Student Number ____

St Catherine's School
Waverley, Sydney

HSC MID COURSE EXAMINATION 2010

Mathematics Extension 2

ASSESSMENT TASK 2 – HSC Course Weighting 30%

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.

Total marks (**81**)

- Attempt Questions 1-5.
- The value of all questions is indicated.
- Start a new answer booklet for each question.
- Put your Student Number and the Question Number(s) on the cover of each answer booklet.

TABLE OF STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}; \quad n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

Note: $\ln x = \log_e x, \quad x > 0$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Question 1 (18 marks) **Marks**

(a) Let $z = \dfrac{3+4i}{1+2i}$. Express z in the form $a+ib$, where a and b are real. **2**

(b) Let $\beta = 1 - i\sqrt{3}$.

(i) Express β in modulus-argument form. **1**

(ii) Hence write the exact value of β^{20} in the form $a+ib$, where a and b are real. **2**

(c) (i) On an Argand diagram shade the region where both $|z-(1+i)| \leq \sqrt{2}$ and $0 \leq \arg z \leq \dfrac{\pi}{2}$ hold. **2**

(ii) Find the exact perimeter and the exact area of the shaded region. **2**

(d) $z = 1+i$ is a root of the equation $z^3 + az^2 + bz + 6 = 0$, where a and b are real numbers.

(i) Find the values of a and b **2**

(ii) Hence find all the roots of the equation. **2**

(e) If $z = r(\cos\theta + i\sin\theta)$, prove that $\arg\left[\dfrac{z^2(3+3i)}{(1-i\sqrt{3})z}\right] = \theta + \dfrac{7\pi}{12}$. **2**

(f) Draw a neat sketch of the locus specified by $z\bar{z} + 2(z+\bar{z}) \leq 0$. **3**

Question 2 (16 marks) **Marks**

(a) Find the parametric coordinates of any point on the ellipse $9x^2 + 81y^2 = 144$. **2**

(b) Consider the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

(i) Find the eccentricity. **1**

(ii) Find the coordinates of the foci **1**

(iii) State the equations of the directrices. **1**

(iv) Show that $x = 4\cos\theta$ and $y = 3\sin\theta$ are the parametric equations of this ellipse **2**

(v) Sketch (neatly) the Ellipse and show clearly the position of the point P with coordinates $(4\cos\frac{\pi}{3}, 3\sin\frac{\pi}{3})$ **1**

(vi) Find the Cartesian equations of the tangent and normal to the Ellipse at the point P in part (v). **4**

(c) The equation $\dfrac{x^2}{4-k} + \dfrac{y^2}{2-k} = 1$ represents a conic section.

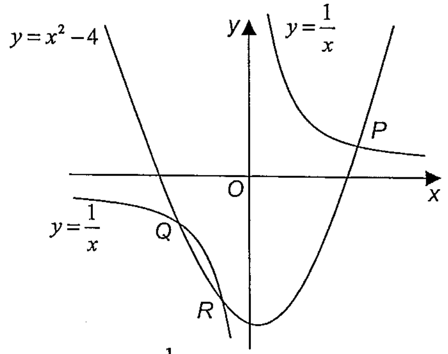
For what values of k is the curve;

(i) an ellipse? **2**

(ii) a hyperbola? **2**

Question 3 **(13 marks)** **Marks**

(a)



The curves $y=x^2-4$ and $y=\frac{1}{x}$ intersect at the points P, Q and R where $x=\alpha$, $x=\beta$ and $x=\gamma$ respectively.

