St George Girls High School

Year 11

Common Test – 1

2008

Mathematics Extension 1

Time Allowed: 65 minutes

Marks: 60

Instructions

1. All questions should be attempted.
2. Show all working.
3. START EACH QUESTION ON A NEW PAGE.
4. Marks will be deducted for careless work or poorly presented solutions.
5. On the cover sheet of the answer booklet clearly show:
   a) your name
   b) your mathematics class and teacher
6. To answer Question 5, use the tear-off grids supplied at end of this examination paper. Insert these inside answer booklet.

Question 1 (10 marks) – **Start a New Page** **Marks**

a) Without using decimal approximations, show that $3\sqrt{5}+4 > 7\sqrt{5}-5$ 2

b) If $g(x) = 2 - x$, find

(i) $g(-3)$ (ii) $g(2-x)$ 1, 1

c) Write down the natural domains of:

(i) $f(x) = \sqrt{x^2-9}$ (ii) $h(x) = \frac{2}{4-x^2}$ 1, 1

d) If $xy + \sqrt{5} = 10 + \sqrt{3y - 10x}$, find the values of x and y, given that they are integers. 4

Question 2 (10 marks) – **Start a New Page** **Marks**

a) On separate number planes, each at least ⅓ page, sketch the following curves, showing all important features.

(i) $y = 2^{x+1}$ 2

(ii) $y = -\sqrt{4 - x^2}$ 2

b) If $P = \{x: -2 \leq x \leq 4\}$ and $Q = \{x: -3 < x < 2\}$, graph on separate number lines:

(i) $P \cap Q$ (ii) $P \cup Q$ 1, 1

c) Solve these simultaneous equations: 4

$2x - 3y + z = -5$

$x + y - z = -2$

$2x + 3y - 5z = -11$

Question 3 (10 marks) – **Start a New Page** **Marks**

a) Express as a simplified fraction: 3

$$\frac{x}{x^2 - 9} - \frac{2x}{2x^2 - 5x - 3}$$

b) (i) Expand $(a^2 + 8)^2$ 1

(ii) Using your answer to part (i), or otherwise, express $a^4 + 64$ as the product of quadratic factors. 2

c) In a group of 40 students, there are 11 who play golf, 27 who play tennis and 5 who play neither. How many play both golf and tennis? 1

d) Solve this pair of simultaneous equations: 3

$x - 3y = 10$

$x^2 - y^2 = -8$

Question 4 (10 marks) – **Start a New Page** **Marks**

a) Find the inverse function $y = f^{-1}(x)$ if 4

(i) $f(x) = 2x + 3$ (ii) $f(x) = \frac{2x-1}{x+3}$

b) (i) What would be a possible restriction on the domain of $y = 2x^2 - 5x - 3$ so that its inverse relation is a function? 3

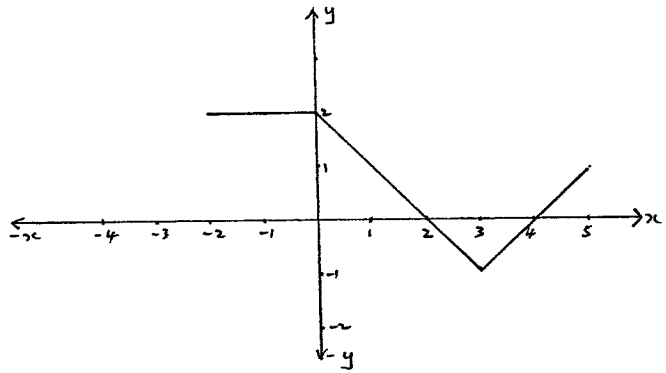
x

(ii) Explain why you have made this restriction.

c) Two boys are running towards each other. They are initially 300m apart. One boy is running at 8 m/s, and the other is running at 6 m/s. How long before the boys meet? (Give answer to nearest tenth of a second). 3

Question 5 (10 marks) – **Start a New Page** **Marks**

The graph of $y = f(x)$ is shown below:



