



**St Catherine's School**  
Waverley

# Year 11 Mathematics

## Preliminary Task 2

27th April 2015

*Time allowed:* 55 minutes

*Total marks:* 55 marks

*Weighting:* 20%

### INSTRUCTIONS

- There are 3 questions each of different value.
- Start each question in a new booklet.
- Marks for each question are indicated.
- All necessary working should be shown.
- Diagrams should be drawn using pencil and ruler.
- Approved scientific calculators may be used.
- Marks may be deducted for careless or badly arranged work.

Question 1	Multiple Choice	/5
Question 2	Trigonometric Ratios	/18
Question 3	Real Functions	/31
<b>TOTAL</b>		<b>/54</b>

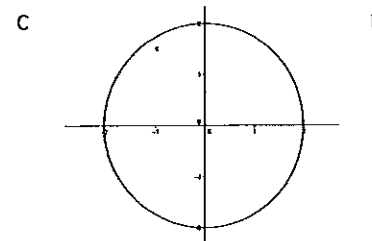
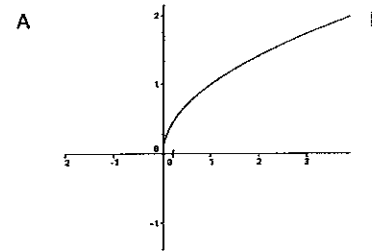
### QUESTION 1

5 Marks

#### Multiple Choice

Choose the correct answer. Answer on the multiple choice answer sheet.

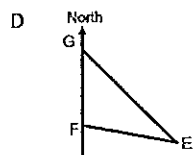
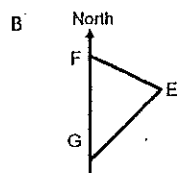
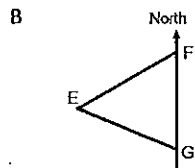
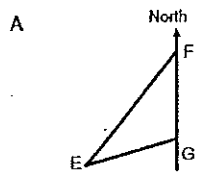
a.) Which of the following curves is not a function? 1



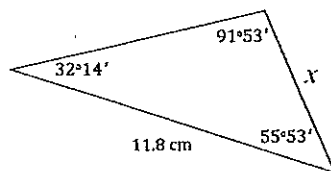
b.) The equation of the circle, centre (5, -3) and radius 6 units, is 1

- A  $(x + 5)^2 + (y - 3)^2 = 36$   
 B  $(x - 5)^2 + (y + 3)^2 = 36$   
 C  $(x - 5)^2 + (y + 3)^2 = 6$   
 D  $(x + 5)^2 + (y - 3)^2 = 6$

- c.) The points E, F and G lie on horizontal ground. Point F is due north of G. The bearing of point F from Point E is  $075^\circ$  and the bearing of point E from point G is  $305^\circ$ . Which of the following diagrams shows the location of E, F and G? 1



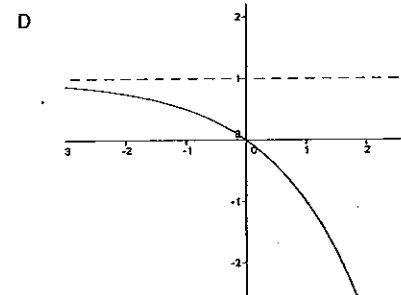
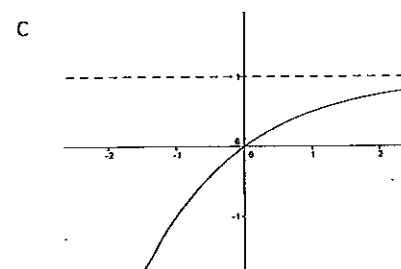
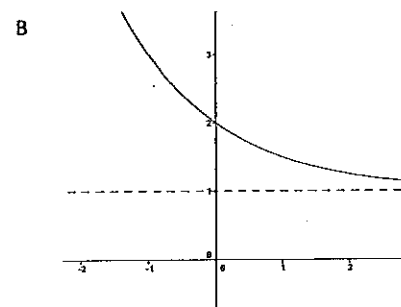
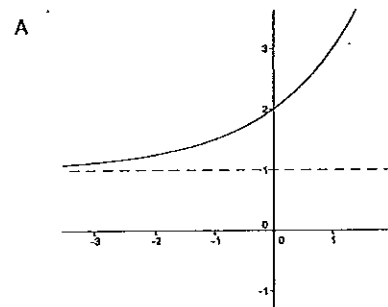
- d.) Choose the correct calculation to find the value of  $x$ . 1



