St. Catherine's School
Waverley

2010

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 3 - 50%

Mathematics Extension 1

Student Number: _

Total Marks - 84

Attempt all questions 1 – 7

Question 1	/12
Question 2	/12
Question 3	/12
Question 4	/12
Question 5	/12
Question 6	/12
Question 7	/12
Total	**/84**

General Instructions

- Working time – 2 hrs
- Write using black or blue pen.
- Board approved calculators may be used.
- Write all answers in answer booklets.
- Start a new page for each question.
- Show all appropriate working.

Question 1 **Start a NEW PAGE in the writing booklet.** **12 marks**

a) Simplify $\dfrac{1}{p^2 - pq} - \dfrac{1}{pq - q^2}$ **2**

b) Solve the inequality $\dfrac{2x+5}{x+1} < 3$ **3**

c) $A(-2,3)$ and $B(6,1)$ are two points. Find the coordinates of the point P which divides the interval AB externally in the ratio 1 : 5. **2**

d) The 4th term of an arithmetic sequence is 74, the 7th term is 53.

i. Find the 1st term a, and common difference d **2**

ii. Find the 100th term **1**

iii. Find the 1st negative term **2**

Question 2 **Start a NEW PAGE in the writing booklet.** **12 marks**

a) Consider the polynomial $P(x) = x^3 - x^2 - 10x - 8$

i. Express $P(x)$ as a product of three linear factors. 3

ii. Hence sketch $P(x)$, clearly indicating all intercepts on the axes. 2

iii. Solve by inspection of the graph where, $x^3 - x^2 - 10x - 8 \geq 0$ 1

b) How many terms of the series $24 + 8 + \frac{8}{3} + \ldots\ldots$ are needed to give a sum of $\frac{320}{9}$? 3

c) The acute angle between the lines $6x - 3y - 4 = 0$, and $kx - y + 5 = 0$ is 45° 3

Find the values of k.

Question 3 **Start a NEW PAGE in the writing booklet.** **12 marks**

a) i. Expand $\cos(A + B)$ 1

ii. Show that $\cos 2x = 2\cos^2 x - 1$ 2

iii. Hence solve the equation $\cos 2x + 3\cos x + 2 = 0$ for $0^\circ < x < 360^\circ$ 3

b) i. Show that $\frac{2x-5}{x-3} = \frac{1}{x-3} + 2$ 1

ii. Hence accurately sketch $\frac{2x-5}{x-3}$ showing any horizontal and vertical asymptotes and any intercepts. 3

iii. On the same axes sketch the function $y = |x-3|$ 1

iv. From the sketch, state how many solutions are there to $\frac{2x-5}{x-3} = |x-3|$? 1

(Do not solve)

Question 4 **Start a NEW PAGE in the writing booklet.** **12 marks**

a) Find the domain and range of the function $f(x)=3\sqrt{4-x^2}$ 2

b) The polynomial $P(x)=ax^3-4bx^2+x-4$ leaves a remainder of 5 when divided by $(x-3)$ and a remainder of 2 when divided by $(x+1)$ 3

Find the value of a and b

c) i. Express $3\cos x+\sin x$ in the form $R\cos(x-\alpha)$ where $R>0$ and $0^\circ<\alpha<90^\circ$, 2

Give the value of R in simplest exact form, and the value of α in degrees correct to two decimal places.

ii. Hence or otherwise solve the equation $3\cos x+\sin x=-2$. 2

Give your answer as a general solution.

d) $P(x,y)$ is a variable point which moves in the number plane so that its distance from the point $A(3,3)$, is twice its distance from the point $(0,0)$. 3

Find the equation of the locus of P.