(i) Show that α, β and γ are the roots of the equation $x^3-4x-1=0$. **1**

(ii) Find the polynomial equation with numerical coefficients which has roots α^2, β^2 and γ^2. **2**

(iii) Find the polynomial equation with numerical coefficients which has roots $\frac{1}{\alpha^2}$, $\frac{1}{\beta^2}$ and $\frac{1}{\gamma^2}$. **2**

(iv) Hence find the numerical value of $OP^2+OQ^2+OR^2$ **2**

(b) (i) Show that $\frac{x^2+6}{x^2+x-6}=1-\frac{x-12}{x^2+x-6}$ **1**

(ii) Hence express $\frac{x^2+6}{x^2+x-6}$ as the sum of partial fractions. **2**

(c) When the polynomial $P(x)$ is divided by $(x-3)$ and $(x-4)$ the remainders are 5 and 12 respectively. **3**

What is the remainder when $P(x)$ is divided by $(x-3)(x-4)$?

Question 4 (16 marks)

(a) A hyperbola has asymptotes $y=x$ and $y=-x$ and passes through the point $P(3, 2)$.

(i) Find the equation of the hyperbola **2**

(ii) Determine the eccentricity and the foci **2**

(iii) Find the equation of the tangent at P **2**

(b)

Diagram here

The distinct points $P\left(cp,\frac{c}{p}\right)$ and $Q\left(cq,\frac{c}{q}\right)$ are on the same branch of the hyperbola H with equation $xy=c^2$. The tangents to H at P and Q meet at the point T

(i) Show that the equation of the tangent at P is $x+p^2y=2cp$ **2**

(ii) Show that T has coordinates $\left(\frac{2cpq}{p+q},\frac{2c}{p+q}\right)$ **2**

(iii) Suppose P and Q move so that the tangent at P intersects the x axis at $(cq,0)$. **3**

Show that the locus of T is a hyperbola, and state its eccentricity

(c) Find the complex numbers z_1 and z_2 which satisfy the simultaneous equations below giving your answer in the form $a+ib$ (a, b real) **3**

$$z_1-iz_2=6-6i$$
$$2z_1+z_2=7-2i$$

Question 5 **(18 marks)** **Marks**

(a) Each term T_n, $n = 1, 2, 3, \ldots..$, of a sequence is given by $T_1 = 2$, $T_2 = 3$ and **4**

$T_n = 3T_{n-1} - 2T_{n-2}$ for all integers $n \geq 3$

Prove by Mathematical induction that $T_n = 2^{n-1} + 1$ for $n \geq 1$

(b) It is known that $\sin^{-1} x$, $\cos^{-1} x$ and $\sin^{-1}(1-x)$ are acute.

(i) Show that $\sin(\sin^{-1} x - \cos^{-1} x) = 2x^2 - 1$ **2**

(ii) Solve the equation $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(1-x)$ **2**

(c) If $P(x) = 1 - x + \frac{x^2}{2!} - \ldots + (-1)^n \frac{x^n}{n!}$, show that $P(x)$ has no multiple zero **3**

for $n \geq 2$

(d) Using De Moivre's theorem on the expression $(\cos\theta + i\sin\theta)^5$

(i) Show that $\tan 5\theta = \dfrac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$ **2**

(ii) Hence show that $\tan\frac{\pi}{5}$, $\tan\frac{2\pi}{5}$, $\tan\frac{3\pi}{5}$, $\tan\frac{4\pi}{5}$ are the roots of **2**

the equation $x^4 - 10x^2 + 5 = 0$

(e) Solve for x: $\tan^{-1} x + \tan^{-1}(1-x) = \tan^{-1}\frac{9}{7}$ **3**

MCE MATHEMATICS EXTENSION 2 SOLUTIONS 2010.