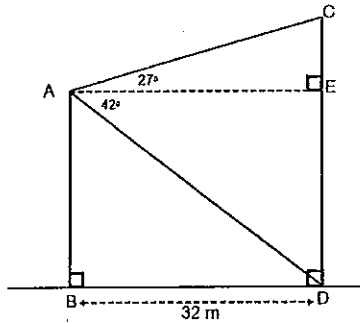
$$\frac{x}{\sin 32^\circ 14'} = \frac{11.8}{\sin 91^\circ 53'}$$

- A  $x = \frac{11.8 \sin 32^\circ 14'}{\sin 91^\circ 53'}$   
 B  $x = \frac{\sin 91^\circ 53'}{11.8 \sin 32^\circ 14'}$   
 C  $x = \frac{\sin 55^\circ 53'}{11.8 \sin 91^\circ 53'}$   
 D  $x = \frac{11.8 \sin 91^\circ 53'}{\sin 55^\circ 53'}$

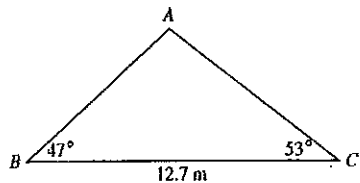
- e.) Which of the following graphs could have the equation  $y = -2^x + 1$ ? 1



- a.) In the diagram below, two vertical buildings AB and CD are 32 metres apart. From A, the angle of elevation of C is  $27^\circ$  and the angle of depression to D is  $42^\circ$ .

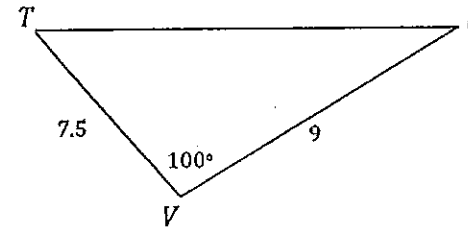


- (i) Find the height of building AB, correct to two significant figures. 2
- (ii) Find the height of building CD correct to two significant figures. 1
- b.) Find the length of AB correct to 1 decimal place. 2



- c.) Simplify 2
- $\sin x \cot x + \cos x$

- d.) In the diagram below,  $TV = 7.5$  m,  $UV = 9$  m and  $\angle V = 100^\circ$ .



- (i) Find the length of TU correct to 1 decimal place. 2
- (ii) Find the area of  $\triangle TVU$  correct to 1 decimal place. 1

- e.) Find the exact value with a rational denominator of 3
- $\sin 60^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

- f.) A ship sails from port P on a bearing of  $055^\circ$  to point Q. It then turns and sails on a bearing of  $153^\circ$  for 29.1 km to point R, when it is due east of the port.
- (i) Draw a diagram that includes the information above. 2
- (ii) What is the value of  $\angle PQR$ ? 1
- (iii) How far to 1 decimal place is the ship from its starting point? 2

a.) For the function

$$f(x) = \begin{cases} x - 3, & x < -4 \\ x^2, & -4 \leq x < 0 \\ 7, & x \geq 0 \end{cases}$$

Calculate:

- (i)  $f(-5)$  1
- (ii)  $f(-3)$  1
- (iii)  $f(0)$  1
- (iv)  $f(a^2 + 1)$  1

b.) Sketch each of the following curves on separate number planes, clearly labelling all essential features, including any intercepts and asymptotes.

State the natural domain and range for each graph.

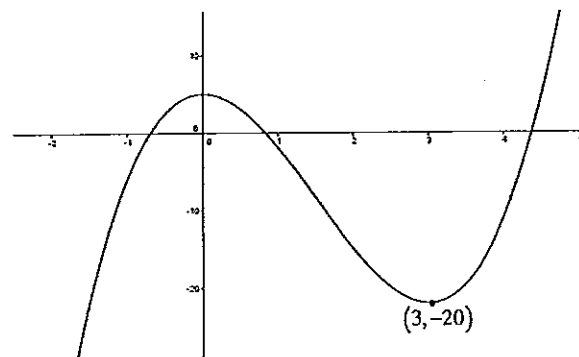
- (i)  $y = -\frac{2}{x+3}$  3
- (ii)  $y = \sqrt{49 - x^2}$  3
- (iii)  $y = x^3 + 8$  3

c.) Consider the function below.