Question 5 **Start a NEW PAGE in the writing booklet.** **12 marks**

a) The polynomial $P(x)=x^3+2x^2-4x-1$ has zeros $\alpha,\ \beta$ and γ. Find the value of:

i. $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$ 2

ii. $(1-\alpha)(1-\beta)(1-\gamma)$. 2

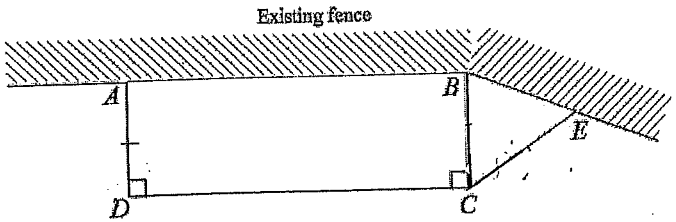
iii. $\alpha^2+\beta^2+\gamma^2$ 2

b) Find the coordinates of the point P on the curve $y=x\sqrt{x+3}$ where the tangent is parallel to the x-axis. 3

c) Use the substitution $t=\tan\frac{x}{2}$ to show that $\frac{1+\sin x}{1-\cos x}=\cot\frac{x}{2}+\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$ 3

Question 6 Start a NEW PAGE in the writing booklet. 12 marks

a)



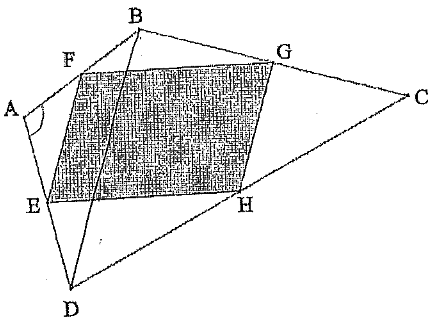
A farmer needs to construct 2 holding paddocks, one rectangular and the other triangular. The figure shows that he uses an existing fence as part of the boundary.

If he has only 440 metres of fencing and if $AD = BC = CE = x$ and $\angle BCE = 30^\circ$

i. Show that the Area, of the 2 paddocks is given by $A = 440x - \frac{11}{4}x^2$. **3**

ii. Find the maximum total area of holding paddocks he can construct. **3**

b)



$ABCD$ is a quadrilateral and E, F, G, H are the midpoints of AD, AB, BC, and CD respectively.

i. Join the interval BD, and show that ΔFAE is similar to ΔBAD. **2**

ii. Hence show that $FE \parallel BD$ and $FE = \frac{1}{2}BD$ **2**

iii. Hence or otherwise show that $EFGH$ is a parallelogram. **2**

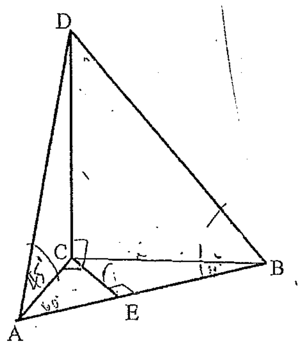
Question 7 Start a NEW PAGE in the writing booklet. 12 marks

a) Show that the equation $x^2 + (k+2)x + k = 0$ has two real roots for all real values of k **2**

b) The interval QR is divided internally at point $P(2.9, -2.5)$ in the ratio $7 : 3$. If Q is $(-2,1)$, find the coordinates of R. **2**

c) Prove that $\dfrac{4\cos\theta - 3\sec\theta}{1 - 2\sin\theta} + \dfrac{1 - 2\sin\theta}{\cos\theta} = 2\sec\theta$ **4**

d)



CD is a vertical flagpole of height 10 metres. It stands with its base on horizontal ground. A and B are points on the ground due South and due East of C respectively.

The angle of elevation of D is 45° from A and 30° from B.

E is the foot of the perpendicular from C to AB.

i. Show that angle $\angle ABC = 30^\circ$ **2**

ii. Find the angle of elevation of D from E **2**

End of Examination!

Yr 11 Yearly Prelim Ext 1.
Solutions.

Question 1

a) $\frac{1}{p^2 - pq} - \frac{1}{pq - q^2} = \frac{1}{p(p-q)} - \frac{1}{q(p-q)}$

$= \frac{q-p}{pq(p-q)}$ (1)

$= \frac{-(p-q)}{pq(p-q)}$

$= \frac{-1}{pq}$ (1)

b) $\frac{2x+5}{x+1} < 3$ Solve

Method 1

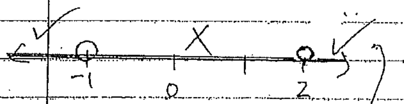
$x \neq -1$

solve $\frac{2x+5}{x+1} = 3$

$2x + 5 = 3x + 3$

$x = 2$

(1)



Test points.