Qn	Solutions	Marks	Comments+Criteria

Q1

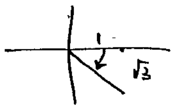
a) $z = \dfrac{3+4i}{1+2i} \times \dfrac{1-2i}{1-2i}$

$= \dfrac{3-6i+4i+8}{1+4}$

$= \dfrac{11}{5} - \dfrac{2}{5}i$

b) $\beta = 1 - i\sqrt{3}$

(i) $2\,\mathrm{cis}\,-\frac{\pi}{3}$



(ii) $\beta^{20} = \left[2\,\mathrm{cis}\left(-\frac{\pi}{3}\right)\right]^{20}$

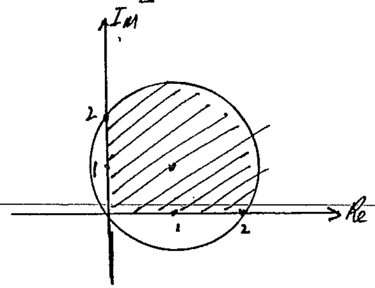
$= 2^{20}\,\mathrm{cis}\left(-\frac{20\pi}{3}\right)$

$= 2^{20}\,\mathrm{cis}\left(-\frac{2\pi}{3}\right)$

$= 2^{20}\left(\dfrac{-1-i\sqrt{3}}{2}\right)$

c).

(i)



(ii) Perimeter: $4 + \sqrt{2}\pi$

Area : $2 + \pi$

d) $z^3 + az^2 + bz + 6 = 0$

(i) roots are $1+i,\ 1-i,\ \alpha$

Sum of roots: $2 + \alpha = -a$

Prod of roots: $2\alpha = -6 \quad \therefore \alpha = -3$

$\therefore a = 1$

Sum of Prod 2x : $2 + (1+i).-3 + (1-i).-3 = b$

$\therefore b = -4$

(ii) Roots are $1+i,\ 1-i,\ -3$

Qn	Solutions	Marks	Comments+Criteria

Q1

e) $z = r(\cos\theta + i\sin\theta)$

LHS $= \arg\left[\dfrac{z^2(3+3i)}{(1-i\sqrt{3})z}\right]$

$= \arg z^2 + \arg(3+3i) - \arg(1-i\sqrt{3}) - \arg z$

$= 2\theta + \frac{\pi}{4} + \frac{\pi}{3} - \theta$

$= \theta + \frac{7\pi}{12}$

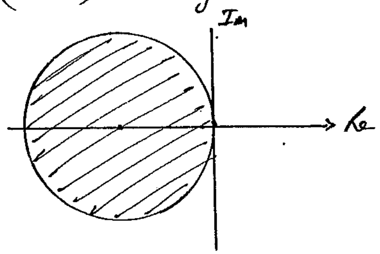
f). $z\bar{z} + 2(z+\bar{z}) \le 0$

$(x+iy)(x-iy) + 2(x+iy+x-iy) \le 0$

$x^2+y^2 \quad + 2(2x) \le 0$

$x^2+4x+4 \quad + y^2 \le 4$

$(x+2)^2 \quad + y^2 \le 4$



Qn	Solutions	Marks	Comments+Criteria

Q2 a) $9x^2+81y^2=144$

$\frac{x^2}{16}+\frac{9y^2}{16}=1$

i.e. $\frac{x^2}{16}+\frac{y^2}{\frac{16}{9}}=1$

$\therefore a=4 \quad b=\frac{4}{3}$

$\therefore x=4\cos\theta \quad y=\frac{4}{3}\sin\theta$

b) $\frac{x^2}{16}+\frac{y^2}{9}=1$

(i) $b^2=a^2(1-e^2)$

$9=16(1-e^2)$

$e^2=1-\frac{9}{16}$

$e=\frac{\sqrt{7}}{4}$

(ii) foci $(\pm\sqrt{7},0)$

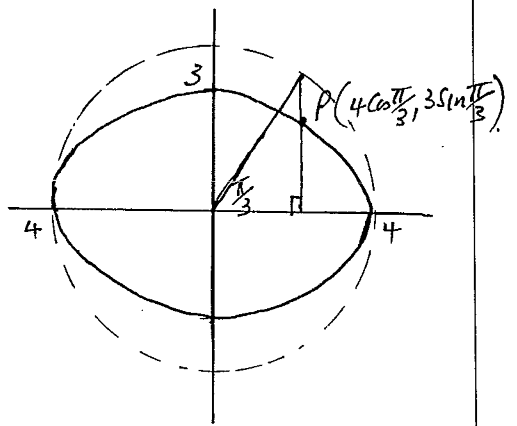
(iii) $x=\pm\frac{16\sqrt{7}}{7}$

(iv) $x=4\cos\theta \quad y=3\sin\theta$

$\cos\theta=\frac{x}{4} \quad \sin\theta=\frac{y}{3}$

$\cos^2\theta+\sin^2\theta=\frac{x^2}{16}+\frac{y^2}{9}=1$

(v)