$$g(x) = |x| - 3$$

- (i) Use the graph paper provided to sketch the function clearly labelling the  $x$  and  $y$  intercepts. 2
  - (ii) Show that  $g(x)$  is even. 1
  - (iii) On the same number plane sketch  $y = -2x$  1
  - (iv) Hence solve graphically 1
- $$|x| - 3 = -2x$$

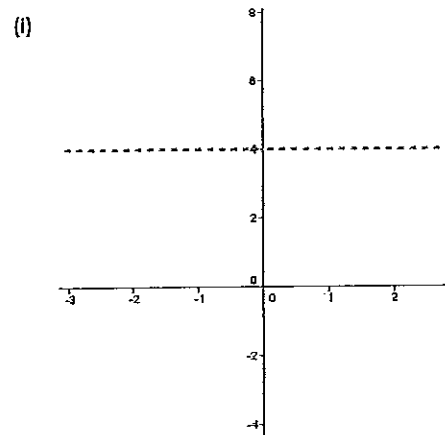
d.) State the domain over which the function below is increasing. 1



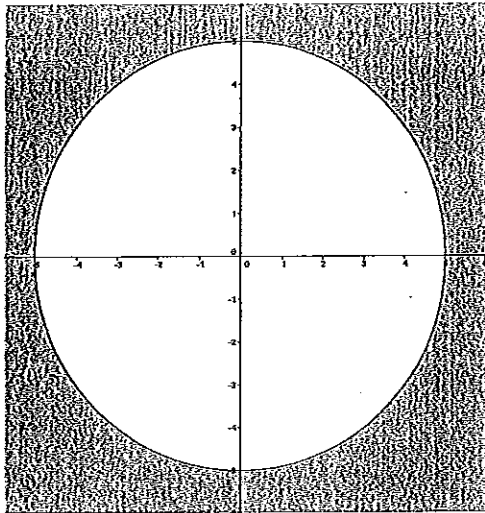
e.) State the domain of the function below. 2

$$y = \frac{1}{\sqrt{3x-2}}$$

f.) Write an inequality to show the regions defined below. 1



(ii)



1

g.) Consider the function below.

$$f(x) = x^2 - 4x + 3$$

(i) Sketch  $y = f(x)$  clearly labelling the  $x$  and  $y$  intercepts. 2

(ii) Find the range of  $f(x)$ . 2

(iii) On the same number plane, sketch  $x = 4$ . 1

(iv) On the same graph, sketch  $y = x + 3$  clearly labelling the  $x$  and  $y$  intercepts. 1

(v) Hence shade the region defined by: 2

$$y \leq x^2 - 4x + 3 \text{ and } y \leq x + 3 \text{ and } x \leq 4$$

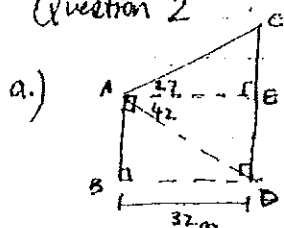
End of Question 3

END OF TASK

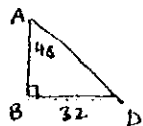
Question 1. (Multiple Choice)

- a.) C
- b.) B
- c.) B
- d.) A
- e.) D

Question 2



(i) In  $\triangle ABD$ ,  $\angle BAD = 48^\circ$



$$\tan 48^\circ = \frac{32}{AB}$$

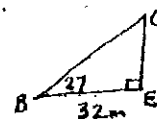
$$AB = \frac{32}{\tan 48^\circ}$$

$$= 28.8$$

$$= 29 \text{ m (to 2 sig fig)}$$

(2)

(ii) In  $\triangle AEC$ ,



$$\tan 27^\circ = \frac{CE}{32}$$

$$32 \tan 27^\circ = CE$$

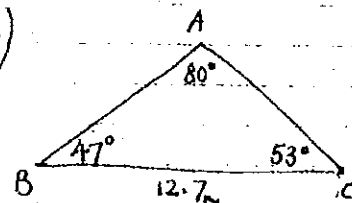
$$16.3 = CE$$

$$16 \text{ m} = CE \text{ (to 1 dp)}$$

$$\therefore CD = 16 + 29$$

$$= 45 \text{ m}$$

b.)



