**
$f(x) = -3x$ when $x \geq 2$

Evaluate $f(-4) + f(0) + f(2)$

Question 7 (12 marks) **Start a new booklet** **Marks**



a) In the diagram above WXYZ is a parallelogram
PZ bisects $\angle WZY$ and XQ bisects $\angle \cancel{WXQ}$ WXY

Copy the diagram onto your answer page

i) Prove that ΔWZP and ΔYXQ are congruent **3**

ii) If WY=20cm and QY=8cm find the length PQ giving reasons for your answer **1**

b) If 2α and $\frac{2}{\alpha}$ are the solutions of $3x^2 + 8x + k = 0$ find the size of k **2**

c) i) Sketch the function $y = x^2$ for values of x such that $x \geq 0$ and write down the range for these x values. **1**

ii) If y in i) is part of an odd function $f(x)$ defined for all real values of x sketch this function $f(x)$ on a separate diagram. **1**

d) i) For $0^0 \leq \theta \leq 360^0$ sketch the graphs of y=sinθ and y=2cosθ on the same axes indicating the main features of the graphs **2**

iii) Using these graphs or otherwise state the number of solutions to the equation $2\cos\theta - \sin\theta = 0$ for $0^0 \leq \theta \leq 360^0$ **2**

END OF PAPER

– SOLUTIONS –

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Total marks – 84

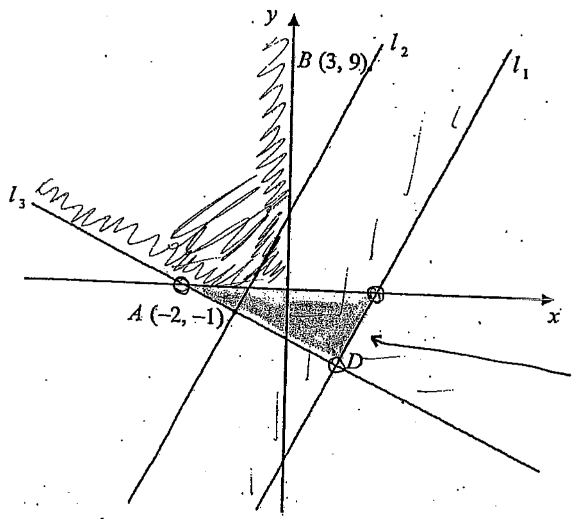
- Attempt all questions.

Qn	Solutions	Marks	Comments+Criteria
1 a)	$\dfrac{\pi}{\sqrt{2.6+1.91}} = 1.5$ (1 d.p)	2	
b)	$\sqrt{18}+\sqrt{8} = 3\sqrt{2}+2\sqrt{2}$ $= 5\sqrt{2}$ 1 $= \sqrt{50}$ $= \sqrt{a} \quad \therefore a = 50$	2	$5\sqrt{2}$ = 1 MARK
c)	$\dfrac{\sqrt{2}+1}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1} = 3+2\sqrt{2}$	2	$\frac{1}{2}$ OFF ANY ERROR
d)	After 60% discount 40% is paid $\therefore 0.4x = \$260$ $\therefore x = \$260 \div 0.4 = \$650$ Original premium = \$650	2	\$433.33 = 1 MARK 0 FOR INCREASING BY 40% OR 60%
e)	$\dfrac{4(a+2b)-3(2a-b)}{12}$ $= \dfrac{4a+8b-6a+3b}{12} = \dfrac{11b-2a}{12}$	2	$\frac{1}{2}$ OFF ANY ERROR
f)	$ab^2 - b + a^2b - a$ $= b(ab-1) + a(ab-1)$ $= (ab-1)(b+a)$	2	

