



St Clare's College
Year 12
Mathematics

Term 2
2009

TIME ALLOWED: 50 MINUTES

INSTRUCTIONS:

Attempt all questions.
Approved calculators may be used.
Write in blue or black pen only.
Show all necessary working.
Marks may be deducted for careless or badly arranged work.

Question 1

Find the derivatives of each of the following in simplest form:

a) e^{4x} 1

b) $\ln(3x^2 - 2x)$ 1

c) $x^2 e^x$ 2

Question 2

Integrate with respect to x :

a) $\int \frac{dx}{3x-5}$ 2

b) $\int \frac{x^3 + 1}{x} dx$ 2

Question 3

Find the volume of the solid generated by rotating about the x -axis, the area bounded by the curve $y = e^x$, the x -axis, $x = 0$ and $x = 1$. 3

Question 4

Find the area of a sector with radius 8 cm and angle at the centre of $\frac{2\pi}{9}$ radians. 2

Question 5

Solve for $0 \leq x \leq 2\pi$, $\cos x = 1 - \cos x$ 2

Question 6Differentiate with respect to x :

a) $y = \sin 4x$ 1

b) $y = \frac{x^2}{\tan x}$ 2

c) $y = e^{\sin x}$ 2

Question 7Integrate with respect to x :

a) $\int \sin \frac{x}{4} dx$ 2

b) $\int_0^{\frac{\pi}{4}} (\cos 2x) dx$ 3

c) $\int (\sin x - 4x^2) dx$ 2

Question 8Find the equation of the tangent to the curve $y = x \cos x$ at the point $(\pi, -\pi)$. 4**Question 9**Consider the trigonometric function $y = 3 \cos 2x$.

a) State the amplitude of $y = 3 \cos 2x$. 1

b) State the period of $y = 3 \cos 2x$. 1

c) Draw a neat accurate graph of $y = 3 \cos 2x$ for $0 \leq x \leq \pi$. 2

d) On the same diagram, sketch the line $y = x$. Hence, find the number of points of intersection of the curve $y = 3 \cos 2x$ and the line $y = x$. 2**Question 10**Find the area bounded by the curve $y = \sin x$ and the x -axis between $x = 0$ and $x = \pi$. 4**Question 11**Find the volume of the solid formed when $y = \sec x$ between $x = 0$ and $x = \frac{\pi}{4}$ is rotated around the x -axis. 4**Question 12**A particle starts from rest at the origin and moves along a straight line so that the position is given by $x = 6t^2 - t^3$ where position is measured in cm and time t , in seconds.

a) Find when the particle is at the origin again. 2

b) Find the velocity of the particle at 3 seconds. 2

c) Show that the particle comes to rest at time is 4 seconds. 2

d) Find the acceleration at this time. 2

e) Find the time when the maximum velocity occurs. 2

f) Draw a sketch of the velocity of the particle against time. 2

Year 10 - Half yearly 2009

SYD GIRLS ?

Marks

Marks

Question 1 (17 Marks)

Question 2 (17 Marks)

- a) Factorise and hence solve
- i) $x^2 - 9 = 0$ 2
- ii) $a^2 - 2a = 0$ 2
- b) Solve $2x^2 + 7x - 15 = 0$ 3
- c) Use the quadratic formula to solve (leave answer in exact form).
 $2x^2 - 11x + 24 = 0$ 3
- d) Mary has 3 children
- i) Draw a tree diagram to illustrate all the possible combinations of her children 2
- ii) What is the probability that:
- α) Mary has 2 girls 1
- β) Mary has at least 2 girls. 2
- e) Find the simple interest earned on \$8000 invested at 12% p.a. for 4 years. 2

- a) Three cards are marked 1, 2 and 3. One card is selected and then returned, another card is then selected.
- i) Draw a tree diagram to illustrate all possibilities. 2
- ii) Find the probability that both cards show the same number. 1
- iii) Find the probability that both cards are different. 2
- b) Find the compound interest earned on \$21400 invested at 7.5% p.a. for 5 years. 3
- c) Sketch the following curves showing all relevant points.
- i) $x^2 + y^2 = 9$ 2
- ii) $xy = -2$ 2
- iii) $y = 3^x$ 2
- iv) $y = -(x - 1)^2$ 3

