St George Girls High School

Trial Higher School Certificate Examination

2008

Mathematics

Total Marks – 120

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Attempt ALL questions.
- Begin each question on a new booklet
- Write your student number on each page
- All necessary working must be shown.
- Diagrams are not to scale.
- Board-approved calculators may be used.
- The mark allocated for each question is listed at the side of the question.

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Question 1 – (12 marks) Marks

a) If $a = 0.52$ find the value of $\frac{2+a^2}{2-a^2}$ to 3 significant figures. 2

b) Factorise $36m^2 - 9n^2$ completely. 2

c) Convert $54°$ to radians giving your answer in terms of π 2

d) Solve $|3x - 2| \le 10$ 2

e) Find the primitive of $3 - e^{-2x}$ 2

f) Simplify $x(2-y) - y(3-x)$ 2

Question 2 – (12 marks)

Marks

a) The points $A(3,-3)$, $B(-3,-4)$ and $C(0,3)$ are the vertices of a triangle ABC.

(i) Plot these points on the number plane. 1

(ii) Find the gradient of AC. 1

(iii) Find the angle of inclination of AC to the positive x-axis to the nearest degree. 1

(iv) Show that the equation of AC is $2x+y-3=0$ 1

(v) Calculate the perpendicular distance of B from the side AC. 1

(vi) Hence find the area of ΔABC 2

(vii) Find the coordinates of D such that $ABCD$ is a parallelogram. 1

b) For what values of p will x^2+5x+p be positive definite? 2

c) Find all the values of x between 0 and 2π for which $\sin x = -\frac{\sqrt{3}}{2}$ 2

Question 3 – (12 marks)

Marks

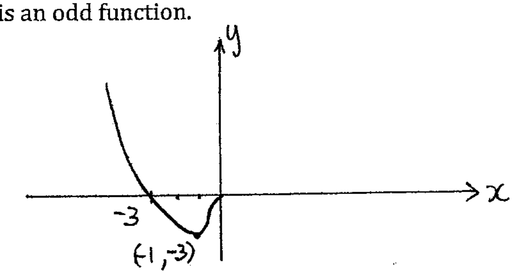
a) Differentiate

(i) $\log_e(3x-2)^2$ 2

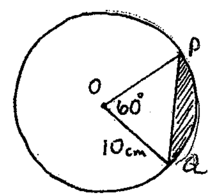
(ii) $x^2 e^x$ 2

(iii) $\frac{1}{4x^4}$ 2

b) The following diagram shows the graph of $y=f(x)$ for $x \le 0$. It is known that $f(x)$ is an odd function.



Copy the diagram onto your answer booklet and complete the graph for $x > 0$ 2



From the diagram given of the circle centre O and radius 10cm. Find the exact value of:

(i) the length of the minor arc QP. 2

(ii) the area of the minor segment cut-off by the chord QP. 2

Question 4 – (12 marks) **Marks**

a) Find:

(i) $\int_4^9 x\sqrt{x}\,dx$ 2

(ii) $\int \frac{3x}{x^2+1}\,dx$ 2

b) Differentiate

(i) $\sin(5x+3)$ 2

(ii) $\log_e(\cos x)$ 2

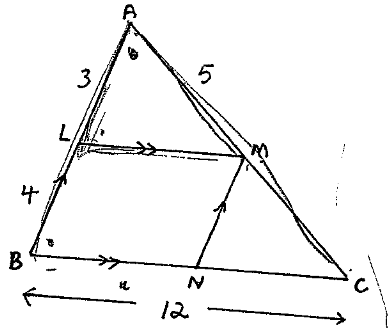
(iii) $e^x \tan 3x$ 2

c) Find the value of 2

$$\sum_{n=1}^{4}(3n^2+2)$$

Question 5 – (12 marks) **Marks**

a) In the triangle ABC, L, M and N are on AB, AC and BC respectively so that LM is parallel to BC and MN parallel to AB 4



Find giving reasons:

(i) the length of MC.

