Student Name:____________

St Catherine's School

Waverley

2014

HSC ASSESSMENT TASK 1

Weighting 15%

Extension 1 Mathematics

Reading Time:	5 minutes
Time allowed:	55 minutes
Total marks:	50 marks

INSTRUCTIONS

SECTION 1 – Multiple Choice (Questions 1-5) (5 marks)

- Answer either A, B, C or D to each question on the answer sheet provided.

SECTION 2 – Written Response (Questions 6 and 7) (45 marks)

- Marks for each part of a question are indicated.
- The two questions are of not of equal value.
- Both questions should be attempted in the booklets provided.
- All necessary working should be shown in each question.
- Start each question in a new booklet and clearly label all parts of a question.
- Approved scientific calculators may be used.
- Diagrams should be drawn using pencil and ruler.

Section A

Multiple choice (5 marks)

Place your answer on the multiple choice answer sheet provided.

1) The polynomial $P(x) = x^3 + 6x^2 - 8x + k$ has a factor of $(x+2)$.

What is the value of k?

a) -32 b) -16 c) 16 d) 0

2) *If* $\tan\theta = \frac{3}{4}$ and $0° < \theta < 90°$, then the value of $\sin 2\theta$ is:

a) $-\frac{3}{5}$ b) $-\frac{24}{25}$ c) $\frac{2\sqrt{7}}{13}$ d) $\frac{24}{25}$

3) $P(x)$ and $Q(x)$ are polynomials of degree m and n respectively, where $m > n$.

The degree of the polynomial $P(x) + Q(x)$ is equal to

a) mn b) $m+n$ c) m d) n

4) $\sin x \cos x \cos 2x =$

a) $\sin x \cos 3x$ b) $\frac{\cos 3x}{2}$ c) $\frac{\sin 4x}{4}$ d) $\tan^2 2x$

5) The exact value of $\tan 75°$ is

a) $2+\sqrt{3}$ b) $\frac{3+\sqrt{3}}{3}$ c) $\sqrt{3}-1$ d) $\frac{3-2\sqrt{2}}{2}$

Section B (45 marks)

Question 6 (22 marks) — marks

a) i) It is given that $(x-3)^2$ is a factor of $P(x) = x^4 - 5x^3 + x^2 + 21x - 18$. Use long division to find its other factors and hence fully factorise $P(x)$. **2**

ii) Draw a sketch of $P(x) = x^4 - 5x^3 + x^2 + 21x - 18$. (You will not need a scale on the y-axis) **2**

iii) Hence solve the inequality $x^4 - 5x^3 + x^2 + 21x - 18 \leq 0$ **2**

b) Let $P(x) = (x-2)(x+3)Q(x) + a(x-2) + b$ where $Q(x)$ is a polynomial and a and b are real numbers.

When $P(x)$ is divided by $(x-2)$ the remainder is 11. When $P(x)$ is divided by $(x+3)$ the remainder is -4.

i) What is the value of b? **1**

ii) What is the remainder when $P(x)$ is divided by $(x-2)(x+3)$? **2**

c) The polynomial equation $2x^3 - 5x^2 + 9x - 12 = 0$ has roots $x = \alpha, \beta, \gamma$.

Find the value of:

i) $\alpha + \beta + \gamma$ **1**

ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **2**

iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

d) i) Show that $x^3 - 7x + 2 = 0$ has a root that lies between $x = 2$ and $x = 3$. **1**

ii) By taking $x = 2.5$ as a first approximation to this root of $x^3 - 7x + 2 = 0$, use Newton's method once to find a second approximation to this root. Give your answer correct to 4 significant figures. **2**

e) It is given that two of the roots of $3x^3 + kx^2 - 37x + 12 = 0$ are reciprocals of each other.

i) Find the value of the other root. **1**

ii) Find the value of k. **1**

iii) Find all the roots of the polynomial equation. **3**

Question 7 (23 marks)

a) Prove by mathematical induction that:

$8^n - 3^n$ is divisible by 5 for all positive integers n, i.e. when $n \geq 1$. **4**

b) If $\cos\alpha = \frac{8}{17}$ where $0° < \alpha < 90°$ and $\sin\beta = -\frac{24}{25}$ where $180° < \beta < 270°$, find the value of:

i) $\sin(\alpha - \beta)$ **2**

ii) $\cot 2\beta$ **2**

c) Find the general solution to $\tan\theta + 1 = 0$. **1**

d) i) Express $\sqrt{3}\sin\theta - \cos\theta$ in the form $r\sin(\theta - \alpha)$. **2**

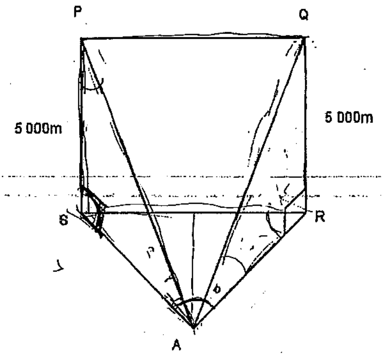
ii) Hence solve $\sqrt{3}\sin\theta - \cos\theta = 1$ for $0° \leq \theta \leq 360°$. **2**

e) Prove that $\frac{1+\sin 2\theta}{\cos 2\theta} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$ **3**

f) Solve $\sin 2\theta = 2sin^2\theta$ for $0° \leq \theta \leq 360°$. **3**

g) A plane is flying horizontally from the point P to Q at an altitude of 5 000m. From a point A on the ground, the angle of elevation to P is 20°, and the angle of elevation to Q is 15°.