Question 3 (17 Marks)

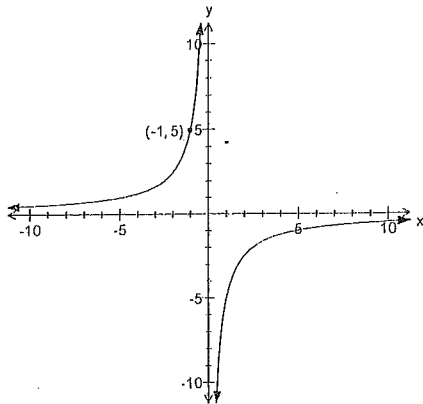
- a) A positive integer, when increased by 30 is 12 less than its square.
 i) Form a quadratic equation
 ii) Find the positive integer

- b) Find the surface area of each solid (correct to 2 decimal places).

- i)  ii) Closed Hemisphere 

- c) A parabola has a vertex $(-3,0)$ and y -intercept equal to 2. Find its equation.

- d) Write the equation of the given graph?(show all working out)



Marks

1

3

4

3

3

3

Question 4 (17 Marks)

- a) Using the formula $D = ut + \frac{1}{2}at^2$

3

Find the positive value of t if $u = 4, a = 10$ and $D = 9$

- b) Find the value of a car after 4 years if its original value is \$35000

2

and it depreciates at a rate of 3.2% per year.

- c) For the parabola $y = 2x^2 - 8x - 10$. Find:

- i) The equation of the axis of symmetry
 ii) The co-ordinates of the vertex
 iii) The x and y intercepts
 iv) The minimum value or the maximum value.(state whether Max./Min.)

2

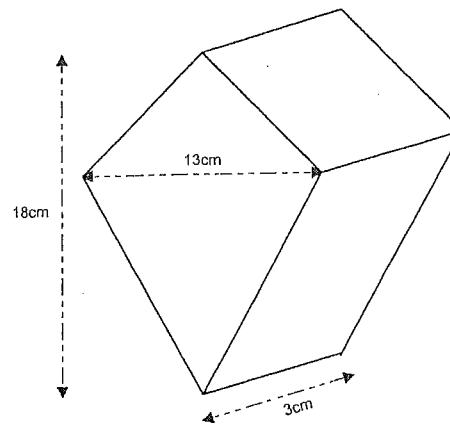
2

3

2

- d) Find the volume of the following solid (correct to 3 significant figures).

3



Question 5 (17 Marks)

- a) In a class of 30 girls all students study History or Geography or both.
 If 16 study History and 18 study Geography and one student is selected at random,
 find
- i) The probability that she studies both subjects
 - ii) The probability that she studies only Geography
 - iii) The probability that she studies either History or Geography but not both.

b) What sum of money must I invest today at 6% p.a. compounded monthly if it is to amount to \$20000 in 10 years time?

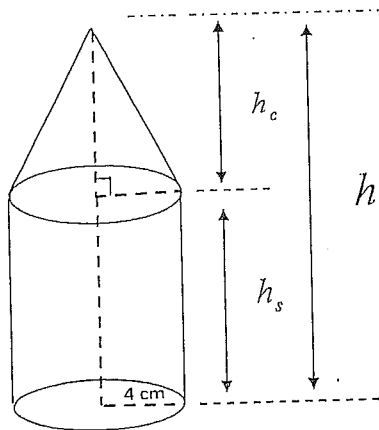
c) If a cone is to have a volume of 6π units³ and radius of 2 cm find the slant height of the cone in exact form.

d) In the solid below the height h_c (the height of the cone) and the height h_s (the height of the cylinder) are in the ratio $h_c : h_s = 1 : 3$.

i) Explain why $h_c = \frac{1}{4}h$ and $h_s = \frac{3}{4}h$

ii) Show that the total volume of the solid

is $\frac{40}{3}\pi h$ cm³.