Test $x = 0$

$\frac{0+5}{0+1} < 3$ (false)

Test $x = 3$

$\frac{6+5}{4} < 3$ (true)

(1)

$\therefore x < -1$ and $x > 2$ (1)

Method 2

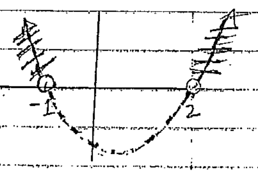
$\frac{2x+5}{x+1} < 3$

$(x+1)(2x+5) < 3(x+1)^2$

$3(x+1)^2 - (2x+5)(x+1) > 0$

$(x+1)[3x + 3 - 2x - 5] > 0$

$(x+1)(x-2) > 0$ (1)



$\boxed{\therefore x < -1 \text{ and } x > 2}$ (1)

c) $A(-2, 3)$ $B(6, 1)$ $P(x, y)$
$\quad (x_1, y_1) \quad (x_2, y_2)$

$m : n$ (external)

$-1 : 5$

$x = \frac{nx_1 + mx_2}{m+n}$ $\quad y = \frac{ny_1 + my_2}{m+n}$

$= \frac{(5)(-2) + (-1)(6)}{4}$ $\quad = \frac{(5)(3) + (-1)(1)}{4}$ (1)

$= -4$ $\quad = \frac{14}{4} = 3\frac{1}{2}$

$\therefore P(-4, 3\frac{1}{2})$ (1)

d) $T_4 = 74$

$T_7 = 53$

(1) $74 = a + 3d$ —(1)

$53 = a + 6d$ —(2)

(2) − (1)

$-21 = 3d$

$\boxed{-7 = d}$ (1)

sub $d = -7$ into (1)

$74 = a + 3(-7)$

$\boxed{\therefore a = 95}$ (1)

(ii) $T_n = 95 + (n-1)(-7)$

$\therefore T_{100} = 95 - 7(99)$

$= -598$ (1)

(iii) 1st negative term when $T_n < 0$

$95 - 7(n-1) < 0$ (1)

$95 - 7n + 7 < 0$

$102 - 7n < 0$

$-7n < -102$

$\therefore n > 14.57\ldots$ ($\frac{1}{2}$)

$\therefore n = 15.$

The 1st negative term is the 15th term

$T_{15} = 95 + (14)(-7) = -3$ ($\frac{1}{2}$)

Question 2

a) (i) $p(x) = x^3 - x^2 - 10x - 8$

test for zeros.

$p(-2) = (-2)^3 - (-2)^2 - 10(-2) - 8$

$= 0$

$\therefore (x+2)$ is a factor of $p(x)$ (1)

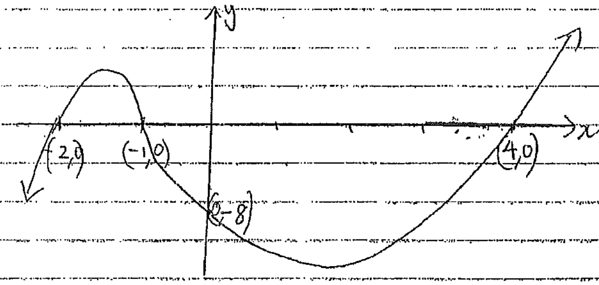
find other factors by long division or other methods.

$$
\begin{array}{r|l}
 & x^2 - 3x - 4 \\
x+2 & x^3 - x^2 - 10x - 8 \\
 & -\;(x^3 + 2x^2) \\
 & \quad -3x^2 - 10x \\
 & \quad -\;(-3x^2 - 6x) \\
 & \qquad -4x - 8 \\
 & \qquad -\;(-4x - 8)
\end{array}
$$
(1)

$\therefore x^3 - x^2 - 10x - 8 = (x+2)(x^2 - 3x - 4)$

$= (x+2)(x-4)(x+1)$ (1)

(ii)



(1) for x-int.