On the tear-off sheets supplied at the end of this paper, draw neat sketches of:

a) $y = f(x) + 1$ 2

b) $y = f(x + 1)$ 2

c) $y = f(-x)$ 2

d) $y = -f(x)$ 2

e) $y = f(2x)$ 2

(Use the spare grids supplied if any mistakes are made)

Question 6 (10 marks) – **Start a New Page** **Marks**

a) Expand and simplify: $(2a-b)^3-(a^2+2b)^2$ 2

b) (i) Show that $\frac{4x-11}{x-3}$ may be written as $4+\frac{1}{x-3}$ 1

and (ii) hence, sketch $y=\frac{4x-11}{x-3}$, showing all important features such as the intercepts. 3

c) Factorise fully

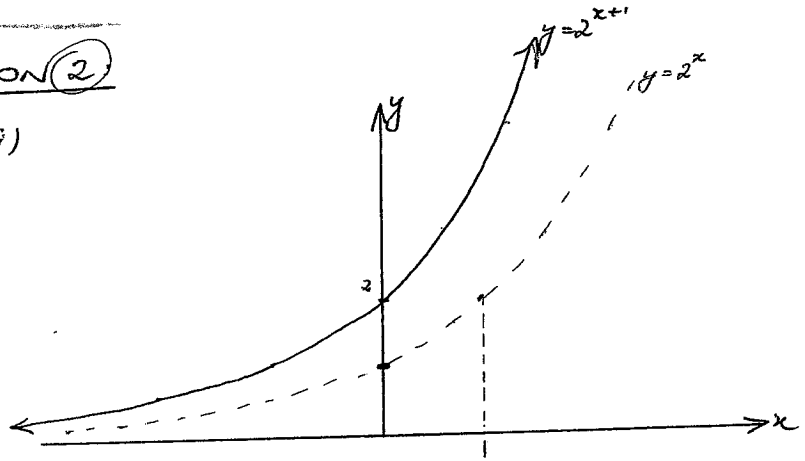
(i) $3x^4-48$ 2

(ii) $y^2-4y+4-x^2$ 2

End of Paper

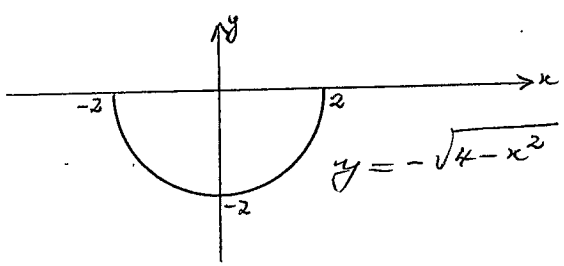
QUESTION (2)

(a) (i)



2

(ii)



2

QUESTION 1:

(a) $3\sqrt{5}+4-(7\sqrt{5}-5) = 3\sqrt{5}+4-7\sqrt{5}+5$

$= 9-4\sqrt{5}$

$= \sqrt{81}-\sqrt{80}$

> 0

2

$\therefore 3\sqrt{5}+4 > 7\sqrt{5}-5$

(b) $g(x) = 2-x$

(i) $g(-3) = 2-(-3)$

$= 5$

1

(ii) $g(2-x) = 2-(2-x)$

$= x$

1

(c) (i) Require $x^2-9 \geqslant 0$

$x^2 \geqslant 9$

$\therefore x \geqslant 3$ or $x \leqslant -3$

1

Domain $= \{x: x \geqslant 3 \text{ or } x \leqslant -3\}$

(ii) Domain $= \{x: x \text{ is real}, x \neq 2, x \neq -2\}$

1

(d) $xy = 10$ ——— (1)

$3y - 10x = 5$ ——— (2)

from (1): $y = \frac{10}{x}$ sub in (2)

$\frac{30}{x} - 10x = 5$

$30 - 10x^2 = 5x$

$\therefore 10x^2 + 5x - 30 = 0$

$2x^2 + x - 6 = 0$

$(x+2)(2x-3) = 0$

$\therefore x = -2, \frac{3}{2}$ sub in (1)

$y = -5, \frac{20}{3}$

4

x, y integral

$\therefore x=-2,$

$y=-5.$

$x=-2$? -3 ?