END OF EXAM

Yr 10 2009 Q1

a) i) $(x-5)(x+5) = 0$

$x = 5$ or -5

ii) $a(a-2) = 0$

$a = 0$ or 2

b) $2x^2 + 10x - 3x - 15 = 0$

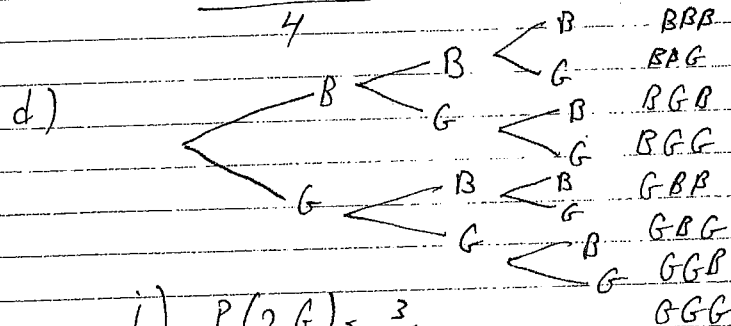
$2x(x+5) - 3(x+5) = 0$

$(x+5)(2x-3) = 0$

$x = -5$ or $\frac{3}{2}$

c) $x = \frac{11 \pm \sqrt{121 - 4 \times 2 \times 24}}{4}$

$= \frac{11 \pm \sqrt{-71}}{4}$ no solution



i) $P(2G) = \frac{3}{8}$

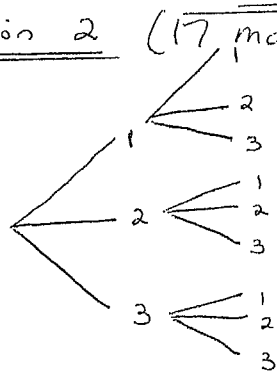
ii) $P(\text{at least 1G}) = \frac{1}{2}$

e)

$I = 8000 \times \frac{12}{100} \times 4$
 $= \$3840$

Question 2 (17 marks) Year 10

a) i)



11	1	X	6	X
12	2	X	7	X
13	3	X	8	X
21	4	X	9	X
22	5	X	10	X
23				
31				
32				
33				

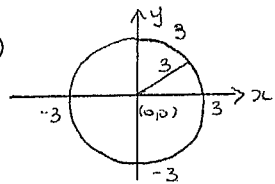
ii) $P(\text{same number}) = \frac{3}{9} = \frac{1}{3}$

iii) $P(\text{different}) = \frac{2}{3}$

b) Compound Amount = $P(1+r)^n$
 $= 21400 \left(1 + \frac{7.5}{100}\right)^5$
 $= \text{\$} 30\,722.47$

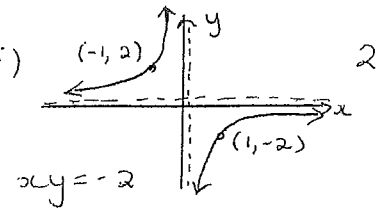
\therefore Interest = $\text{\$} 30\,722.47 - \text{\$} 21\,400$
 $= \text{\$} 9\,322.47$

c) i)



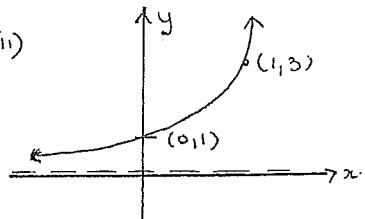
$x^2 + y^2 = 9$

ii)



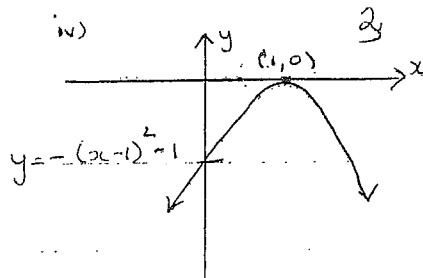
$xy = -2$

iii)



$y = 3^x$

iv)



$y = -(x-1)^2 - 1$

Question 3

a) i) let the integer be x

$x + 30 = x^2 - 12$

ii) $x + 30 = x^2 - 12$

$x^2 - x - 42 = 0$

$(x-7)(x+6) = 0$

$\therefore x = 7$ or $x = -6$

However since we are looking for a positive integer, $x = 7$.

b) i) S.A. = $\frac{1}{2} \times 2 \times 6 \times 8 + 6 \times 12 + 12 \times 8 + 12 \times 10$