Qn	Solutions	Marks	Comments+Criteria

(vi) $\frac{x^2}{16}+\frac{y^2}{9}=1$ / $x=4\cos\theta \quad y=3\sin\theta$

$P\left(2,\frac{3\sqrt{3}}{2}\right)$

Differentiating wrt x

$\frac{2x}{16}+\frac{2y}{9}\cdot\frac{dy}{dx}=0$

$\frac{dy}{dx}=-\frac{2x}{16}\cdot\frac{9}{2y}$

$=-\frac{9x}{16y}$

$\therefore$ gradient at P: $-\frac{18}{24\sqrt{3}}=-\frac{3}{4\sqrt{3}}=-\frac{3\sqrt{3}}{12}$

$=-\frac{\sqrt{3}}{4}$

$\therefore$ tangent at P: $y-\frac{3\sqrt{3}}{2}=-\frac{\sqrt{3}}{4}(x-2)$

$4y-6\sqrt{3}=-\sqrt{3}x+2\sqrt{3}$

OR $\sqrt{3}x+4y-8\sqrt{3}=0$

OR $3x+4\sqrt{3}y-24=0$

OR Method 2 using $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

$\frac{2x}{16}+\frac{3\sqrt{3}y}{18}=1$

$\frac{x}{8}+\frac{\sqrt{3}y}{6}=1$

$3x+4\sqrt{3}y=24$

$3x+4\sqrt{3}y-24=0$

Qn	Solutions	Marks	Comments+Criteria
	Gradient of Normal: $\frac{4\sqrt{3}}{3}$		
	$y - \frac{3\sqrt{3}}{2} = \frac{4\sqrt{3}}{3}(x-2)$		
	$6y - 9\sqrt{3} = 8\sqrt{3}x - 16\sqrt{3}$		
	$\underline{8\sqrt{3}x - 6y - 7\sqrt{3} = 0}$		
	<u>Method 2</u> using $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$		
	$\frac{16x}{2} - \frac{18y}{3\sqrt{3}} = 7$		
	$8x - \frac{6y}{\sqrt{3}} = 7$		
	$8\sqrt{3}x - 6y - 7\sqrt{3} = 0$		
c)	$\frac{x^2}{4-k} + \frac{y^2}{2-k} = 1$		
	(i) for ellipse $4-k>0$ $k<4$		
	<u>and</u> $2-k>0$ $k<2$		
	$\therefore$ for ellipse $k<2$		
	(ii) for hyperbola $4-k>0$		
	<u>and</u> $2-k<0$		
	$\therefore$ $k<4$ and $k>2$		
	$\therefore$ for hyperbola $2<k<4$		
	[<u>Note</u>: there are no values of k for which $4-k<0$ and $2-k>0$]		