If $\angle RAS = 60°$;



i) Show that $AS = 5\,000 \tan 70°$. 1

ii) Calculate the distance PQ to the nearest metre. 3

END OF TEST

① A

$-8+24+16+k=0$

$k=-32$

② D

3, 5, 4, θ

$2\sin\theta\cos\theta$

$=2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$=\frac{24}{25}$

③ C

④ C

$\frac{1}{2}\sin 2x\cos 2x=\frac{1}{4}\sin 4x$

⑤ A

$\tan 75=\frac{\tan 45+\tan 30}{1-\tan 45\tan 30}$

$=\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$

$=\frac{\sqrt{3}+1}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1}$

$=\frac{2\sqrt{3}+4}{2}$

$=\sqrt{3}+2$

⑥

a) i) $(x^2-6x+9)\overline{)x^4-5x^3+x^2+21x-18}$ quotient x^2+x-2 1

$x^4-6x^3+9x^2$

x^3-8x^2+21x

x^3-6x^2+9x

$-2x^2+12x-18$

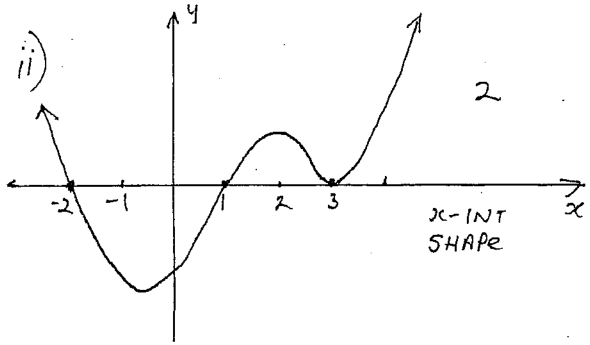
$-2x^2+12x-18$

0

$P(x)=(x-3)^2(x^2+x-2)$ $\frac{1}{2}$

$=(x-3)^2(x+2)(x-1)$ $\frac{1}{2}$

ii)



iii) $-2\le x\le 1$ OR $x=3$ 1 1

b) $P(x)=(x-2)(x+3)Q(x)+a(x-2)+b$

i) $P(2)=b=11$ 1

ii) $P(-3)=-5a+11=-4$ $\frac{1}{2}$

$5a=15$

$a=3$ $\frac{1}{2}$

$\therefore$ Rem $=3(x-2)+11$ $\frac{1}{2}$

$=3x+5$ $\frac{1}{2}$

c) $2x^3-5x^2+9x-12=0$

$x=\alpha,\beta,\gamma$

i) $\alpha+\beta+\gamma=-\frac{b}{a}$

$=\frac{5}{2}$ 1

ii) $\Sigma\alpha\beta=\frac{c}{a}$ $\alpha\beta\gamma=-\frac{d}{a}$

$=\frac{9}{2}$ $=6$

$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$ $\frac{1}{2}$

$=\frac{9/2}{6}$ $\frac{1}{2}$ $\frac{1}{2}$

$=\frac{3}{4}$ $\frac{1}{2}$

iii) $\alpha^2+\beta^2+\gamma^2=(\Sigma\alpha)^2-2(\Sigma\alpha\beta)$ $\frac{1}{2}$

$=\left(\frac{5}{2}\right)^2-2\left(\frac{9}{2}\right)$ 1

$=-\frac{11}{4}$ $\frac{1}{2}$

d) i) $P(x)=x^3-7x+2$

$P(2)=-4<0$

$P(3)=8>0$ 1

$\therefore$ SINCE $P(x)$ IS CONTINUOUS AND CHANGES SIGN AT $x=2$ AND $x=3$, A ROOT MUST LIE BETWEEN THEM.

