St Clare's College

Term 1 Examination 2009

Mathematics

Year 12

Total Marks - 80

General Instructions:

- Reading time: 5 minutes
- Working time: 2 hours
- Use black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of the paper

STUDENT NUMBER.....

QUESTION 1 12 marks

a) The distance from the Earth to the Sun is *149 492 000* km. Write this number in scientific notation, correct to 4 significant figures. 2

b) Solve $3 - 2x \geq 7$. 1

c) Solve the simultaneous equations 2

$$x + y = 1$$
$$2x - y = 5.$$

d) Solve $|x + 3| < 2$. 2

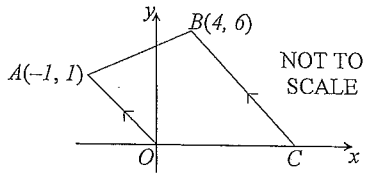
e) Simplify $\frac{x}{x^2 - 4} + \frac{2}{x - 2}$. 2

f) Solve $\frac{x-5}{3} - \frac{x+1}{4} = 5$. 2

g) Factorise $x^3 - 27$. 1

QUESTION 2 14marks

a)



In the Diagram, $OABC$ is a trapezium with $OA \parallel CB$. The coordinates of O, A and B are $(0, 0)$, $(-1, 1)$ and $(4, 6)$ respectively.

i. Calculate the length of OA. 1

ii. Write down the gradient of the line OA. 1

iii. What is the size of $\angle AOC$? 1

iv. Find the equation of the line BC, and hence find the coordinates of C. 3

v. Find the perpendicular distance from O to the line BC. 2

b) Evaluate the following integrals:

i $\int_1^2 \frac{1}{x^2}dx$ 2

ii $\int_0^3 e^{4x}dx$ 2

iii $\int_0^1 \frac{dx}{x+4}$. 2

QUESTION 3 14 marks

a) Differentiate:

i. $5(4-x)^9$ 2

ii. $\frac{\log_e x}{x}$ 2

b) Differentiate:

i. $3x^4 - 2x + \frac{1}{x^2} + 1$ 2

ii. xe^{2x} 2

c) Differentiate with respect to x:

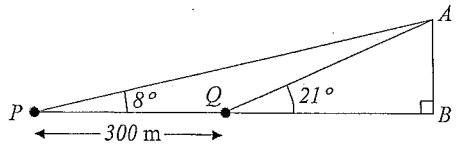
i. $ln(x^2 - 9)$ 2

ii. $\frac{x}{e^x}$ 2

d) Solve the equation $2\cos x = \sqrt{3}$, where $0° \leq x \leq 360°$ 2

QUUESTION 4 13 marks

a)



From position P, Anne finds that the angle of elevation of the top A of a rock pillar AB is $8°$. After walking 300 m directly towards the pillar to the point Q she finds that the angle of elevation of A is $21°$.

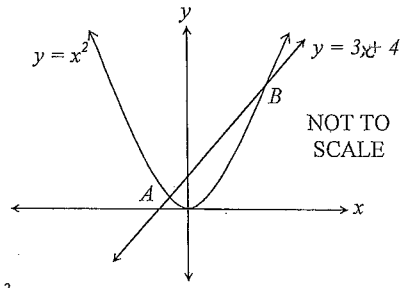
i. Copy the diagram into your Writing Booklet and find $\angle PAQ$. 1

ii. Calculate the length AQ. 2

iii. Find the height of the rock pillar AB. 2

b) Find the equation of the tangent to the curve $y = 2\log_e x$ at $(1, 0)$. 3

c)



i. The curve $y = x^2$ and the line $y = 3x + 4$ intersect at the points A and B as shown in the diagram above. Find the x coordinates of the points A and B. 2

ii. Find the area bounded by the curve $y = x^2$ and the line $y = 3x + 4$. 3

QUESTION 5 14 marks

a) The following table lists the values of a function for three values of x.

x	1·0	2·0	3·0
$f(x)$	1·7	9·0	4·3

Use these function values to estimate $\int_1^3 f(x)dx$ by:

i. Simpson's rule 2

ii. the trapezoidal rule. 2

b) i. Sketch the parabola P which has focus $(2, 3)$ and directrix $y = -1$.
State the coordinates of the vertex. 2

ii. Find the equation of P. 2

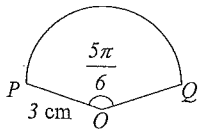
c) The area under the curve $y = \frac{1}{\sqrt{x}}$, for $1 \le x \le e^2$, is rotated about the x axis. Find the exact volume of the solid of revolution. 3

d) Convert $\frac{3\pi}{5}$ radians to degrees. 1

e) Write down the exact value of $135°$ in radians. 1

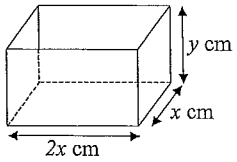
QUESTION 6 13 marks

a)



