St George Girls High School

Year 11

Common Test – 1

2007

Mathematics Extension 1

Time Allowed: 70 minutes

Marks: 60

Instructions

1. All questions should be attempted,
2. Show all working.
3. START EACH QUESTION ON A NEW PAGE.
4. Marks will be deducted for careless work or poorly presented solutions.
5. On the cover sheet of the answer booklet clearly show:
   a) your name
   b) your mathematics class and teacher

Question 1 (12 marks) – **Start a New Page** **Marks**

a) Factorise: 4

(i) y^4+3y^2-4

(ii) x^2-6x-y^2+6y

b) Simplify 5

(i) $\dfrac{x^3+8}{x^2+4x+4}\div\dfrac{x^3-2x^2+4x}{x^3+2x^2}$

(ii) $\dfrac{2}{3x-x^2}+\dfrac{x}{x^2-9}$

c) Simplify, giving your answer with a rational denominator 3

$$\frac{1}{\sqrt{3}-1}+\frac{2}{2\sqrt{3}+3}$$

Question 2 (12 marks) – **Start a New Page** **Marks**

a) Draw neat sketches (8 – 10 lines) of the graphs of: 6

(i) $x^2 + y^2 = 8$

(ii) $y = x^2 - 4x + 5$

(iii) $x = \sqrt{9 - y^2}$

b) Form a pair of simultaneous equations and solve them to find x and y, given they are rational $xy + \sqrt{x - y} = 28 + 2\sqrt{3}$ 3

c) Sketch the region $\left\{(x, y) : y \le \frac{1}{x}\right\}$ 3

Question 3 (12 marks) – **Start a New Page** **Marks**

a) Solve the following for x:

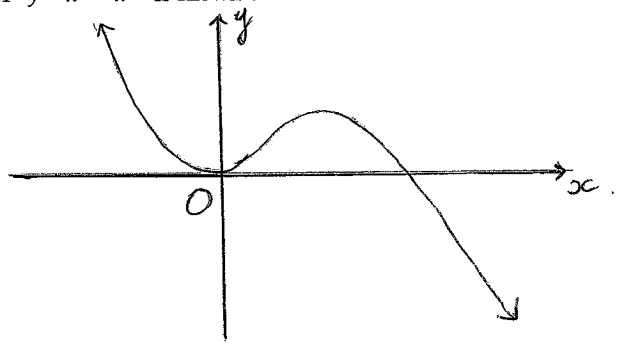
(i) $-2 \le 2 - 3x < 7$ 2

(ii) $|2x - 1| = 5$ 2

(iii) $x^2 \le 3x + 4$ 3

(iv) $\frac{2x - 1}{x - 3} > 1$ 3

b) The graph of $y = x^2 - x^3$ is shown below. 2



Using the graph or otherwise solve $x^3 \ge x^2$

Question 4 (12 marks) – **Start a New Page** **Marks**

a) What is the natural domain of the function $y=\log_2(4-x)$? 1

b) Find the range of the function $y=x^2-2x,\ 0\le x\le 5$ 3

c) (i) Sketch the graph of $y=\sqrt{x-1}$ 2

(ii) Hence sketch $y=\dfrac{1}{\sqrt{x-1}}$ on the same diagram. 2

(iii) If $f(x)=\sqrt{x-1}$ explain why the inverse of $f(x)$ is a function. 1

(iv) Find $f^{-1}(x)$ 1

d) Two trains 750km apart travel towards each other. The speeds of the trains are 80km/h and 65 km/h respectively. By solving an equation find how long will it be before they meet? 2