Qn	Solutions	Marks	Comments+Criteria
<u>Q3</u>	a)(i) $y = x^2 - 4$ $y = \frac{1}{x}$		
	Solving simultaneously		
	$\frac{1}{x} = x^2 - 4$		
	$\therefore x^3 - 4x - 1 = 0$ roots α, β, γ		
	(ii) let $y = x^2$ $\therefore x = \pm\sqrt{y}$		
	$(\pm\sqrt{y})^3 - 4(\pm\sqrt{y}) - 1 = 0$		
	$\pm\left[(\sqrt{y})^3 - 4\sqrt{y}\right] = 1$		
	Squaring both sides		
	$\underline{y^3 - 8y^2 + 16y - 1 = 0}$		
	(iii) let $y = \frac{1}{x^2}$ $\therefore x = \pm\frac{1}{\sqrt{y}}$		
	$\left(\pm\frac{1}{\sqrt{y}}\right)^3 - 4\left(\pm\frac{1}{\sqrt{y}}\right) = 1$		
	$\pm\left[\left(\frac{1}{\sqrt{y}}\right)^3 - 4\left(\frac{1}{\sqrt{y}}\right)\right] = 1$		
	Squaring both sides		
	$\frac{1}{y^3} - \frac{8}{y^2} + \frac{16}{y} = 1$		
	$\times y^3$ $1 - 8y + 16y^2 = y^3$		
	$\therefore$ polynomial is		
	$\underline{y^3 + 8y - 16y^2 - 1 = 0}$		
	or $\underline{y^3 - 16y^2 + 8y - 1 = 0}$		

Qn	Solutions	Marks	Comments+Criteria

Now $P\left(\alpha, \frac{1}{\alpha}\right) \therefore OP^2 = \alpha^2 + \frac{1}{\alpha^2}$

$Q\left(\beta, \frac{1}{\beta}\right) \therefore OQ^2 = \beta^2 + \frac{1}{\beta^2}$

$R\left(\gamma, \frac{1}{\gamma}\right) \therefore OR^2 = \gamma^2 + \frac{1}{\gamma^2}$

$\therefore OP^2 + OQ^2 + OR^2 = \alpha^2 + \beta^2 + \gamma^2 + \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\right)$

$= 8 + 16$

$= 24$

b) (i) $RHS = 1 - \frac{x-12}{x^2+x-6}$

$= \frac{x^2+x-6-x+12}{x^2+x-6}$

$= \frac{x^2+6}{x^2+x-6} = LHS$

(ii) $\therefore \frac{x^2+6}{x^2+x-6} = 1 - \left[\frac{A}{x+3} + \frac{B}{x-2}\right]$

$= 1 - \left[\frac{A(x-2) + B(x+3)}{x^2+x-6}\right]$

$\therefore A(x-2) + B(x+3) = x - 12$

let $x=2$: $5B = -10$, $B = -2$

let $x=-3$: $-5A = -15$, $A = 3$

$\therefore \frac{x^2+6}{x^2+x-6} = 1 - \frac{3}{x+3} + \frac{2}{x-2}$

Qn	Solutions	Marks	Comments+Criteria

c) $P(x) = (x-3)(x-4)\,Q(x) + (ax+b)$

$P(3) = 3a + b = 5$ —— ①

$P(4) = 4a + b = 12$ —— ②

$a = 7$

$b = -16$

$\therefore$ remainder is $(7x - 16)$

Q4 a) rectangular hyperbola of form

(i) $x^2 - y^2 = a^2$ passes through $P(3,2)$

$\therefore 9 - 4 = a^2$

$a^2 = 5$

(ii) eccentricity $= \sqrt{2}$ $\quad a = \sqrt{5}$

foci $(\pm ae, 0) \Rightarrow (\pm\sqrt{10}, 0)$

directrices $x = \pm\frac{a}{e} \Rightarrow x = \pm\sqrt{\frac{5}{2}}$

(iii) $x^2 - y^2 = 5$

$2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{y}$

at P $\frac{dy}{dx} = \frac{3}{2}$

$\therefore$ equation of tangent at P is:

$y - 2 = \frac{3}{2}(x - 3)$

$2y - 4 = 3x - 9$

$3x - 2y - 5 = 0$

Qn	Solutions	Marks	Comments+Criteria
Q4	b)(i) $P\left(cp, \frac{c}{p}\right)$ $\quad xy = c^2$		

differentiating: $y + x\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\frac{y}{x}$$

$$= -\frac{1}{p^2}$$

equation of tangent at $P\left(cp, \frac{c}{p}\right)$

$$y - \frac{c}{p} = -\frac{1}{p^2}(x - cp)$$

$$p^2y - cp = -x + cp$$

$$\underline{x + p^2y = 2cp} \;\; ''$$

(ii) intersecting tangents:

$$x + p^2y = 2cp \quad \text{—— ①}$$

$$x + q^2y = 2cq \quad \text{—— ②}$$

① - ② $\quad y(p^2 - q^2) = 2c(p - q)$

$$y = \frac{2c}{p+q}$$

Sub in ① $\quad x + \frac{2cp^2}{p+q} = 2cp$

$$(p+q)x + \cancel{2cp^2} = \cancel{2cp^2} + 2cpq$$

$$x = \frac{2cpq}{p+q}$$

$$\therefore T\left(\frac{2cpq}{p+q}, \frac{2c}{p+q}\right).$$

Qn	Solutions	Marks	Comments+Criteria
Q4	b) iii) tangent at P: $x + p^2y = 2cp$		

passes through $(cq, 0)$

$$\therefore \quad cq = 2cp \Rightarrow q = 2p$$

now at T $\quad x = \frac{2cpq}{p+q} \quad y = \frac{2c}{p+q}$

but $q = 2p$

$$\therefore x = \frac{4cp^2}{3p} \Longrightarrow x = \frac{4cp}{3} \quad \text{—— ①}$$

$$y = \frac{2c}{3p} \quad \text{—— ②}$$

① × ② $\quad xy = \frac{8c^2}{9}$

$\therefore$ locus of T is hyperbola (rectangular) with eccentricity $\sqrt{2}$

c) $\quad z_1 - iz_2 = 6 - 6i \quad \text{—— ①}$

$$2z_1 + z_2 = 7 - 2i \quad \text{—— ②}$$

① × 2 $\quad 2z_1 - 2iz_2 = 12 - 12i \quad \text{—— ③}$

② - ③ $\quad (1 + 2i)z_2 = -5 + 10i$

$$z_2 = \frac{-5 + 10i}{1 + 2i} \times \frac{1 - 2i}{1 - 2i}$$

$$= \frac{-5 + 10i + 10i + 20}{5}$$

$$\therefore z_2 = 3 + 4i$$

Sub in ① $\quad z_1 = 6 - 6i + i(3 + 4i)$

$$\underline{z_1 = 2 - 3i}$$

Qn	Solutions	Marks	Comments+Criteria

Q5 (a) $T_1 = 2 \quad T_2 = 3 \quad T_n = 3T_{n-1} - 2T_{n-2}$

$$\left(T_n = 2^{n-1} + 1\right)$$

1. for $n=1$ $\quad T_1 = 1+1 = 2$ true
   for $n=2$ $\quad T_2 = 2+1 = 3$ true
   $\therefore$ true for $n=1$ and $n=2$

2. assume true for $n=k$ and $n=k-1$, $k \geq 3$
   ie assume $T_k = 2^{k-1}+1 \quad T_{k-1} = 2^{k-2}+1$

3. Aim to prove true for $n=k+1$
   ie. A.T.P. $T_{k+1} = 2^k + 1$

now $T_{k+1} = 3T_k - 2T_{k-1}$

$= 3\left(2^{k-1}+1\right) - 2\left(2^{k-2}+1\right)$

$= 3.2^{k-1} + 3 - 2.2^{k-1}.\frac{1}{2} - 2$

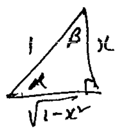
$= 3.2^{k-1} + 3 - 2^{k-1} - 2$

$= 2.2^{k-1} + 1$

$= 2^k + 1$

4. Therefore true for $n=k+1$ if true for $n=k$. Since true for $n=1$ and $n=2$ then true for $n = 3, 4, 5, \ldots\ldots$ and by MI true for all $n \geq 1$

b) $\sin^{-1}x = \alpha \qquad \cos^{-1}x = \beta$



now $\sin\left(\sin^{-1}x - \cos^{-1}x\right)$

$= \sin(\alpha - \beta)$

$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= x^2 - \sqrt{1-x^2}.\sqrt{1-x^2}$