$$\angle BAC = 180 - (47 + 53)$$

$$= 80^\circ \quad (2)$$

$$\frac{AB}{\sin 53^\circ} = \frac{12.7}{\sin 80^\circ}$$

$$\therefore AB = \frac{12.7 \sin 53^\circ}{\sin 80^\circ}$$

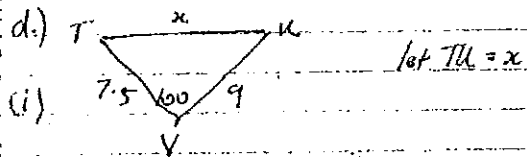
$$= 10.3 \text{ m (to 1 dp)}$$

c.) Simplify

$$\sin x \cot x + \cos x = \frac{\sin x \cdot \cos x}{\sin x} + \cos x \quad (1)$$

$$= \cos x + \cos x$$

$$= 2 \cos x \quad (1)$$



Using Cosine Rule

$$x^2 = 7.5^2 + 9^2 - 2(7.5)(9) \cos 100^\circ \quad (1)$$

$$= 160.69$$

$$\therefore x = \sqrt{160.69}$$

$$\therefore TU = 12.7 \text{ m (to 1 dp)} \quad (1)$$

(ii) Area  $\Delta TUV = \frac{1}{2} (7.5)(9) \sin 100^\circ \quad (1)$

$$= 33.2 \text{ m}^2 \text{ (to 1 dp)}$$

e.)  $\sin 60^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \quad (1)$$

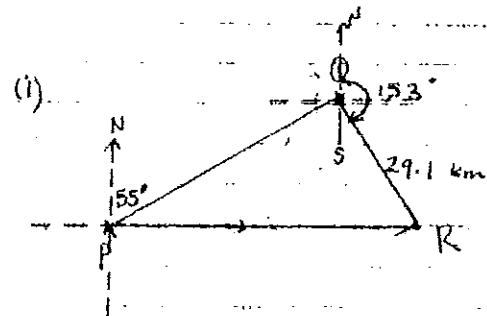
$$= \frac{3}{4} - \frac{1}{2\sqrt{2}}$$

$$= \frac{3\sqrt{2}}{4\sqrt{2}} - \frac{2}{4\sqrt{2}}$$

$$= \frac{(3\sqrt{2} - 2) \cdot \sqrt{2}}{4\sqrt{2} \cdot \sqrt{2}} \quad (\text{Rationalise})$$

$$= \frac{6 - 2\sqrt{2}}{8} = \frac{3 - \sqrt{2}}{4}$$

f.)



(ii)  $\angle PQS = 55^\circ$  (alt  $\angle$ s on || lines)

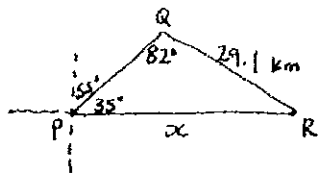
$$\angle QRS = 180 - 153 \quad (\text{Supplementary } \angle\text{'s})$$

$$= 27^\circ$$

$$\therefore \angle PQR = 55 + 27$$

$$= 82^\circ$$

(iii)



let  $PR = x$

$$\angle QPR = 90 - 55 \quad (\text{complementary } \angle\text{s}) \quad (1)$$

$$= 35^\circ$$

$$\frac{x}{\sin 82} = \frac{29.1}{\sin 35}$$

$$\therefore x = \frac{29.1 \sin 82}{\sin 35}$$

$$x = 50.2 \text{ km (to 1 dp)} \quad (1)$$

$\therefore$  Ship is 50.2 km from its starting point.

### Question 3

a.) (i)  $f(-5) = -5 - 3$   
 $= -8 \quad (1)$

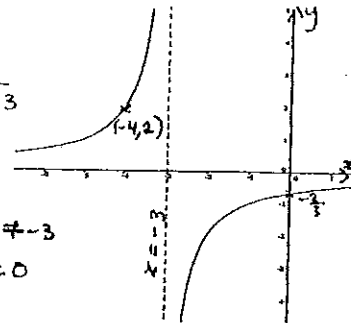
(ii)  $f(-3) = (-3)^2$   
 $= 9 \quad (1)$

(iii)  $f(0) = 7 \quad (1)$

(iv)  $f(a^2 + 1) = 7 \quad (1)$

b.)