750 km

combined speed = 145 km/h

Question 5 (12 marks) – **Start a New Page** **Marks**

a) Show that $g(x)=\dfrac{x^3}{3x^2-1}$ is an odd function. 2

b) Without using approximate values from your calculator, determine which of the numbers $12-\sqrt{5}$ and $3+3\sqrt{5}$ is greater. 2

c) Describe how the graph of $y=3^{1-x}$ can be obtained from the graph of $y=3^x$ 2

d) (i) Sketch the graphs of $y=|3x|$ and $y=x+2$ on the same diagram. 3

(ii) Solve $|3x|>x+2$ 3

$x=3^y$

$y=x$

End of Paper

Q1

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Solutions

Q1 a)(i) y^4+3y^2-4

y^2 × 4, y^2 × -1

$=(y^2+4)(y^2-1)$ 1

$=(y+1)(y-1)(y^2+4)$ 1 — 2

(ii) x^2-6x-y^2+6y

$=x(x-6)-y(y-6)$

$=x^2-y^2-6(x-y)$

$=(x+y)(x-y)-6(x-y)$ 1

$=(x-y)(x+y-6)$ 1 — 2

b)(i) $\dfrac{x^3+8}{x^2+4x+4} \div \dfrac{x^3-2x^2+4x}{x^3+2x^2}$

$=\dfrac{(x+2)(x^2-2x+4)}{(x+2)^2} \times \dfrac{x^2(x+2)}{x(x^2-2x+4)}$ 1½ 1½

$=x$ — 5

(ii) $\dfrac{2}{3x-x^2}+\dfrac{x}{x^2-9}$

$=\dfrac{2}{x(3-x)}+\dfrac{x}{(x+3)(x-3)}$

$=\dfrac{-2(x+3)+x^2}{x(x+3)(x-3)}$ 1

$=\dfrac{x^2-2x-6}{x(x^2-9)}$ 1

c) $\dfrac{1}{\sqrt{3}-1}+\dfrac{2}{2\sqrt{3}+3}$

$=\dfrac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}-1)}+\dfrac{2(2\sqrt{3}-3)}{(2\sqrt{3}+3)(2\sqrt{3}-3)}$

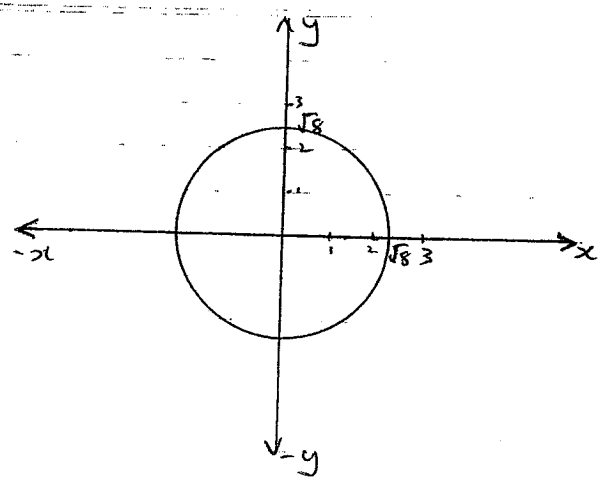
$=\dfrac{\sqrt{3}+1}{2}+\dfrac{4\sqrt{3}-6}{12-9}$

$=\dfrac{3(\sqrt{3}+1)+2(4\sqrt{3}-6)}{6}$

$=\dfrac{11\sqrt{3}-9}{6}$ — (3)

Q2

a) (i)