JESTION 3:

(a) $\frac{x}{(x+3)(x-3)} - \frac{2x}{(x-3)(2x+1)}$

$= \frac{x(2x+1) - 2x(x+3)}{(x+3)(x-3)(2x+1)}$

$= \frac{2x^2 + x - 2x^2 - 6x}{(x+3)(x-3)(2x+1)}$

$= -\frac{5x}{(x+3)(x-3)(2x+1)}$ 3

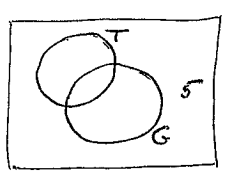
(b) (i) $(a^2+8)^2 = a^4 + 16a^2 + 64$ 1

(ii) $a^4 + 64 = (a^2+8)^2 - 16a^2$

$= (a^2+8+4a)(a^2+8-4a)$

$= (a^2+4a+8)(a^2-4a+8)$ 2

(c)



$n(T \cup G) = n(T) + n(G) - n(T \cap G)$

$35 = 27 + 11 - n(T \cap G)$

$\therefore n(T \cap G) = 3$ 1

(d) $x - 3y = 10$ ——— ①

$x^2 - y^2 = -8$ ——— ②

from ①: $x = 10 + 3y$ sub in ②

$\therefore 100 + 60y + 9y^2 - y^2 = -8$

$8y^2 + 60y + 108 = 0$

$2y^2 + 15y + 27 = 0$

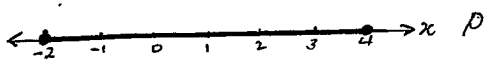
$(y+3)(2y+9) = 0$

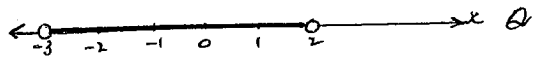
$y = -3, -\frac{9}{2}$ sub in ①

$x = 1, -\frac{7}{2}$ 3

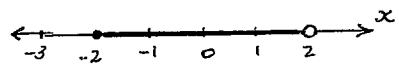
$x = 1$ } $x = -\frac{7}{2}$ }

(a)



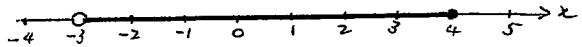


(i) $P \cap Q$:



1

(ii) $P \cup Q$:



1

(c) $2x - 3y + z = -5$ ——— ①

$x + y - z = -2$ ——— ②

$2x + 3y - 5z = -11$ ——— ③

② × 3: $3x + 3y - 3z = -6$ ——— ④

① + ④: $5x - 2z = -11$ ——— ⑤

③ + ③: $4x - 4z = -16$ ——— ⑥

⑤ × 2: $10x - 4z = -22$ ——— ⑦

⑦ − ⑥: $6x = -6$

$x = -1$ sub in ⑤

$-5 - 2z = -11$

$6 = 2z$

$z = 3$ sub in ②

$-1 + y - 3 = -2$

$y = 2$

$\therefore x = -1, y = 2, z = 3$ 4

③ − ① $by - 6z = -6$

(c) b)(i) $y = 2x^2 - 5x - 3$

axis of symmetry: $x = -\frac{b}{2a}$

$= \frac{5}{4}$ 2

$\therefore$ Possible restriction is $x \leq \frac{5}{4}$ OR $x \geq \frac{5}{4}$

(ii) Horizontal line test will hold ie each y has at most one x value. OR vertical line test will hold on inverse ie each x has at most one y-value. 1

c) Let t = time taken before boys meet (in seconds)

$S = \frac{D}{T}$

$\therefore D = ST$

$8t + 6t = 300$

$14t = 300$

$t = \frac{300}{14}$ 3

$\doteq 21.4$ s.

ie boys meet after 21.4 s.