($\frac{1}{2}$) for y-int

($\frac{1}{2}$) for 1st & 3rd quad

(iii) From the graph in (ii) (1)

$-2 \le x \le -1$ and $x \ge 4$

b) $24 + 8 + \frac{8}{3} + \ldots$ is a Geometric series with

$a = 24$ and $r = \frac{1}{3}$ (1)

$S_n = \dfrac{a(1-r^n)}{1-r}$ for $r < 1$ $\quad S_n = \frac{320}{9}$

$\dfrac{24\left(1 - (\frac{1}{3})^n\right)}{1 - \frac{1}{3}} = \dfrac{320}{9}$ (1)

$324\left(1 - (\frac{1}{3})^n\right) = 320$

$1 - (\frac{1}{3})^n = \frac{80}{81}$

$(\frac{1}{3})^n = \frac{1}{81}$

$3^{-n} = 3^{-4}$ (1)

$\therefore n = 4$ (4 terms are required to give a sum of $\frac{320}{9}$)

e.) $6x - 3y - 4 = 0$ $\quad kx - y + 5 = 0$

$3y = 6x - 4$ $\quad y = kx + 5$

$y = 2x - \frac{4}{3}$ (1)

$m_2 = k$

$m_1 = 2$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan 45 = \left|\frac{2 - k}{1 + 2k}\right|$

$1 = \left|\frac{2-k}{1+2k}\right|$ (1)

Solve.

$\frac{2-k}{1+2k} = 1$ $\quad \frac{2-k}{1+2k} = -1$

$2 - k = 1 + 2k$ $\quad 2 - k = -1 - 2k$

$3k = 1$ $\quad k = -3$

$k = \frac{1}{3}$

$\therefore k = \frac{1}{3}, -3$ (1)

Question 3

a.) (i) $\cos(A+B) = \cos A\cos B - \sin A\sin B$ (1)

(ii) $\cos(x+x) = \cos x\cos x - \sin x\sin x$

$\cos 2x = \cos^2 x - \sin^2 x$ (1)

$= \cos^2 x - (1 - \cos^2 x)$ $\quad \cos^2 x + \sin^2 x = 1$

$= 2\cos^2 x - 1$ (1)

(iii) $\cos 2x + 3\cos x + 2 = 0$ $\quad 0 \le x \le 360$

$2\cos^2 x - 1 + 3\cos x + 2 = 0$

$2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$ (1)

$2\cos x = -1$ $\quad \cos x = -1$

$x = \cos^{-1}\left(\frac{1}{2}\right)$ $\quad x = \cos^{-1}(-1)$

acute $\angle = 60°$

$x = 180 - 60, 180 + 60$ $\quad x = 180$ (2)

$\therefore x = 120°, 180°, 240°$ $\quad -\frac{1}{2}$ for not having 240°

b.) (i) RHS $\frac{1}{x-3} + 2 = \frac{1 + 2(x-3)}{x-3}$

$= \frac{1 + 2x - 6}{x - 3}$

$= \frac{2x - 5}{x - 3}$ (1)

$= LHS$

(7)

(ii) Vertical asymptote

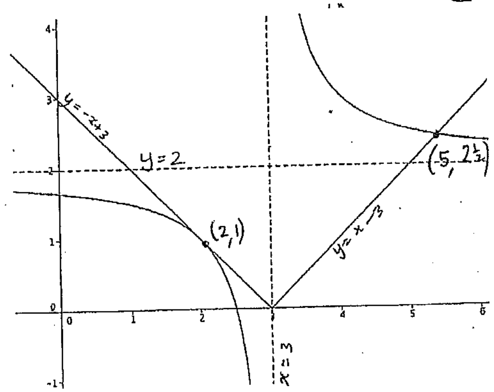
$x \neq 3$

Horizontal asymptot

$y \neq 2$

$\lim_{x \to \infty^+} \frac{1}{x-3} + 2 = 2$

$\lim_{x \to \infty^-} \frac{1}{x-3} + 2 = 2$



x-int where $y = 0$

$\frac{1}{x-3} + 2 = 0$

$1 + 2x - 6 = 0$

$-5 + 2x = 0$

$2x = 5$

$x = \frac{5}{2}$

y-int $x = 0$

$y = \frac{1}{-3} + 2$

$= \frac{5}{3}$

(1) mark asympt

(1) mark intercepts

(1) mark for graph approach asymptote ~~appro~~ appropriately.