(ii) the length of BN.

b) (i) Show that $\frac{\sec^2 x}{\tan x} = \frac{1}{\sin x \cos x}$ 2

and hence

(ii) Find the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin x \cos x}\,dx$ 3

c) (i) Copy and complete the table of values for $y = \log_e(x+2)$ in your booklet. 1

x	0	0.5	1	1.5	2
y	0.69	0.92	1.1	1.3	1.4

(ii) By using Simpson's rule with 5 function values, estimate the value of the integral $\int_0^2 \log_e(x+2)\,dx$ 2

Question 6 – (12 marks) **Marks**

a) The personal assistant to the CEO of Spendupbig Retail chain starts on an annual salary of $30 000 with an annual increase of $2 000.

(i) Show that his salary forms an arithmetic sequence and write down a formula for determining his salary after n years with the company. 2

(ii) Find his total earnings after he has worked for the company for 15 years. 2

(iii) In which year will his salary be $42 000. 1

b) Find the value of the smallest term of the geometric series $4 + 10 + 25 + \ldots$ that is greater than 10^{20}. Write your answer in scientific notation correct to 3 significant figures. 3

c) (i) Find $\frac{d}{dx}\left(\frac{1}{2}x \sin 2x\right)$ 2

(ii) Hence or otherwise find $\int x \cos 2x \, dx$ 2

Question 7 – (12 marks) **Marks**

a) (i) For what values of x does this geometric series have a limiting sum? 2

$2x + 6x^2 + 18x^3 + \ldots$

(ii) Write an expression for the limiting sum and hence find the limiting sum if $x = \frac{1}{4}$ 2

b) For the equation $y = 2\cos 3x$ find:

(i) the period. 1

(ii) the amplitude. 1

c) For the parabola $y = -3 - 4x - x^2$ find:

(i) the vertex. 2

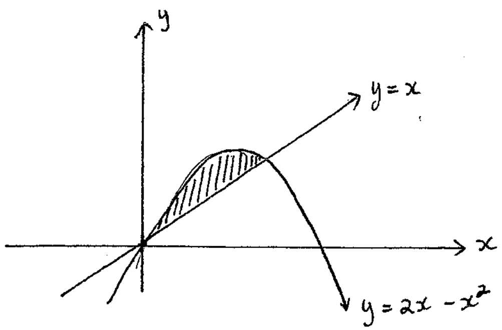
(ii) the focal length. 1

(iii) the focus. 1

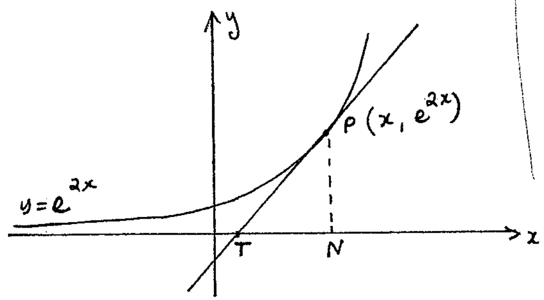
d) Sketch the region $x^2 + y^2 > 25$ 2

Question 9 – (12 marks) Marks

a) Find the volume of the solid formed when the region shaded in the diagram given is rotated around the x-axis. 4



b)



In the diagram above, $P(x_1, e^{2x_1})$ is a variable point on the curve $y = e^{2x}$. The tangent at P crosses the x-axis at T. The perpendicular from P to the x-axis meets the x-axis at N.

(i) Find the equation of the tangent to $y = e^{2x}$ at $P(x_1, e^{2x_1})$ 2

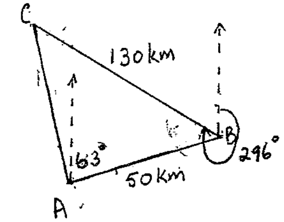
(ii) Find the coordinates of T. 1

(iii) Show that for all positions of P the length of TN is constant. 2

c) Find the solutions of $2\sin^2\theta - \sin\theta = 0 \qquad 0 \le \theta \le 2\pi$ 3

Question 8 – (12 marks) Marks

a) A ship sails 50km from Port A to Port B on a bearing of $063°T$ then sails 130km from Port B to Port C on a bearing of $296°T$ as shown in the diagram.