Qn	Solutions	Marks	Comments+Criteria
2 i)	$A(-2,-1)\quad B(3,9)$ $\therefore$ gradient $l_2 = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{9+1}{3+2} = 2$	1	
(ii)	Slope $l_3 = -\frac{1}{2}$ since perpendicular to l_2 $\therefore$ using $y-y_1 = m(x-x_1)$ $y+1 = -\frac{1}{2}(x+2)$ $2y+2 = -x-2$ $\therefore x+2y+4=0$ is required equation	2	→ 1 MARK → 1 MARK.
(iii)	D solve $y=2x-7$ and $x+2y+4=0$ $\therefore x+2(2x-7)+4=0$ $x+4x-14+4=0$ $5x=10 \quad x=2$ $\therefore y=4-7=-3$ $\therefore$ D is $(2,-3)$	2	$x=2$ $1\frac{1}{2}$ MARKS $y=-3$ $\frac{1}{2}$ MARK
(iv)	length $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(3+2)^2+(9+1)^2} = \sqrt{125} = 5\sqrt{5}$	2	$\sqrt{125} =$ $1\frac{1}{2}$ MARKS
v)	$d = \left\lvert \dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}} \right\rvert = \left\lvert \dfrac{2\times 3-9-7}{\sqrt{2^2+1^2}} \right\rvert = \dfrac{10}{\sqrt{5}}$ $= \dfrac{10\sqrt{5}}{5}$ $= 2\sqrt{5}$	2	$\frac{1}{2}$ OFF ANY ERROR
vi)	substituting $(5,3)$ into $y=2x-7$ $3 = 2\times 5-7$ which is true Hence $(5,3)$ lies on line	1	

Qn	Solutions	Marks	Comments+Criteria

Question 2 (12 marks) **Start a new page** **Marks**



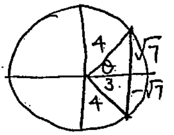
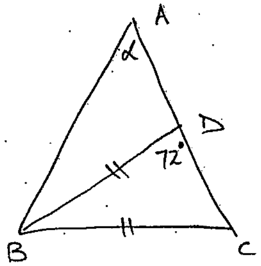
NOT TO SCALE

REQUIRED REGION

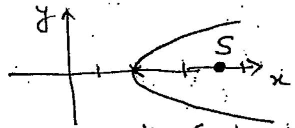
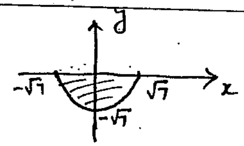
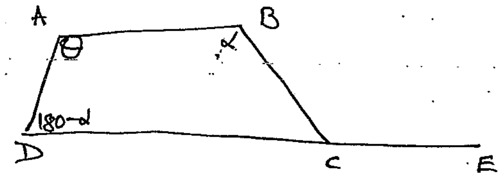
$\frac{1}{2}$ FOR EACH REGION

