Student Number

St. Catherine's School
Waverley

2010

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 1 – 15%
TUESDAY 16th FEBRUARY2010

Mathematics

General Instructions

- Working time – 55 minutes
- Write using black or blue pen
- Board approved calculators may be used
- All necessary working should be shown
- Start each question in a new booklet

Total marks - 48

Attempt questions 1-3
All questions are of equal value

TABLE OF STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}; \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

Note: $\ln x = \log_e x, \quad x > 0$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Total marks – 48
Attempt Questions 1-3
All questions are of equal value

Begin each question in a NEW booklet. Extra writing booklets are available.

Question 1 (16 Marks) **Marks**

(a) For the parabola, $(x+1)^2 = 32(y-3)$, write down:

(i) the coordinates of the vertex — 1

(ii) the focal length — 1

(iii) the equation of the directrix — 1

(iv) the coordinates of the focus — 1

(v) the equation of the line through the focus parallel to the x- axis. (i.e. the latus rectum). — 1

(b) A parabola has equation $y^2 - 12x = 24$.

(i) Express this equation in the form $(y-k)^2 = 4a(x-h)$. — 2

(ii) Find the coordinates of all points of intersection of the parabola with the x and y axes. — 3

(c) (i) Show that the equation of the locus of a point that moves so that its distance from the point $A(2,1)$ is twice its distance from the point $(-4,-5)$ is:

$$x^2 + 12x + y^2 + 14y = -53.$$ — 3

(ii) Describe the locus geometrically. — 3

Question 2 (16 Marks) **Start a NEW booklet.** **Marks**

(a) Differentiate with respect to x:

(i) $x^3 - 6x + 8$ — 1

(ii) $(1 - x + x^2)^8$ — 1

(iii) $\dfrac{1}{7x^2}$ — 1

(iv) $\dfrac{1}{(2-9x)^4}$ — 2

(v) $\dfrac{x^2}{5-x}$ — 2

(b) Show that $\dfrac{d}{dx}x(x+1)^3 = (x+1)^2(4x+1)$ — 2

(c) Use the definition of the derivative $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

to find the derivative of the function $f(x) = x^2 + 1$. — 2

(d) (i) Find the gradient of the tangent to the curve $y = \sqrt{5-x^2}$ at $x = 1$. — 2

(ii) Hence, find the equation of the tangent at $x = 1$. — 2

(iii) Find the point on the curve where the tangent has gradient 0. — 1

Question 3 (16 Marks) **Start a NEW booklet.** **Marks**

(a) Evaluate $\sum_{n=2}^{5} n^2 - 1$. **1**

(b) An author writes a manuscript, so that on the first day she writes 54 pages, on the second day 36 pages and on each day after, she writes $\frac{2}{3}$ the number of pages of the preceding day.

(i) How many pages does she write on the 6^{th} day? **1**

(ii) What is the maximum number of pages she will write? **1**

(c) The third term of an arithmetic series is 7 and the seventh term is 31. Find the 50^{th} term. **3**

(d) For what values of x does the geometric series $(5+x)+(5+x)^2+(5+x)^3+\ldots$ have a limiting sum? **2**

(e) A car dealership has a car for sale for a cash price of \$20 000. It can also be bought on terms over three years where interest is charged at the rate of 1% per month on the balance owing for that month. Repayments are to be made in equal monthly instalments of \$$M$ with the first repayment applied at the end of the first month. A customer agrees to buy the car on these terms.

Let \$$A_n$ be the amount owing at the end of the nth month.