(i) Show that $A\hat{B}C = 53°$ 2

(ii) Find, correct to the nearest km, the distance of Port A to Port C. 2

(iii) Find the bearing of Port A from Port C. 2

b) The cost C (in dollars per hour) of running a boat depends on the speed v km/h of the boat according to the formula $C = 500 + 40v + 5v^2$

(i) Show that the total cost of the trip of 100km is 2

$$T = \frac{50\,000}{v} + 4000 + 500v$$

(ii) What speed will minimise the total cost of the trip. 4

Question 10 – (12 marks) **Marks**

a) For the function $y = x\log_e x$

(i) Write down the domain of the function. 1

(ii) Write down the x-intercept of the function. 1

(iii) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ 2

(iv) Hence find any stationary points and determine their nature. 2

(v) Given that $y \to 0^-$ as $x \to 0^+$ sketch the curve showing all relevant features. 2

b) At the beginning of each month Jane deposits \$600 into a bank account which pays 9% p.a. calculated monthly.

(i) How much will be in her account after four years. 2

(ii) Jane needed \$50 000. How much should she have deposited each month into this account for her to have reached her goal in the four years. 2

End of Paper

Mathematics trial Solutions 2008.

Question 1.

a) $\frac{2+(0.52)^2}{2-(0.52)^2} = 1.31267$

$= 1.31$ (3 sig. figs.)

) $36m^2 - 9n^2 = 9(2m-n)(2m+n)$

) $54° = 54 \times \frac{\pi}{180}$ radians

$= \frac{3\pi}{10}$

) $|3x-2| \leq 10$

$-10 \leq 3x - 2 \leq 10$

$-8 \leq 3x \leq 12$

$-\frac{8}{3} \leq x \leq 4$

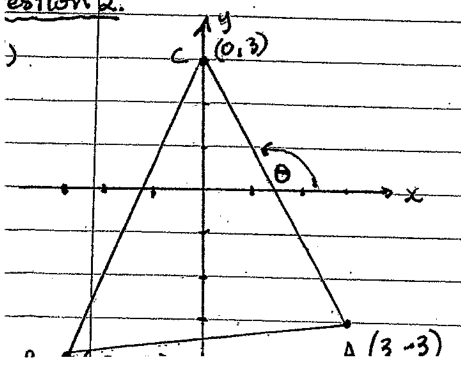
$\int 3 - e^{-2x}\,dx = 3x + \frac{e^{-2x}}{2} + c$

$x(2-y) - y(3-x) = 2x - xy - 3y + xy$

$= 2x - 3y$

estion 2.

)



(ii) $m_{AC} = \frac{-3-3}{3-0}$

$= -2$

(iii) $\tan\theta = -2$

$\theta = 116°34'$

$\approx 117°$

(iv) $y - 3 = -2(x-0)$

$y - 3 = -2x$

$2x + y - 3 = 0$

(v) $d = \frac{|2\times 3 + 1\times -4 \;\; -3|}{\sqrt{2^2+1^2}}$

$d = \frac{13}{\sqrt{5}}$

(vi) $AC = \sqrt{(3-0)^2 + (-3-3)^2}$

$= \sqrt{9+36}$

$= \sqrt{45}$

$= 3\sqrt{5}$

Area $= \frac{bh}{2}$

$= \frac{1}{2} \times 3\sqrt{5} \times \frac{13}{\sqrt{5}}$

$= \frac{39}{2}$ square units

(vii) $D = (6, 4)$

b) $b^2 - 4ac < 0$; $a > 0$

$x^2 + 5x + p \rightarrow a = 1$

$\therefore a > 0$

$5^2 - 4 \times 1 \times p < 0$

$25 - 4p < 0$

$-4p < -25$

$p > \frac{25}{4}$

) $\sin x = -\frac{\sqrt{3}}{2}$

elated angle $= \frac{\pi}{3}$

$\therefore x = \pi + \frac{\pi}{3}, \; 2\pi - \frac{\pi}{3}$

$= \frac{4\pi}{3}, \; \frac{5\pi}{3}$

Question 3.