$\frac{1}{2}$ FOR FINAL ANSWER

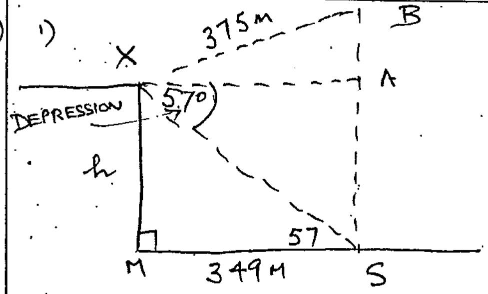
3

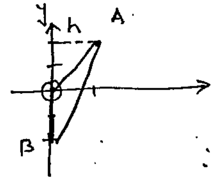
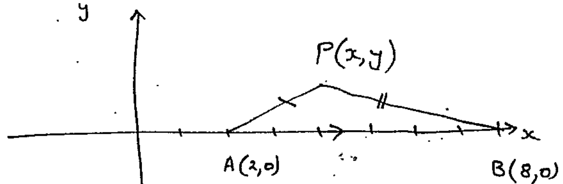
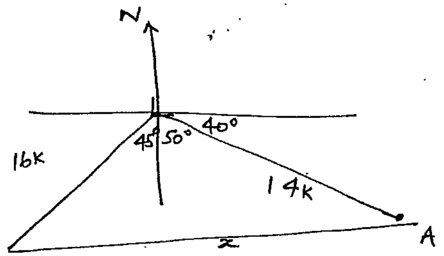
Qn	Solutions	Marks	Comments+Criteria
a)	$\lvert x+3\rvert > 2$ $\therefore x+3>2$ or $-(x+3)>2$ $x>-1$ or $-x>5$ $x<-5$ Hence $x>-1$ OR $x<-5$	2	$x>1$; $-x>5$ $1\frac{1}{2}$ MKS $x=-1$, $x=-5$ 1 MK
b)	 $\sin\theta = \frac{\sqrt{7}}{4}$ OR $-\frac{\sqrt{7}}{4}$	2	1 MARK EACH ANSWER
c)	$\triangle BCD$ is isosceles (given) Hence $\angle BCD = 72^\circ$ (base angles) But $\triangle ABC$ is also isosceles Hence $\angle ABC = 72^\circ = \angle ACB$ $\therefore$ In $\triangle ABC$ $\alpha = 180^\circ - (2\times 72^\circ)$ $= 36^\circ$	3	
d)	For one real solution $\Delta = 0$ $\therefore (k-6)^2 - 4\times 1\times 4 = 0$ $k^2 - 12k + 20 = 0$ $(k-2)(k-10) = 0 \Rightarrow k=2$ or $k=10$	2	1 MARK FOR $k^2-12k+20=0$
e)	If $y = x^2+5x-1$ and $y = 2x^2+3$ then $2x^2+3 = x^2+5x-1$ $\therefore x^2-5x+4=0$ $(x-4)(x-1)=0$ $\therefore x=4$ or $x=1$ and $y=35$ or $y=5$ by substitution	3	1 MARK OFF IF y values NOT FOUND

4

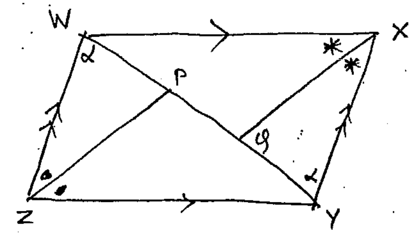
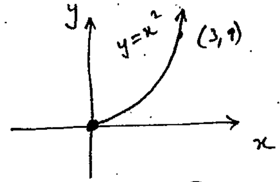
Qn	Solutions	Marks	Comments+Criteria
a)	$y^2 = 6(x-2)$ $\therefore$ Vertex is $(2,0)$ and focal length $= 6\div 4 = 1\frac{1}{2}$ Sketch Hence Focus is $(3\frac{1}{2}, 0)$	2	VERTEX $\frac{1}{2}$ MARK Focus $(0, 3\frac{1}{2})$ $1\frac{1}{2}$ MARKS
b)	i) LHS $= \sec A \sin A = \frac{1}{\cos A}\times \sin A$ $= \frac{\sin A}{\cos A} = \tan A = $ RHS ii) Hence $\sec A \sin A + 1 = 0$ becomes $\tan A + 1 = 0$ $\therefore \tan A = -1$ S A / T C $\therefore A = 135^\circ, 315^\circ, 135^\circ+360^\circ, 315^\circ+360^\circ$ $= 135^\circ, 315^\circ, 495^\circ, 675^\circ$	1 2	$\frac{1}{2}$ MARK
c)	i) Factors of x^3+y^3 are $(x+y)(x^2-xy+y^2)$ Hence $27+8a^3 = (3+2a)(9-6a+4a^2)$ ii) $-9m^2+64 = 64-9m^2 = (8-3m)(8+3m)$	1 1	
d)	i)  ii) $A = \frac{1}{2}\pi R^2$ $= \frac{1}{2}\times\pi\times(\sqrt{7})^2$ $= \frac{7\pi}{2}$ u^2	i) 1 ii) 1	$\frac{1}{2}$ MARK USING $\frac{1}{2}\pi R^2$
e)	angle sum $ABCD = 360^\circ$ If $\angle ABC + \angle ADC = 180^\circ$ then $\angle DAB + \angle BCD = 180^\circ$ Let $\angle DAB = \theta$ $\therefore \angle BCD = 180-\theta$ But $\angle BCD + \angle BCE = 180^\circ$ $\therefore \angle BCE =$	3	$\frac{1}{2}$ MARK OFF IF 360° NOT MENTIONED.