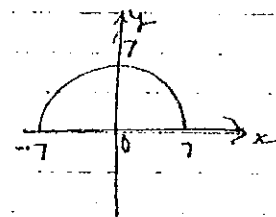
$$y = \frac{-2}{x+3}$$



domain:  $x \neq -3$   
 range:  $y \neq 0$

(1/2) do  
 (1/2) ra  
 (2) sta

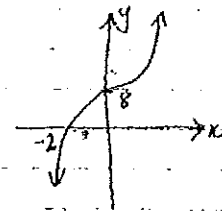
(ii)  $y = \sqrt{49 - x^2}$



domain:  $-7 \leq x \leq 7$   
 range:  $0 \leq y \leq 7$

sketch (1)

(iii)

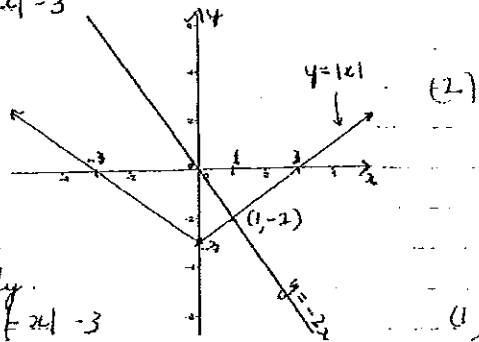


$x$ -int  $y=0$   
 $x^3 + 8 = 0 \quad (2)$   
 $x^3 = -8$   
 $x = \sqrt[3]{-8}$   
 $x = -2$

domain: all real  $x$  (1/2)  
 range: all real  $y$  (1/2)



c.) (i)  $g(x) = |x| - 3$



(ii) Algebraically.  
 $g(x) = |x| - 3$   
 $= |-2x| - 3$  (1)

$\therefore g(x)$  is even since  
 $g(x) = g(-x)$  OR

From the graph:  
 $g(x)$  is even since it is  
 symmetrical about the  
 y-axis (1)

(iii) on graph. (1)

(iv) from graph, point of  
 intersection of  $|x| - 3$  and  $-2x$   
 is  $(1, -2)$  (1)

$\therefore x = 1$

d.) The function is increasing for... (1)

$x < 0$  and  $x \geq 3$

# Note:  $x \geq 3$  and  $x \leq$   
 was also accepted.

e)  $y = \frac{1}{\sqrt{3x-2}}$

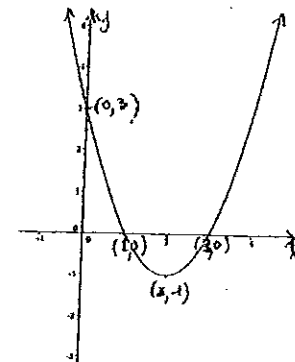
$3x - 2 > 0$  (1)  
 domain:  $x > \frac{2}{3}$  (1)

f.) (i)  $y < 4$  (1)

(ii)  $x^2 + y^2 \geq 25$  (1)

g)  $f(x) = x^2 - 4x + 3$

(i)



x-int solve  $y = 0$   
 $x^2 - 4x + 3 = 0$   
 $(x-3)(x-1) = 0$   
 $\therefore x = 3, 1$

(1) for x  
 (1/2) for y  
 (1/2) for x

y-int solve  $x = 0$   
 $y = 3$

(ii) axis of symmetry  $x = 2$

(1)

$$\begin{aligned} p(x) &= x^2 - 4x + 3 \\ &= 4 - 4 + 3 \\ &= -1 \end{aligned}$$

$\therefore$  range is  $y \geq -1$

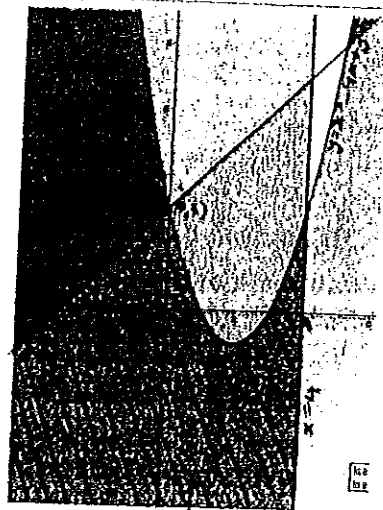
(1)

iii) on graph below

(1)

iv) on graph below

(1)



(2)

$$y \leq x^2 - 4x + 3 \text{ and}$$

$$y \leq x + 3 \text{ and}$$

$$y \leq 4$$