) (i) $y = \log_e(3x-2)^2$

$y = 2\log(3x-2)$

$y' = 2 \cdot \frac{1}{3x-2} \cdot 3$

$= \frac{6}{3x-2}$

(ii) $y = x^2 e^x$

$y' = 2x \cdot e^x + e^x \cdot x^2$

$= e^x(x^2 + 2x)$

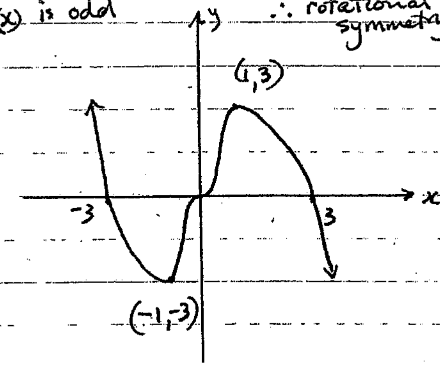
(iii) $y = \frac{1}{4x^4}$

$y = \frac{1}{4}x^{-4}$

$y' = -\frac{4}{4}x^{-5}$

$= \frac{-1}{x^5}$

(b) f(x) is odd $\therefore$ rotational symmetry.



(c)(i) $l = r\theta$ $\qquad 60° = \frac{\pi}{3}$ radia

$= 10 \times \frac{\pi}{3}$

$= \frac{10\pi}{3}$

(ii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2} \times 10^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$

$= 50\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$

Question 4.

(i) $\int_4^9 x\sqrt{x}\,dx = \int_4^9 x^{3/2}\,dx$

$= \frac{2}{5}x^{5/2}\Big]_4^9$

$= \frac{2}{5}(3^5 - 2^5)$

$= \frac{2}{3}(243 - 32)$

$= \frac{422}{5}$

$= 84\frac{2}{5}$

$\int \frac{3x}{x^2+1}\,dx = \frac{3}{2}\int \frac{2x}{x^2+1}\,dx$

$= \frac{3}{2}\log_e(x^2+1) + C$

(i) $y = \sin(5x+3)$

$y' = 5\cos(5x+3)$

(ii) $y = \log_e(\cos x)$

$y' = \frac{1}{\cos x} \cdot -\sin x$

$= -\frac{\sin x}{\cos x}$

$= -\tan x$

(iii) $y = e^x \tan 3x$

$y' = e^x \tan 3x + 3\sec^2 3x \cdot e^x$

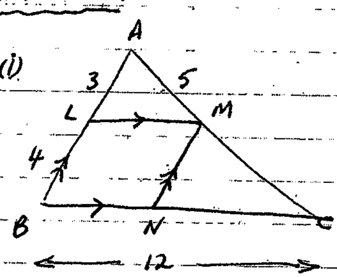
$= e^x(\tan 3x + 3\sec^2 3x)$

(c) $\sum_{n=1}^{4}(3x^2+2) = 5 + 14 + 29 + 50$

$= 98.$

Question 5.

(a) (i)



$\frac{5}{MC} = \frac{3}{4}$ (intercepts on // lines)

$MC = \frac{20}{3}$

(ii) $BN : NC = AM : MC$ (intercepts on // lines)

Divide 12 in the ratio $5 : \frac{20}{3}$
$= 3:4$

$\therefore \frac{3}{7}$ of $12 = 5\frac{1}{7}$

$\therefore$ $BN = 5\frac{1}{7}$

b) (i) $\frac{\sec^2 x}{\tan x} = \frac{1}{\sin x \cos x}$

LHS $= \frac{\sec^2 x}{\tan x}$

$= \frac{1}{\cos^2 x} \cdot \cot x$

$= \frac{1}{\cos^2 x} \cdot \frac{\cos x}{\sin x}$

$= \frac{1}{\sin x \cos x}$

= RHS.