In the diagram, PQ is an arc of a circle with centre O. The radius $OP = 3$ cm and the angle POQ is $\frac{5\pi}{6}$ radians. Find the length of the arc PQ. 2

b) An open rectangular box has four sides and a base, but no lid, as in the figure.



The dimensions of the base of the box are x cm, $2x$ cm, and the height is y cm.

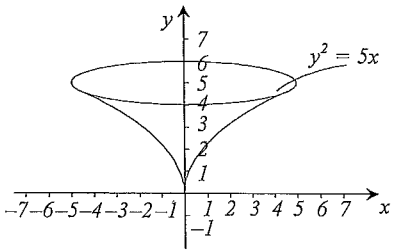
i) Show that the formula for the area A cm^2 of the outer surface of the box is given by $A = 2x^2 + 6xy$. 2

ii) Given that $A = 150$ cm^2, eliminate y and obtain a formula V for the volume as a function of x. Hence show that 2

$$V = 2x^2\left(\frac{25}{x} - \frac{1}{3}x\right)$$

iii) Hence find the value of x for which V is maximum. 3

c)



The diagram shows the shape of a vessel obtained by rotating about the y axis, the part of the parabola $y^2 = 5x$ between $y = 0$ and $y = 5$. Show that the volume of the vessel is 25π units^3. 4

$\int x^{-2}\,dx$

$\frac{x^{-1}}{-1}$

$\left[-\frac{1}{x}\right]_1^2$

$\left(-\frac{1}{2}\right)-(-1)$

$-\frac{1}{2}+1$

$\frac{1}{2}$

$\int e^{4x}\,dx$

$\left[\frac{e^{4x}}{4}\right]_0^3$

$\frac{1}{4}(e^{12}-e^{0})$

$\frac{1}{4}(e^{12}-1)$

$\int \frac{dx}{x+4}$

$\left[\log_e(x+4)\right]_0^1$

$\log_e 5 - \log_e 4$

$\log_e \frac{5}{4}$

Question 3

c) $45(4-x)^8 \times -1$

$-45(4-x)^8$

b) quotient rule

$y' = \frac{x \times \frac{1}{x} - \log_e x \times 1}{x^2}$

$\frac{1-\log_e x}{x^2}$

b) $y' = 12x^3 - 2 - 2x^{-3}$

$= 12x^3 - 2 - \frac{2}{x^3}$

ii) product rule

$y' = x \times 2e^{2x} + 1 \times e^{2x}$

$= 2xe^{2x} + e^{2x}$

c) $\frac{2x}{x^2-9}$

ii) $\frac{e^x \times 1 - xe^x}{2x}$

$= \frac{e^x(1-x)}{e^{2x}}$

$= \frac{1-x}{e^x}$

Question 1

c) 1.49492×10^8

1.495×10^8

b) $3 - 2x \geq 7$

$-2x \geq 4$

$x \leq -2$

a) $x = 1-y$ ①

$2x - y = 5$ ②

sub ① in ②

$2(1-y) - y = 5$

$2 - 2y - y = 5$

$2 - 3y = 5$

$-3y = 3$

$y = -1$

$x = 1 + 1 = 2$

d) $|x+3| < 2$

$x+3 < 2$ or $-x-3 < 2$

$x < -1$ $\quad -x < 5$

$x > -5$

e) $\frac{x}{(x+2)(x-2)} + \frac{2}{x-2}$

$\frac{x}{(\ )(\ )} + \frac{2(x+2)}{(\ )(\ )}$

$\frac{x+2x+4}{(x+2)(x-2)}$

$\frac{3x+4}{(x+2)(x-2)}$

f) $\frac{4x-20}{12} - \frac{3x+3}{12} = 5$

$\frac{x-23}{12} = 5$

$x - 23 = 60$

$x = 83$

g) $(x-3)(x^2+3x+9)$

Question 2

a) i: $d = \sqrt{(1-0)^2 + (-1-0)^2}$

$= \sqrt{2}$

ii $m = \frac{1-0}{-1-0} = -1$

iii $\tan\alpha = -1$

$\alpha = 135°$

iv AO ∥ BC

∴ gradient BC = −1

$y - 6 = -1(x-4)$

$= -x + 4$

$y = -x + 10$ C(−10, 0)

v $x + y - 10 = 0$ (0, 0)