(iii) $y = |x - 3|$ shown on sketch above. (1)

(iv) There ~~are~~ only 2 solution where $\frac{2x-5}{x-3} = |x-3|$ (1)

(8)

Question 4.

a.) $f(x) = 3\sqrt{4-x^2}$

domain:

$4 - x^2 \geq 0$

$x^2 \leq 4$

$(x-2)(x+2) \leq 4$

$\boxed{\therefore -2 \leq x \leq 2}$

range:

$0 \leq y \leq 6$

(1) mark for dom

(1) mark for ran

b.) $p(x) = ax^3 - 4bx^2 + x - 4$

$p(3) = 5$

$a(3)^3 - 4b(3)^2 + 3 - 4 = 5$

$27a - 36b = 6$

$9a - 12b = 2$ —— (1)

$p(-1) = 2$

$a(-1)^3 - 4b(-1)^2 - 1 - 4 = 2$

$-a - 4b = 7$ —— (2)

(½) ~~(1) mark for 1st equation.~~

(½) ~~mark for 2nd equat.~~

Solve simultaneously:

$9a - 12b = 2$ —— (1)

$-a - 4b = 7$ —— (2)

(2) ×3

$-3a - 12b = 21$ —— (3)

(1) − (3)

$12a = -19$

$\therefore a = \frac{-19}{12}$

½

~~(1) mark to find a~~

sub $a = \frac{19}{12}$ into (2)

$\frac{19}{12} - 4b = 7$

$19 - 48b = 84$

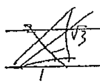
$-48b = 65$

$\boxed{\therefore\ b = \frac{65}{-48}}$ (1) mark for b.

c.) (i) $3\cos x + \sin x = R\cos(x-\alpha)$

$R = \sqrt{1^2+3^2}$

$= \sqrt{10}$



(1) mark for both R & α.

$\tan\alpha = \frac{1}{3}$

$\alpha = \tan^{-1}\left(\frac{1}{3}\right)$

$= 18^\circ 43'$

$\therefore 3\cos x + \sin x = \sqrt{10}\cos(x - 18^\circ 43')$ (1) mark to express as Rcos...

(ii) $\sqrt{10}\cos(x - 18^\circ 43') = -2$

$\cos(x - 18^\circ 43') = \frac{-2}{\sqrt{10}}$

$x - 18^\circ 43' = 360n \pm 129.23$

$x = 360n \pm 129.23 + 18.43$

d) $P(x,y)$ $A(3,3)$ $O(0,0)$

locus condition.

$PA = 2 \times PO$

$PA = \sqrt{(x-3)^2+(y-3)^2}$

$PO = \sqrt{x^2+y^2}$ } (1) mark

$\therefore \sqrt{(x-3)^2+(y-3)^2} = 2\left(\sqrt{x^2+y^2}\right)$

$(x-3)^2+(y-3)^2 = 4(x^2+y^2)$ (1) mark

$x^2-6x+9+y^2-6y+9 = 4x^2+4y^2$

$3x^2+6x-9+3y^2+6y-9 = 0$ (1m)

$x^2+2x-3+y^2+2y-3 = 0$

$(x+1)^2+(y+1)^2 = 8$

Question 5.

a.) (i) $\alpha^{-1}+\beta^{-1}+\gamma^{-1} = \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$\alpha+\beta+\gamma = \frac{-b}{a}$

$= -2$

$= \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$

$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$ $= \frac{-4}{1}$

$= -4$ $= -4$

$\alpha\beta\gamma = \frac{-d}{a}$

$= 1$

(x^2+2x-3)

$(x+3)(x-1)$

11

a.) (i) $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$

$= \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$= \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$ (1)

$= \frac{-4}{1}$

$= -4$ (1)