OR

LHS $= \frac{\sec^2 x}{\tan x}$

$= \frac{1 + \tan^2 x}{\tan x}$

$= \frac{1}{\tan x} + \tan x$

$= \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$

$= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$

$= \frac{1}{\sin x \cos x}$

= RHS.

(ii) $\int_{\pi/6}^{\pi/3} \frac{1}{\sin x \cos x}\,dx = \int_{\pi/6}^{\pi/3} \frac{\sec^2 x}{\tan x}\,dx$

$= \log_e(\tan x)\Big]_{\pi/6}^{\pi/3}$

$= \log(\tan\frac{\pi}{3}) - \log(\tan\frac{\pi}{6})$

$= \log\sqrt{3} - \log\frac{1}{\sqrt{3}}$

$= \log\frac{\sqrt{3}}{\frac{1}{\sqrt{3}}}$

$= \log 3$

(c) (i)

x	0	0.5	1	1.5	2
y	ln2	ln2.5	ln3	ln3.5	ln4 1.39

$A \approx \frac{\frac{1}{2}}{3}\left[\ln 2 + \ln 4 + 4(\ln 2.5 + \ln 3.5) + 2\ln\right.$

$\approx \frac{0.5}{3}[12.9]$

$= 2.16$

Question 6

(i)

$A_1 = 30000 \quad A_2 = 32000 \quad A_3 = 34000$

$A_n = 30000 + (n-1)2000$

$= 28000 + 2000n$

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$= \frac{15}{2}\left(2 \times 30000 + 14 \times 2000\right)$

$= \$660\,000$

$T_n = 42000$

$28000 + 2000n = 42000$

$2000n = 14000$

$n = 7$

$\therefore$ after 7 years.

$4 + 10 + 25$

$a = 4 \quad r = 2.5$

$ar^{n-1} > 10^{20}$

$4 \times (2.5)^{n-1} > 10^{20}$

$(2.5)^{n-1} > \frac{10^{20}}{4}$

$(n-1)\log(2.5) > \log\left(\frac{10^{20}}{4}\right)$

$n-1 > \frac{\log\left(\frac{10^{20}}{4}\right)}{\log(2.5)}$

$n > \frac{\log\left(\frac{10^{20}}{4}\right)}{\log(2.5)} + 1$

$n > 49.7$

$\therefore$ the 50th term.

$T_n = ar^{n-1}$

$T_{50} = 4 \times (2.5)^{49}$

$= 1.26 \times 10^{20}$

(c)(i) let $y = \frac{1}{2}x\sin 2x$

$y' = \frac{x}{2}\cdot\cos 2x \cdot 2 + \frac{1}{2}\sin 2x$

$= \frac{1}{2}\sin 2x + x\cos 2x$

(ii) $\int \frac{1}{2}\sin 2x + x\cos 2x \, dx = \frac{1}{2}x\sin 2x$

$\int \frac{1}{2}\sin 2x\,dx + \int x\cos 2x\,dx = \frac{1}{2}x\sin 2x$

$\int x\cos 2x\,dx = \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x\,dx$

$\int x\cos 2x\,dx = \frac{1}{2}x\sin 2x - \frac{1}{2}\cdot -\frac{\cos 2x}{2} +$

$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$

Question 7.