QUESTION 4:

a) (i) $f: y = 2x + 3$

$f^{-1}: x = 2y + 3$

$2y = x - 3$

$y = \frac{x-3}{2}$ 2

$\therefore f^{-1}(x) = \frac{x-3}{2}$

(ii) $f: y = \frac{2x-1}{x+3}$

$f^{-1}: x = \frac{2y-1}{y+3}$

$xy + 3x = 2y - 1$

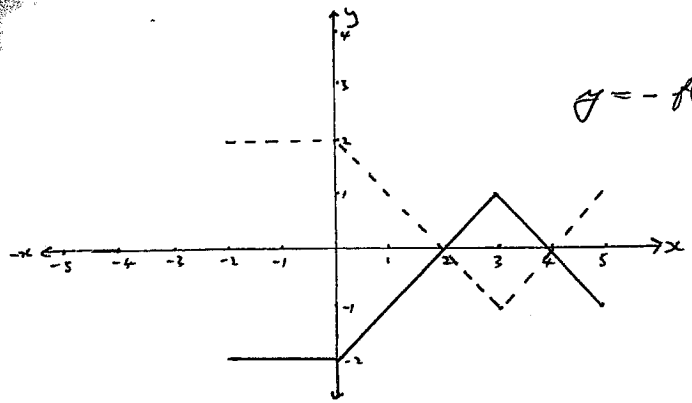
$3x + 1 = 2y - xy$

$3x + 1 = y(2 - x)$ 2

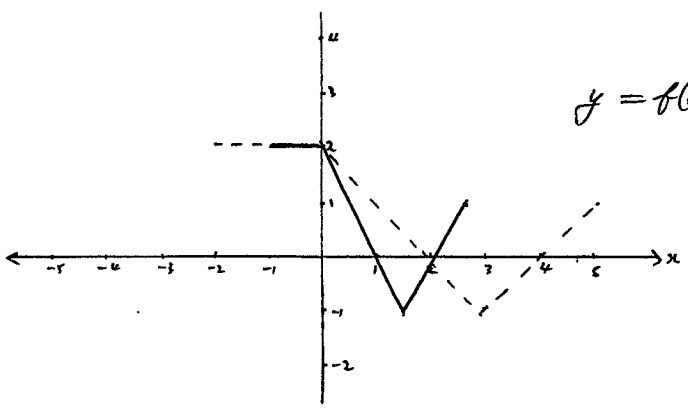
$\therefore y = \frac{3x+1}{2-x}$

ie $f^{-1}(x) = \frac{3x+1}{2-x}$

d)

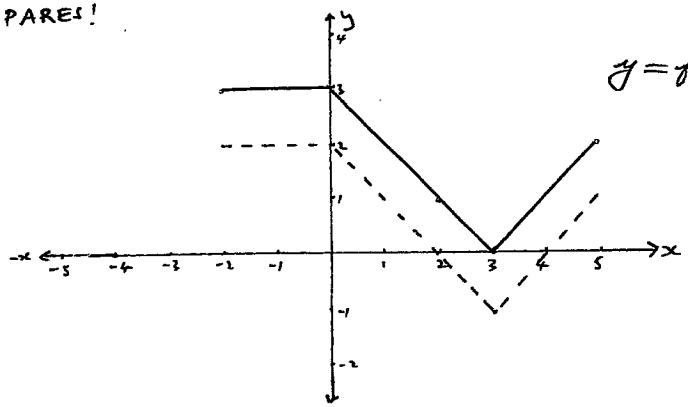


e)

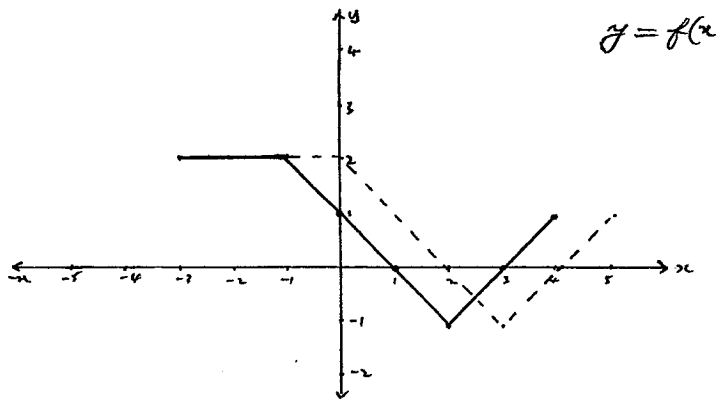


SPARES!