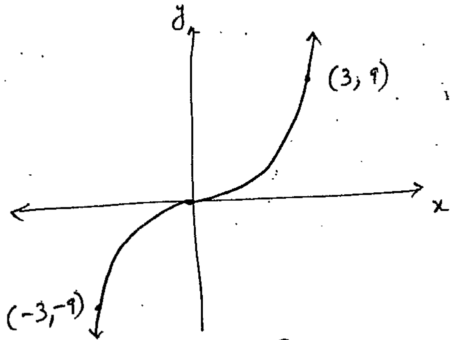
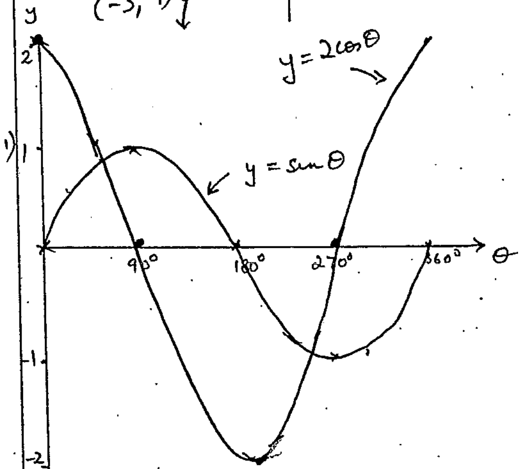
$180-(180-\theta) = \theta$
Hence $\angle BCE = \angle BAD$

Qn	Solutions	Marks	Comments+Criteria
5 a)	i) 	1	0 MARKS FOR FINDING XS
	ii) In $\triangle XSM$ $\angle XSM = 57°$ since it is alternate to the angle of depression with the horizontal lines being parallel Hence $\tan 57° = \frac{h}{349} \Rightarrow h = \underline{537.4\text{ m}}$ (1 d.p)	1	OR $\tan 33° = \frac{349}{h}$
	iii) BA can be found using Pythagoras in $\triangle BXA$ where $BA^2 = 375^2 - 349^2 \Rightarrow BA = 137.2\text{ m}$ Hence $BS = 137.2 + 537.4 = \underline{\underline{674.6\text{ m}}}$ (1dp)	2	$\frac{1}{2}$ MARK FINDING BA
b)	i) $y = 2x^2 + 3 - 6x^{-1}$ $\quad \frac{dy}{dx} = 4x + 6x^{-2}$ $= 4x + \frac{6}{x^2}$	2	
	ii) $y = (3x^2 - x)^4$ $\quad \frac{dy}{dx} = 4(3x^2-x)^3 \times (6x-1)$ $= 4(6x-1)(3x^2-x)^3$	2	$4(3x^2-x)^3 \times 6x = 1$ MARK
	iii) $y = \frac{4x^3+5}{2x-9}$ $\quad U = 4x^3+5 \quad U' = 12x^2$ $V = 2x-9 \quad V' = 2$ $\therefore dy/dx = \frac{VU' - UV'}{V^2} = \frac{(2x-9) \times 12x^2 - (4x^3+5)\times 2}{(2x-9)^2}$ $= \frac{16x^3 - 108x^2 - 10}{(2x-9)^2}$	2	$\frac{1}{2}$ MARK $\frac{1}{2}$ MARK $\frac{1}{2}$ MARK $\frac{1}{2}$ MARK
	iv) $y = 2x(4x+3)^{\frac{1}{2}}$ $\quad U = 2x \quad U' = 2$ $V = (4x+3)^{1/2} \quad V' = 2(4x+3)^{-\frac{1}{2}}$ $\frac{dy}{dx} = UV' + VU'$ $= \frac{2x \times 2}{\sqrt{4x+3}} + \sqrt{4x+3} \times 2 = \frac{4x}{\sqrt{4x+3}} + 2\sqrt{4x+3}$	2	$\frac{1}{2}$ MARK $\frac{1}{2}$ MARK