(a)(i) $2x + 6x^2 + 18x^3 + \ldots$

$r = 3x \qquad |r| < 1$

$-1 < 3x < 1$

$-\frac{1}{3} < x < \frac{1}{3}$

(ii) $x = \frac{1}{4}$

$S_\infty = \frac{\frac{3}{4}}{1 - \frac{3}{4}}$

$= 2$

b) (i) period $= \frac{2\pi}{3}$

(ii) amplitude $= 2$

) $y = -3 - 4x - x^2$

$x^2 + 4x = -y - 3$

$x^2 + 4x + 4 = -y - 3 + 4$

$(x+2)^2 = -y + 1$

$(x+2)^2 = -(y-1)$

i) vertex $(-2, 1)$

i) $x^2 = 4ay$

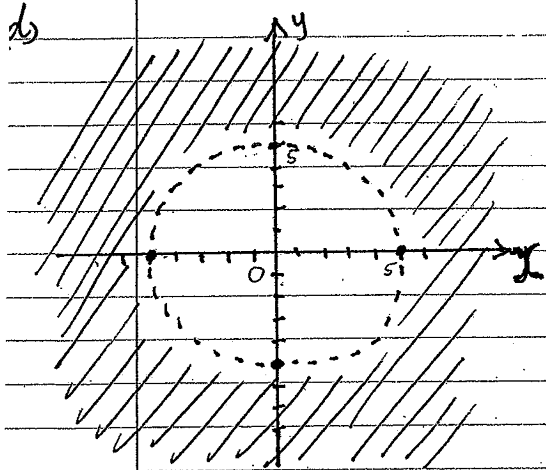
$4a = -1$

$a = -\frac{1}{4}$

$\therefore$ focal length $= \frac{1}{4}$

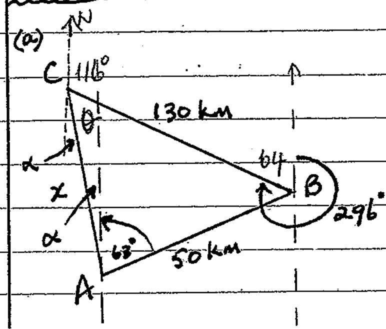
iii) focus $= \left(-2, \frac{3}{4}\right)$

d)



Question 8

(a)



(i) Using alternate angles on $\parallel$ lines,

$A\hat{B}C = 116 - 63$

$= 53°$

(ii) $x^2 = 50^2 + 130^2 - 2 \times 50 \times 130 \times \cos 53°$

$x^2 = 11576.4$

$x = 107.59$

$= 108$ km (nearest km)

(iii) $\frac{\sin\theta}{50} = \frac{\sin 53}{108} \quad (\theta < 53°)$

$\sin\theta = \frac{\sin 53}{108} \times 50$

$\theta = \sin^{-1}\left(\frac{\sin 53}{108} \times 50\right)$

$= 21.6\ldots$

$\approx 22°$

$\angle NCB = 116°$

$\therefore$ bearing $= 116° + 22°$

$= 138°$

$\therefore$ bearing of A from C $= 138$

(b) time taken $= \frac{100}{v}$

total cost = cost/h × time

$T = c \times \frac{100}{v}$

$= (500 + 40v + 5v^2) \times \frac{100}{v}$

$T = \frac{50000}{v} + 4000 + 500v$

$\frac{dT}{dv} = ?$

$T = 50000v^{-1} + 4000 + 500v$

$T' = -50000v^{-2} + 500$

$= \frac{-50000}{v^2} + 500$

$T' = 0$ for max or min

$0 = \frac{-50000}{v^2} + 500$

$\frac{50000}{v^2} = 500$

$50000 = 500v^2$

$100 = v^2$

$\pm 10 = v$

$T'' = \frac{100000}{v^3}$

$v = 10$ $T'' > 0$ ∴ min.

minimum cost when speed = 10 km/h.

Question 9.