$= 2x^2 - 1$

Qn	Solutions	Marks	Comments+Criteria

Q5 b)(ii) $\sin^{-1}x - \cos^{-1}x = \sin^{-1}(1-x)$

$\therefore \quad 1 - x = \sin\left(\sin^{-1}x - \cos^{-1}x\right)$

$\therefore \quad 1 - x = 2x^2 - 1$ from (i)

$2x^2 + x - 2 = 0$

$x = \frac{-1 \pm \sqrt{17}}{2}$

c) $P(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} \ldots + (-1)^n \frac{x^n}{n!}$

$P'(x) = -1 + x - \frac{x^2}{2!} + \ldots + (-1)^n \frac{x^{n-1}}{(n-1)!}$

let $x = \alpha$ be a multiple root

$\therefore P'(\alpha) = 0$ and $P(\alpha) = 0$

now $P(\alpha) = 1 - \alpha + \frac{\alpha^2}{2!} - \frac{\alpha^3}{3!} + . + (-1)^n \frac{\alpha^{n-1}}{n+1!} + \frac{(-1)^n\alpha^n}{n!}$

$\therefore P(\alpha) = -P'(\alpha) + \frac{(-1)^n\alpha^n}{n!}$

this implies $\frac{(-1)^n\alpha^n}{n!} = 0$

which is only true if $\alpha = 0$

but $\alpha = 0$ is not a root

$\therefore$ there are no multiple roots.

d)(i) $(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$

also $(c + is)^5 = c^5 + 5c^4si - 10c^3s^2 - 10c^2s^3i + 5cs^4 + s^5i$

where $c = \cos\theta \quad s = \sin\theta$

$\therefore \frac{\sin 5\theta}{\cos 5\theta} = \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta}$ — *

$\therefore \tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$ — dividing * by $\cos^5\theta$

Qn	Solutions	Marks	Comments+Criteria
Q5	d)(ii) $\tan 5\theta = \dfrac{\tan\theta(5-10\tan^2\theta+\tan^4\theta)}{1-10\tan^2\theta+5\tan^4\theta}$ $\tan 5\theta = 0 \Rightarrow \theta = \frac{k\pi}{5}$ $\theta = \frac{k\pi}{5}$ $k=0,1,2,3,4$ $\therefore$ Solutions of $5-10\tan^2\theta+\tan^4\theta=0$ are $\frac{k\pi}{5}$ where $k=1,2,3,4$ letting $x=\tan\theta$ Solution of $5-10x^2+x^4=0$ are $\tan\frac{k\pi}{5}$ where $k=1,2,3,4$ ie $\tan\frac{\pi}{5}, \tan\frac{2\pi}{5}, \tan\frac{3\pi}{5}, \tan\frac{4\pi}{5}$		
	e) $\tan^{-1}x + \tan^{-1}(1-x) = \tan^{-1}\frac{9}{7}$ let $\alpha = \tan^{-1}x$ $\therefore x = \tan\alpha$ let $\beta = \tan^{-1}(1-x)$ $\therefore (1-x) = \tan\beta$ tan of both sides $\tan[\tan^{-1}x + \tan^{-1}(1-x)] = \tan(\tan^{-1}\frac{9}{7})$ $\therefore \tan(\alpha+\beta) = 9/7$ $\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{9}{7}$ $\therefore \dfrac{\cancel{x}+1-\cancel{x}}{1-x(1-x)} = \dfrac{9}{7}$ $\therefore 9-9x+9x^2=7$		

Qn	Solutions	Marks	Comments+Criteria
5e)	cont $9x^2-9x+2=0$ $(3x-1)(3x-2)=0$ $\therefore x = \frac{1}{3}, \frac{2}{3}$ END		