Qn	Solutions	Marks	Comments+Criteria
6 a)	i) To find m $dy/dx = 6x - 2 \Rightarrow f'(1) = 6 - 2 = 4$ (1) Using $y - y_1 = m(x - x_1)$ $y - 2 = 4(x-1)$ $y = 4x - 2$ as required (1)	2	
	ii) On y axis $x = 0$ $\therefore y = -2$ ($\frac{1}{2}$) $\quad$ B is $(0,-2)$ (1)	1	
	iii)  FOR $\triangle AOB$ Calling OB the base BASE = 2 HEIGHT = 1 $\therefore A = \frac{1}{2} \times 2 \times 1 = 1\text{ u}^2$ (1)	1	Some students used the lengthy method of finding perp. dist from (0,0) to $y = 4x - 2$ and base AB. may gain <u>some marks</u> & If correct result <u>full marks</u>.
b)	 Now $PA = \frac{1}{2} PB$ (1) $\therefore \sqrt{(x-2)^2 + y^2} = \frac{1}{2}\sqrt{(x-8)^2 + y^2}$ $\therefore x^2 - 4x + 4 + y^2 = \frac{1}{4}\left[x^2 - 16x + 64 + y^2\right]$ (1) $4x^2 - 16x + 16 + 4y^2 = x^2 - 16x + 64 + y^2$ $3x^2 + 3y^2 = 48$ ✓ (1) $x^2 + y^2 = 16$ ✓	3	-1 for not squaring the $\frac{1}{2}$ in $(\frac{1}{2}PB)^2$
c)	i) 	1	marks awarded even if 45° not marked on diagram of correct shape
	ii) Using cosine rule $x^2 = 16^2 + 14^2 - 2 \times 16 \times 14 \times \cos 95°$ $\Rightarrow x = 22.160\text{ KM}$	2	$-\frac{1}{2}$ for not correct accuracy.

Qn	Solutions	Marks	Comments+Criteria
6 d)	$f(x) = 2x-1$ when $x<2$ $\quad\;\; = -3x$ when $x \geqslant 2$ $\therefore f(-4) = 2\times -4-1 = -9$ (½) $f(0) = 2\times 0 - 1 = -1$ (½) $f(2) = -3\times 2 = -6$ (½) $\therefore f(-4) + f(0) + f(2) = -16$ (½)	2	

Qn	Solutions	Marks	Comments+Criteria
7 a)			
i)	Since opposite angles parm are equal then $\angle WZP = \angle YX\varphi$ [half of equal angles] In Δ's WZP and $YX\varphi$ $\angle ZWP = \angle XY\varphi$ (alternate and $WZ \parallel XY$) $WZ = XY$ (opp sides parm $=$) $\angle WZP = \angle YX\varphi$ [see above] $\therefore \Delta WZP \equiv \angle YX\varphi$ (AAS)	3	$\frac{1}{2}$ MARK $\frac{1}{2}$ MARK 1 MARK $\frac{1}{2}$ MARK $\frac{1}{2}$ MARK
ii)	Now $WP = \varphi Y$ (corresponding sides congruent triangles) $\therefore P\varphi = 20 - (8+8) = 4$ cm	1	$\frac{1}{2}$ MARK $\frac{1}{2}$ MARK
b)	$3x^2 + 8x + k = 0$ If solutions are 2α and $\frac{2}{\alpha}$ Product $= 2\alpha \times \frac{2}{\alpha} = 4$ But product $= \frac{c}{a} = \frac{k}{3}$ $\therefore \frac{k}{3} = 4 \Rightarrow k = 12$	2	$\frac{1}{2}$ MARK 1 MARK $\frac{1}{2}$ MARK
c) i)	 Range is $y \geqslant 0$	1	$\frac{1}{2}$ MARK GRAPH $\frac{1}{2}$ MARK RANGE

Qn	Solutions	Marks	Comments+Criteria
7 c ii)	 	1	
d i)	 	2	1 MARK EACH CURVE.
ii)	$2\cos\theta - \sin\theta = 0$ $\therefore\ 2\cos\theta = \sin\theta$ From the graph there are 2 points of intersection meaning 2 solutions OR $2\cos\theta = \sin\theta$ divide through by $\cos\theta$ $\tan\theta = 2$ If $0^\circ \leq \theta \leq 360^\circ$ there are 2 solutions to $\tan\theta = 2$ S A T C	2	