(a) $y = x$

$y = 2x - x^2$

$x = 2x - x^2$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

$Vol = \pi\int_0^1 (2x-x^2)^2 - x^2 \, dx$

$= \pi\int_0^1 4x^2 - 4x^3 + x^4 - x^2 \, dx$

$= \pi\int_0^1 3x^2 - 4x^3 + x^4 \, dx$

$= \pi\left[\frac{3x^3}{3} - \frac{4x^4}{4} + \frac{x^5}{5}\right]_0^1$

$= \pi\left[x^3 - x^4 + \frac{x^5}{5}\right]_0^1$

$= \pi\left[(1 - 1 + \frac{1}{5}) - 0\right]$

$= \frac{\pi}{5}$ cubic units.

(b) (i) $y' = 2e^{2x}$ $\quad P(x_1, e^{2x_1})$

∴ $m = 2e^{2x_1}$

$y - y_1 = m(x - x_1)$

$y - e^{2x_1} = 2e^{2x_1}(x - x_1)$

(ii) let $y = 0$

∴ $-e^{2x_1} = 2e^{2x_1}(x - x_1)$

$-\frac{1}{2} = (x - x_1)$

$x = x_1 - \frac{1}{2}$

$T = (x_1 - \frac{1}{2}, 0)$

$T = (x_1 - \frac{1}{2}, 0)$

$N = (x_1, 0)$

$TN = x_1 - (x_1 - \frac{1}{2})$

$= \frac{1}{2}$

TN is independent of x_1

∴ TN is a constant.

ie $TN = \frac{1}{2}$ for all x_1

c) $2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$ $\qquad 2\sin\theta - 1 = 0$

$\theta = 0, \pi, 2\pi$ $\qquad \sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

∴ $\theta = 0, \pi, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}$

Question 10.

a) $y = x \log_e x$

(i) domain $x > 0$

(ii) x-int $y = 0$

$x\log_e x = 0$

$\log_e x = 0$

$x = e^0$

$= 1$

∴ $(1, 0)$

(iii) $y = x\log x$

$\frac{dy}{dx} = 1.\log x + x.\frac{1}{x}$

$= \log x + 1$

$\frac{d^2y}{dx^2} = \frac{1}{x}$

(iv) $y' = 0$ for stationary pts.

$\log x + 1 = 0$

$\log x = -1$

$x = e^{-1}$

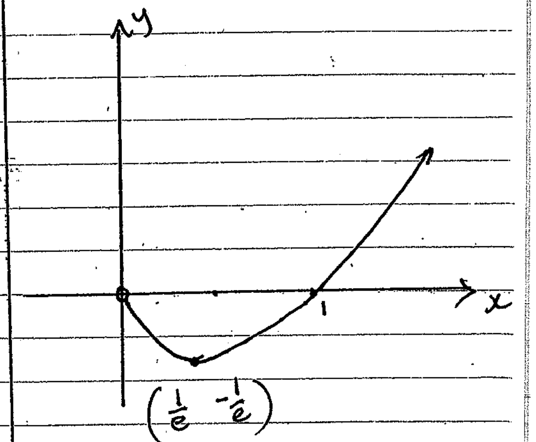
$x = \frac{1}{e}$

$y'' = \frac{1}{x}$

$= \frac{1}{1/e}$

$= e$ > 0 ∴ min.

$\left(\frac{1}{e}, -\frac{1}{e}\right)$



2) \$600 $9\% = \frac{9}{12}\%$ per month

$= 0.75\%$

$A_1 = 600(1.0075)^{48}$

$A_2 = 600(1.0075)^{47}$

$A_3 = 600(1.0075)^{46}$

$\vdots$

$A_{48} = 600(1.0075)$

$a = 600(1.0075)$ $r = 1.0075$ $n = 48$

$$S_{48} = \frac{600(1.0075)(1.0075^{48} - 1)}{1.0075 - 1}$$

$= \$34721.27$

$$50000 = \frac{x(1.0075)(1.0075^{48} - 1)}{1.0075 - 1}$$

$$x = \frac{50000 \times 0.0075}{(1.0075)(1.0075^{48} - 1)}$$

$= \$862.78$