(ii) $(1-\alpha)(1-\beta)(1-\gamma)$

$= (1-\beta-\alpha+\alpha\beta)(1-\gamma)$

$= 1-\gamma-\beta+\beta\gamma-\alpha+\alpha\gamma+\alpha\beta-\alpha\beta\gamma$

$= 1-(\gamma+\alpha+\beta)+(\beta\gamma+\alpha\gamma+\alpha\beta)-\alpha\beta\gamma$ (1)

$= 1-(-2)+(-4)-1$

$= -2$ (1)

(iii) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma)$ (1)

$= (-2)^2-2(-4)$

$= 4+8$

$= 12$ (1)

12

b.) $y = x(x+3)^{\frac{1}{2}}$ $\quad u=x \quad v=(x+3)^{\frac{1}{2}}$ $\quad u'=1 \quad v'=\frac{1}{2}(x+3)^{-\frac{1}{2}}$

$y' = uv'+vu'$

$= x\cdot\frac{1}{2}(x+3)^{-\frac{1}{2}}+(x+3)^{\frac{1}{2}}\cdot 1$

$= \frac{x}{2\sqrt{x+3}}+\sqrt{x+3}$ (1) mark to differentiate accurately.

tangent ∥ to x-axis when $y'=0$ ($\frac{1}{2}$) mark to state $y'=0$.

$\frac{x}{2\sqrt{x+3}}+\sqrt{x+3}=0$

$x+2(x+3)=0$ (1) mark solve for x.

$x+2x+6=0$

$3x=-6$

$x=-2$

sub $x=-2$ into y. ($\frac{1}{2}$) mark solve for y.

$y=(-2)(-2+3)^{\frac{1}{2}}$

$=-2$

∴ Co-ordinates: $P(-2,-2)$.

c.) $\sin x = \frac{2t}{1+t^2}$ (1) mark to write t results.

$\cos x = \frac{1-t^2}{1+t^2}$

LHS $= \dfrac{1+\frac{2t}{1+t^2}}{1-\frac{1-t^2}{1+t^2}} = \dfrac{\frac{1+t^2+2t}{\cancel{1+t^2}}}{\frac{1+t^2-1+t^2}{\cancel{1+t^2}}}$

$= \frac{1+t^2+2t}{2t^2}$ (1) mark

$= \frac{1}{2t^2}+\frac{1}{2}+\frac{1}{t}$

$= \frac{1}{2}\left(\frac{1}{t^2}+1\right)+\frac{1}{t}$

(13)

but $t = \tan\frac{x}{2}$

$= \frac{1}{2}\left(\frac{1}{\tan^2\frac{x}{2}} + 1\right) + \frac{1}{\tan\frac{x}{2}}$

$= \frac{1}{2}\left(\cot^2\frac{x}{2} + 1\right) + \cot\frac{x}{2}$ Use $\cot^2 x + 1 = \operatorname{cosec}^2 x$ (1) mark.

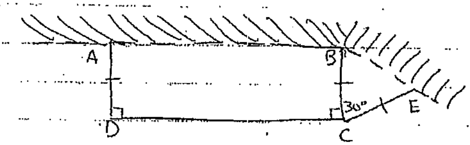
$= \frac{1}{2}\operatorname{cosec}^2\frac{x}{2} + \cot\frac{x}{2}$

$=$ RHS

(14)

Question 6

a.)



(i) let $AD = x$, then DC $440 - 3x$

A of ABCD $= x(440 - 3x)$ (1)

A of $\Delta BCE = \frac{1}{2}x^2 \sin 30$

$= \frac{1}{4}x^2$ (1)

$\therefore A = x(440 - 3x) + \frac{1}{4}x^2$

$= 440x - 3x^2 + \frac{x^2}{4}$

$= 440x - \frac{11}{4}x^2$ (1)

(ii) $\frac{dA}{dx} = 440 - \frac{11}{2}x$ (1)

Stat points when $\frac{dA}{dx} = 0$

$440 - \frac{11}{2}x = 0$

$880 = 11x$

$80 = x$ (1)

$\frac{d^2A}{dx^2} = -\frac{11}{2} < 0$ for all x

$\therefore$ maximum stat pt at $x = 80$.

$\therefore A_{max} = 440(80) - \frac{11}{4}(80)^2 = 17600\,m^2$ (1)