(i) Find an expression for A_1. **1**

(ii) Show that $A_2 = 20000(1.01)^2 - M(1+1.01)$. **2**

(iii) Find an expression for A_{36}. **1**

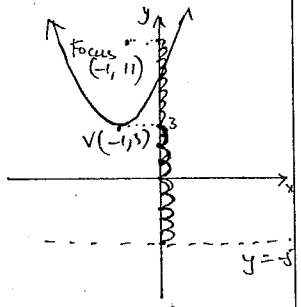
(iv) Find the value of M. **3**

(v) How much interest has the customer paid over the three years? **1**

End of paper

St Catherine's School – HSC 2 unit assessment task 1 – FEB 2010

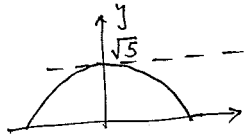
Qn	Solutions	Marks	Comments: Criteria
1 (a)	$(x+1)^2 = 32(y-3)$		
(i)	Vertex $(x-h)^2 = 4a(y-k)$ $V(h,k) = (-1, 3)$ ✓	1	
(ii)	Focal length $4a = 32$ $a = 8$ ✓ $\therefore$ focal length $= 8$ units	1	
(iii)	Equation of the directrix is $y = -5$ ✓	1	* note: −½ if equation of directrix is not written in correct form
(iv)	Focus $(-1, 11)$ ✓ ✓	1	
(v)	Equation of latus rectum is $y = 11$ ✓	1	
(b) (i)	$y^2 = 12x = 24$ $y^2 = 24 + 12x$ ✓ $y^2 = 12(2+x)$ $y^2 = 12(x+2)$ ✓ $(y-0)^2 = 4 \cdot 3(x+2)$ ✓	2	✓ means 1 mark ✗ means ½ mark
(ii)	For the intersection with the * x-axis, let $y=0$ $0^2 = 12(x+2)$ $12x = -24$ ✓ $x = -2$ * y-axis, let $x=0$ $y^2 = 12(0+2)$ ✓ $y^2 = 24$ $y = \pm\sqrt{24} = \pm 2\sqrt{6}$ ✓ $(-2,0)$ ✓, $(0, 2\sqrt{6})$ and ✓ $(0, -2\sqrt{6})$ are the pts of intersection	3	



Qn	Solutions	Marks	Comments: Criteria
(c) (i)	$PA = 2PB$ $\left(\sqrt{(x-2)^2+(y-1)^2}\right)^2 = \left(2\sqrt{(x+4)^2+(y+5)^2}\right)^2$ ✓ $(x-2)^2+(y-1)^2 = 4\left[(x+4)^2+(y+5)^2\right]$ ✓ $x^2-4x+4+y^2-2y+1 = 4(x^2+8x+16+y^2+10y+25)$ ✓ $x^2-4x+y^2-2y+5 = 4(x^2+8x+y^2+10y+41)$ $x^2-4x+y^2-2y+5 = 4x^2+32x+4y^2+40y+164$ ✓ $3x^2+36x+3y^2+42y+159 = 0$ ÷3 ✓ $x^2+12x+y^2+14y+53 = 0$ $x^2+12x+y^2+14y = -53$ as required	3	✓ means 1 mark ✗ means ½ mark
(ii)	$x^2+12x+\square+y^2+14y+\square = -53$ Completing the square $x^2+12x+6^2+y^2+14y+7^2 = -53+6^2+7^2$ $(x+6)^2+(y+7)^2 = 32$ $\therefore$ The locus is a circle with centre $(-6, -7)$ ✓ ✓ and radius $= \sqrt{32}$ $= \sqrt{16\times 2}$ $= 4\sqrt{2}$ units ✓	3	realising that completing the square is necessary and is correct 1.5 mark 1.5 mark for centre and radius and stating that it is a circle * coe means carried on error

Qn	Solutions	Marks	Comments: Criteria
2 (a)	i) $\frac{d}{dx}\left(x^3-6x+8\right) = 3x^2-6$	i) 1	
	ii) $\frac{d}{dx}\left(1-x+x^2\right)^8 = 8\left(1-x+x^2\right)^7(2x-1)$	ii) 1	
	iii) $\frac{1}{7x^2} = \frac{x^{-2}}{7}$ $\therefore \frac{dy}{dx} = \frac{-2x^{-3}}{7} = \frac{-2}{7x^3}$	iii) 1	$\frac{1}{7x^2} \neq 7x^{-2}$ NO MARKS IF THIS ERROR MADE
	iv) $y = (2-9x)^{-4}$ $\therefore \frac{dy}{dx} = -4(2-9x)^{-5} \times -9$ $= 36(2-9x)^{-5}$ $= \frac{36}{(2-9x)^5}$	iv) 2	TRY TO WRITE ANSWER IN SAME FORM AS QUESTION e.g. $y = \frac{1}{(2-9x)^4}$ $\frac{dy}{dx} = \frac{36}{(2-9x)^5}$ not $36(2-9x)^{-5}$ $\frac{1}{2}$ OFF FOR -36
	v) $y = \frac{x^2}{5-x}$ $u = x^2$ $u' = 2x$ $v = 5-x$ $v' = -1$ $\frac{dy}{dx} = \frac{vu'-uv'}{v^2} = \frac{(5-x)\cdot 2x - x^2(-1)}{(5-x)^2}$ $= \frac{10x-2x^2+x^2}{(5-x)^2}$ $= \frac{10x-x^2}{(5-x)^2}$	v) 2	
	vi) $y = x(x+1)^3$ $u = x$ $u' = 1$ $v = (x+1)^3$ $v' = 3(x+1)^2$ $\frac{dy}{dx} = uv' + vu' = x\cdot 3(x+1)^2 + (x+1)^3$ extract H.C.F $(x+1)^2$ $= (x+1)^2(3x+x+1)$ $= (x+1)^2(4x+1)$ Q.E.D.	vi) 2	1 MARK FOR dy/dx 1 MARK FOR CORRECTLY OBTAINING REQUIRED ANSWER