ii) $a_2=a_1-\frac{f(a_1)}{f'(a_1)}$ $f(x)=x^3-7x+2$

$f'(x)=3x^2-7$ $\frac{1}{2}$

$a_2=2.5-\frac{2.5^3-7(2.5)+2}{3(2.5)^2-7}$ 1

$=2.489$ $\frac{1}{2}$

e) $3x^3+kx^2-37x+12=0$

i) Let $x=\alpha,\frac{1}{\alpha},\beta$

$\alpha\beta\gamma=-\frac{d}{a}$

$\alpha\cdot\frac{1}{\alpha}\cdot\beta=-\frac{12}{3}$ $\frac{1}{2}$

$\therefore\beta=-4$ $\frac{1}{2}$

the other root is $x=-4$

ii) $P(-4)=3(-4)^3+k(-4)^2-37(-4)+12=0$

$16k=32$

$k=2$ $\frac{1}{2}$

iii) $\alpha+\frac{1}{\alpha}+\beta=-\frac{2}{3}$

$\alpha+\frac{1}{\alpha}-4=-\frac{2}{3}$ 1

$3\alpha^2+3-12\alpha=-2\alpha$

$3\alpha^2-10\alpha+3=0$ $\frac{1}{2}$

$(3\alpha-1)(\alpha-3)=0$ $\frac{3\alpha}{\alpha}\times\frac{-1}{-3}$ $\frac{1}{2}$

$\alpha=\frac{1}{3},3$.

$\therefore$ roots are

$x=\frac{1}{3},3,-4$. 1

(6)

a) PROVE TRUE FOR $n=1$

$8^1 - 3^1 = 5$

$= 5 \times 1$

$\therefore$ TRUE FOR $n=1$

ASSUME TRUE FOR $n=k$

$8^k - 3^k = 5P$ (P AN INTEGER)

ie $8^k = 5P + 3^k$

PROVE TRUE FOR $n=k+1$

$8^{k+1} - 3^{k+1} = 5Q$ (Q AN INTEGER)

$LHS = 8(8^k) - 3(3^k)$

$= 8(5P + 3^k) - 3(3^k)$

$= 40P + 5(3^k)$

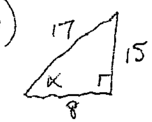
$= 5(8P + 3^k)$

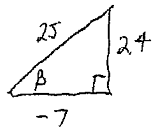
$= 5Q$ ($Q = 8P + 3^k$, AN INTEGER)

$= RHS$

IF TRUE FOR $n=k$, THEN PROVED TRUE FOR $n=k+1$. IT IS TRUE FOR $n=1$, $\therefore$ BY INDUCTION TRUE FOR $n=2,3,4,\ldots$ SO TRUE FOR ALL $n \geq 1$.

b)





3RD Q

i) $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{15}{17}\cdot\left(\frac{-7}{25}\right) - \frac{8}{17}\left(\frac{24}{25}\right)$ 1

$= \frac{-297}{425}$ 1

ii) $\cot 2\beta = \frac{1-\tan^2\beta}{2\tan\beta}$

$= \frac{1-\left(\frac{24}{7}\right)^2}{2\left(\frac{24}{7}\right)}$

$= -\frac{527}{}$

-4.

c) $\tan\theta = -1$

$\theta = 180n + 135°, n \in \mathbb{Z}$ ✓1

d) i) $\sqrt{3}\sin\theta - \cos\theta = r\sin(\theta - \alpha)$

$r = \sqrt{a^2+b^2}$

$= \sqrt{(\sqrt{3})^2 + 1^2}$

$= 2$

$\tan\alpha = \frac{b}{a}$

$= \frac{1}{\sqrt{3}}$

$\alpha = 30°$

$\therefore \sqrt{3}\sin\theta - \cos\theta = 2\sin(\theta - 30°)$ 2

ii) $2\sin(\theta - 30°) = 1$

$\sin(\theta - 30°) = \frac{1}{2}$

$\theta - 30 = 30°, 150°$

$\theta = 60°, 180°$ 2

e) $LHS = \frac{1+\sin 2\theta}{\cos 2\theta}$

$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ ½ ½

$= \frac{(\sin\theta + \cos\theta)^2}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$ ½ ½

$= \frac{\sin\theta + \cos\theta}{\cos\theta - \sin\theta} = RHS$ 1

f) $\sin 2\theta = 2\sin^2\theta$

$2\sin\theta\cos\theta - 2\sin^2\theta = 0$

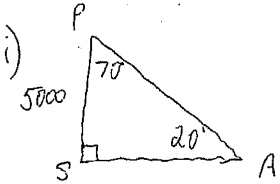
$2\sin\theta(\cos\theta - \sin\theta) = 0$ 1

$\sin\theta = 0$ 1 $\tan\theta = 1$

$\theta = 0°, 180°, 360°, 45°, 225°$

1

g) i)

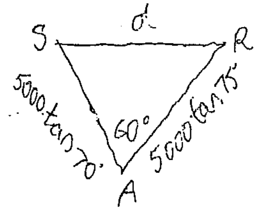


$\tan 70° = \frac{AS}{5000}$

$AS = 5000\tan 70°$ 1

ii) IN $\triangle ARQ$

$AR = 5000\tan 75°$ 1



NOTE $RS = PQ$

$c^2 = a^2 + b^2 - 2ab\cos C$

$d^2 = (5000\tan 70)^2 + (5000\tan 75°)^2 - 2(5000\tan 70°) \times (5000\tan 75)\cos 60$

½ $d^2 = 5000^2(\tan^2 70 + \tan^2 75 - \tan 70°\tan 75)$

$d = 16\,750$ m 1