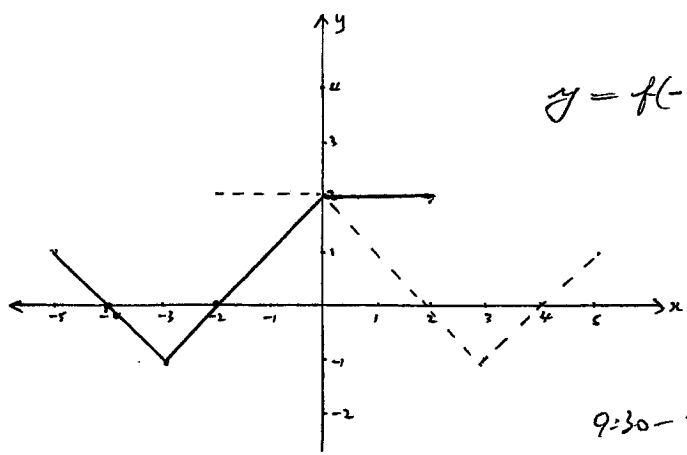
(a)



(b)



(c)



9:30 – 9 (35)

37

41

45

49

54

Q6c)

(i) $3x^4 - 48 = 3(x^4 - 16)$

$= 3(x^2 + 4)(x^2 - 4)$

$= 3(x+2)(x-2)(x^2+4)$ (2)

(ii) $y^2 - 4y + 4 - x^2 = (y-2)^2 - x^2$

$= (y - 2 + x)(y - 2 - x)$ (2)

QUESTION 5:

on sheets

QUESTION 6:

(a)

$(2a - b)^3 - (a^2 + 2b)^2$

$= (2a)^3 - 3(2a)^2(b) + 3(2a)(b)^2 - b^3 - a^4 - 4a^2b - 4b^2$

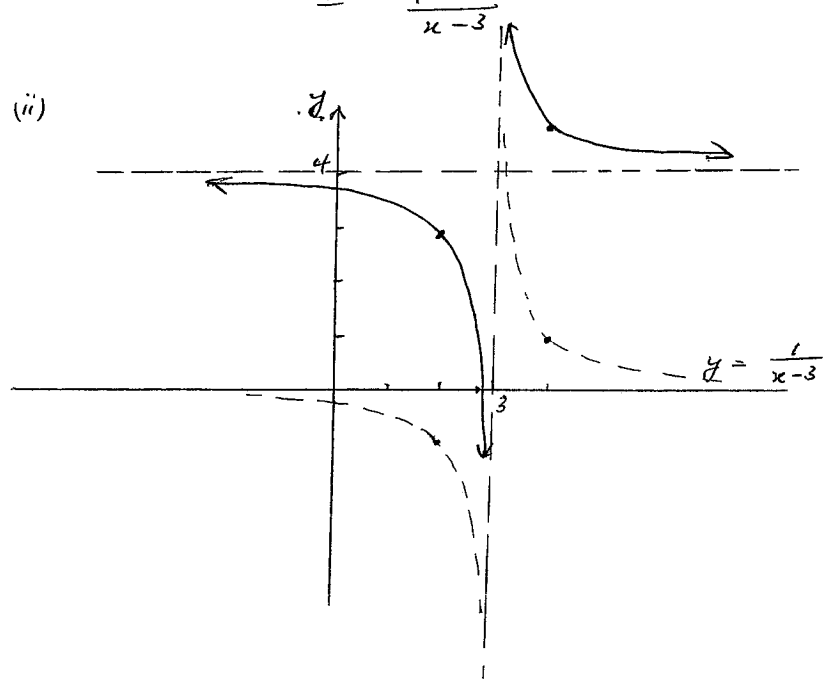
$= 8a^3 - 12a^2b + 6ab^2 - b^3 - a^4 - 4a^2b - 4b^2$

$= -a^4 + 8a^3 - 16a^2b + 6ab^2 - b^3 - 4b^2$ 2

(b) (i) $4 + \frac{1}{x-3} = \frac{4x - 12 + 1}{x-3}$

$= \frac{4x - 11}{x-3}$ 1

(ii)



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