Qn	Solutions	Marks	Comments: Criteria
2c)	$f(x) = x^2+1$ $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0} \frac{(x+h)^2+1-(x^2+1)}{h}$ $= \lim_{h\to 0} \frac{x^2+2xh+h^2+1-x^2-1}{h}$ $= \lim_{h\to 0} \frac{2xh+h^2}{h}$ $= \lim_{h\to 0} (2x+h)$ $= 2x$	2	Correct setting out often did not happen. $\lim_{h\to 0}$ must appear until last line.
d)	i) $y = \sqrt{5-x^2} = (5-x^2)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}(5-x^2)^{-\frac{1}{2}} \times -2x$ $= \frac{-x}{\sqrt{5-x^2}}$ when $x=1$ $\frac{dy}{dx} = \frac{-1}{\sqrt{5-1}} = -\frac{1}{2}$ $\therefore$ gradient tangent $= -\frac{1}{2}$	i) 1 2	1 mark for $\frac{dy}{dx}$ 1 mark for correct substitution substitution into an incorrect but simpler $dy/dx = \frac{1}{2}$ MK

Qn	Solutions	Marks	Comments: Criteria
2 (d)	(ii) By substitution $y=2$ when $x=1$ Hence $y-y_1 = m(x-x_1) \Rightarrow$ $y-2 = -\frac{1}{2}(x-1)$ $2y-4 = -x+1$ $x+2y-5=0$ is required equation	(ii) 2	
	(iii) Let $\frac{dy}{dx}=0$ Now $\frac{dy}{dx} = \frac{-x}{\sqrt{5-x^2}}$ $\therefore \frac{-x}{\sqrt{5-x^2}} = 0$ $\therefore x=0$ by substitution $y=\sqrt{5}$ Hence $(0,\sqrt{5})$ is point	(iii) 1	a large number of students had difficulty solving $\frac{-x}{\sqrt{5-x^2}}=0$ * Remember if a fraction is zero then the numerator is zero. N.B. Two students very cleverly saw that $y=\sqrt{5-x^2}$ has a semi circle as its graph radius $=\sqrt{5}$  Hence horizontal tangent is at $(0,\sqrt{5})$

Question 3 (16 marks)

(a) $\sum_{n=2}^{5} n^2-1$

$= 3+8+15+24$

$= 50$

(a) 1

$\frac{1}{2}$ off if not summed.

(b) $54+36+24+\ldots$

GP, where $r=\frac{2}{3}$, $a=54$, $n=6$.

(i) $T_n = ar^{n-1}$

$T_6 = 54\left(\frac{2}{3}\right)^5$

$= \frac{64}{9}$

$= 7\frac{1}{9}$

(b)(i) 1

If not GP formula
No MARK

Do not give the answer 7.1 pages or 8 pages
The number of pages is $7\frac{1}{9}$

(ii) $S_\infty = \frac{a}{1-r}$

$= \frac{54}{\frac{1}{3}}$

$= 162$

$\therefore$ A maximum of 162 pages will be written

(ii) 1

(c) $T_3 = 7 \quad \therefore a+2d=7$ ①

$T_7 = 31 \quad \therefore a+6d=31$ ②

② − ① $4d = 24$

$\therefore d = 6$

Sub. $d=6$ into ①: $a+12=7$

$\therefore a=-5$

$\therefore T_{50} = -5+49(6)$

$= 289$

(c) 3

(d) 2

½ off for {incorrect / neglecting} sign

eg $-6 < x < 4$

as long as working is correct

(a) For a limiting sum to exist, $|r| < 1$.

i.e. $-1 < r < 1$.

$\therefore -1 < 5 + x < 1$

$\therefore -6 < x < -4$

(e) (i) Use, $A_n = P(1+r)^n - M$

$\therefore A_1 = 20000(1+0.01)^1 - M$

$\therefore A_1 = 20000(1.01) - M$

(e) (i) 1

(ii) $A_2 = A_1(1.01)^1 - M$

$= [20000(1.01) - M](1.01) - M$

$= 20000(1.01)^2 - 1.01M - M$

$= 20000(1.01)^2 - M(1 + 1.01)$, as required.

(ii) 2

(iii) $A_{36} = 20000(1.01)^{36} - M(1 + 1.01 + \ldots + 1.01^{35})$

(iii) 1 ½ off if power 36 on last term.

(iv) Let $A_{36} = 0$ (since car will be paid off)

$\therefore M(1 + 1.01 + \ldots + 1.01^{35}) = 20000(1.01)^{36}$

$M = \dfrac{20000(1.01)^{36}}{1 + 1.01 + 1.01^2 + \ldots + 1.01^{35}}$

(iv) 3

Denominator is a GP, $a = 1$, $r = 1.01$, $n = 36$

$\therefore S_{36} = \dfrac{1(1.01^{36} - 1)}{0.01}$

$\therefore M = 20000(1.01)^{36} \cdot \dfrac{0.01}{1.01^{36} - 1}$

$\therefore M = \$664.29$

(v) Total of repayments $= 36 \times \$664.29$

$= \$23914.30$ (44).

$\therefore$ Interest $= \$23914.30 - \$20000$

$= \$3914.30$ (44)

(v) 1

23914.30 or 23914.44 accepted

also 3914.30 3914.44

.30 is more accurate answer

7. — End of paper —