$= 48 + 72 + 96 + 120$

$= 336 \text{ cm}^2$

ii) S.A = $3\pi r^2$

$= 3 \times \pi \times 2.5^2$

$= 58.90 \text{ cm}^2$

c) $y = ax^2 + bx + 2$

$-\frac{b}{2a} = -3$

$b = 6a$

$\therefore y = ax^2 + 6ax + 2$

substitute (3, 0)

$0 = 9a - 18a + 2$

$a = \frac{2}{9}$

$\therefore y = \frac{2}{9}x^2 + \frac{4}{3}x + 2$

d) $y = \frac{k}{x}$

$5 = \frac{k}{-1}$

$k = -5$

$\therefore y = -\frac{5}{x}$

Year 10: Question Four: (17 Marks)

a) $D = ut + \frac{1}{2}at^2$
 $9 = 4t + \frac{1}{2} \times 10t^2$
 $0 = 5t^2 + 4t - 9$
 $0 = (t-1)(5t+9)$
 $\therefore t = 1$ or $t = -\frac{9}{5}$ ✓✓

But $t > 0 \therefore t = 1$ ✓

b) $A = P(1-r)^n$
 $= 35000(1-0.032)^4$ ✓
 $= 35000(0.968)^4$
 $= \$30730.49$ ✓

c) i) axis of symmetry: $x = -\frac{b}{2a} = \frac{8}{4} = 2 \therefore x = 2$ ✓✓

ii) vertex: $(2, 2(2)^2 - 8(2) - 10) = (2, -18)$ ✓✓

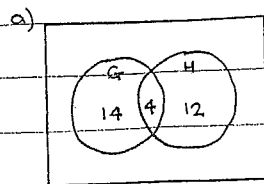
iii) When $x = 0$: $y = -10$ ✓

When $y = 0$: $x = -1.5$ ✓✓

iv) $y = -18$ ✓ is a minimum value ✓

d) $V = \left(\frac{1}{2} \times 13 \times 18\right) \times 3$ ✓✓
 $= 351 \text{ cm}^3$ (to 3 sig figs) ✓

Question 5:



i) $P(\text{both}) = \frac{4}{30}$
 $= \frac{2}{15}$

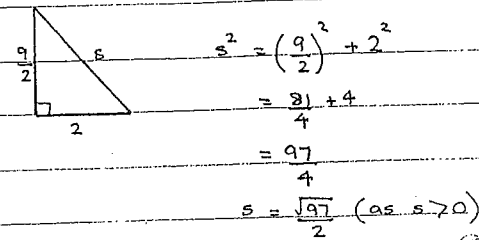
ii) $P(\text{Geog. only}) = \frac{14}{30}$
 $= \frac{7}{15}$ ①

iii) $P(\text{one only}) = 1 - \frac{2}{15}$
 $= \frac{13}{15}$ ②

b) $A = 20000$	$A = P(1+r)^n$
$r = 0.06 \div 12$	$20000 = P(1.005)^{120}$
$= 0.005$	$P = \frac{20000}{1.005^{120}}$
$n = 10 \times 12$	$= 10992.65467...$
$= 120$	

\therefore Approx. \$10992.65 needs to be invested (nearest cent). ③

c) $V = \frac{1}{3}\pi r^2 h$
 $6\pi = \frac{1}{3} \times \pi \times 2^2 \times h$
 $h = \frac{9}{2} \text{ cm}$



②) $h_c : h_s = 1:3$

$\therefore h_c : h = 1:4$

So $h_c = \frac{1}{4}h$

$h_s : h = 3:4$

So $h_s = \frac{3}{4}h$ ②

ii) $V = \frac{1}{3}\pi r_c^2 h_c + \pi r_s^2 h_s$
 $= \frac{1}{3} \times \pi \times \left(\frac{1}{4}h\right)^2 \times \left(\frac{1}{4}h\right) + \pi \times \left(\frac{3}{4}h\right)^2 \times \left(\frac{3}{4}h\right)$
 $= \frac{4\pi h^3}{8} + 12\pi h^3$
 $= \frac{40\pi h^3}{3}$ ③