$a=1 \quad b=1 \quad c=-10$

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

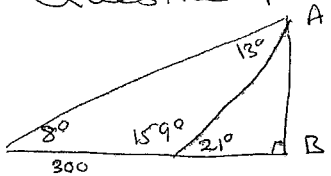
$= \frac{|x + x - 10|}{\sqrt{1+1}}$

$= 10/\sqrt{2}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = 30°, 330°$

Question 4



$159 + 8 = 167°$

$\angle PAQ = 13°$

$\frac{300}{\sin 13°} = \frac{AQ}{\sin 8°}$

$AQ = \frac{300 \sin 8°}{\sin 13°}$

$= 185.6$

iii) $\sin 21° = \frac{AB}{185.6}$

$AB = 66.5$

b) $y = 2 \log_e x$

$\frac{dy}{dx} = \frac{2}{x}$

at $x = 1$ $\frac{dy}{dx} = \frac{2}{1} = 2$

$y - 0 = 2(x - 1)$

$y = 2x - 2$

c) $x^2 = 3x + 4$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

$x = 4, -1$

$A = \int_{-1}^{4} 3x + 4 - \int_{-1}^{4} x^2 \, dx$

$= \int_{-1}^{4} 3x + 4 - x^2 \, dx$

$\left[\frac{3x^2}{2} + 4x - \frac{x^3}{3} \right]_{-1}^{4}$

$\left(\frac{48}{2} + 16 - \frac{64}{3} \right) - \left(\frac{3}{2} - 4 + \frac{1}{3} \right)$

$20\frac{5}{6}$ units^2

Question 5

ci) $h = 1$

$A \sim \frac{1}{3}(1.7 + 4.3 + 4 \times 9)$

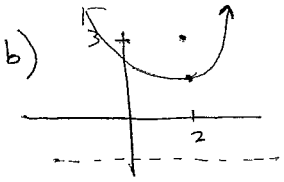
≈ 14

ii) $h = 1$

$A \sim \frac{1}{2}(1.7 + 4.3 + 2 \times 9)$

~ 12

b)



vertex $(2, 1)$

$a = 2$

$(x - 2)^2 = 8(y - 1)$

c $V = \pi \int_1^{e^2} \left(\frac{1}{\sqrt{x}} \right)^2 dx$

$= \pi \int \frac{1}{x} \, dx$

$= \pi \left[\log_e x \right]_1^{e^2}$

$= \pi \left(\log_e e^2 - \log_e 1 \right)$

$= \pi (2 \log_e e - 0)$

$= 2\pi$ units^3

d) $\frac{3\pi}{5} = \frac{3 \times 180}{5} = 108°$

e) $135° = \frac{3\pi}{4}$

Question 6

a) $l = r\theta$

$= 3 \times \frac{5\pi}{6}$

$= \frac{5\pi}{2}$ cm

or 7.85

b) base $2x^2$

sub $xy \times 2$

end $2xy \times 2$

$2x^2 + 2xy + 4xy$

$A = 2x^2 + 6xy$

ii) $2x^2 + 6xy = 150$

$6xy = 150 - 2x^2$

$y = \frac{150 - 2x^2}{6x}$

$= \frac{25}{x} - \frac{1}{3}x$

$V = 2x \times x \times y$

$= 2x^2 y$

$= 2x^2 \left(\frac{25}{x} - \frac{1}{3}x \right)$

iii) $V = 50x - \frac{2}{3}x^3$

$\frac{dV}{dx} = 50 - 2x^2$

Let $\frac{dV}{dx} = 0$

$50 - 2x^2 = 0$

$2x^2 = 50$

$x^2 = 25$

$x = 5$

$\frac{d^2V}{dx^2} = -4x$

when $x = 5$ $\frac{d^2V}{dx^2} = -20$

$\therefore x = 5$ gives a max value

$V = \pi \int f(y)^2 \, dy$

$= \pi \int_0^5 \left(\frac{y^2}{5} \right)^2 dy$

$= \pi \int_0^5 \frac{y^4}{25} \, dy = \pi \left[\frac{y^5}{125} \right]_0^5$