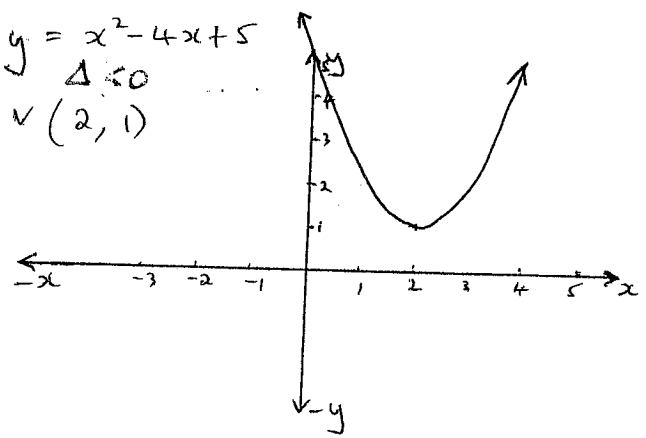


2

(ii) $y = x^2 - 4x + 5$

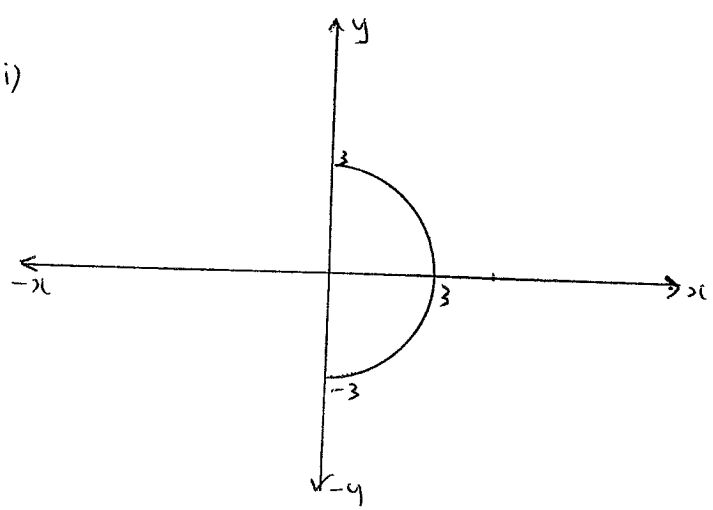
$\Delta < 0$

V (2, 1)



2

(iii)



b) $xy + \sqrt{x-y} = 28 + 2\sqrt{3}$

$= 28 + \sqrt{12}$

$\therefore xy = 28$ ①

$x - y = 12$ ②

From ②, $x = y + 12$

$\therefore$ ① $\Rightarrow y(y+12) = 28$

$y^2 + 12y - 28 = 0$

$(y+14)(y-2) = 0$

$\therefore y = -14, 2.$

when $y = -14$, $x = -2$

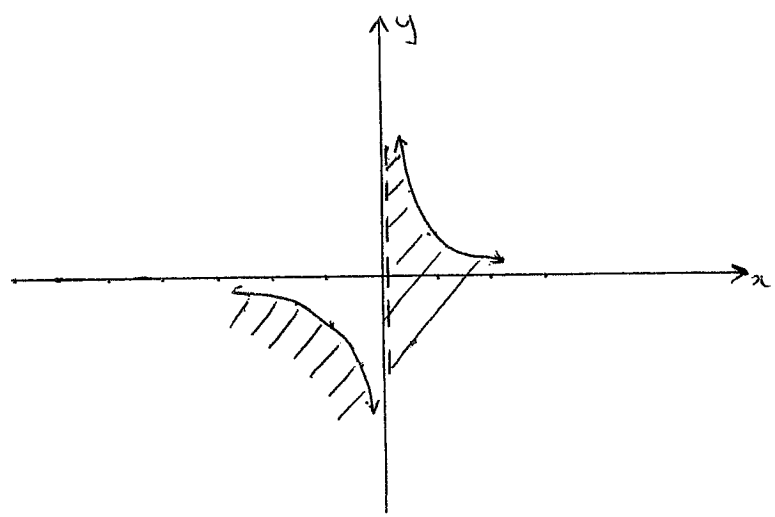
$y = 2$, $x = 14$.

(3)

c) $\{(x, y) : y \leq \frac{1}{x}\}$

ie $xy \leq 1$, $x > 0$

$xy \geq 1$, $x < 0$



(3)

Q3

a)(i) $-2 \le 2-3x < 7$ (-2)

$-4 \le -3x < 5$ $(\div -3)$

$\frac{4}{3} \ge x > -\frac{5}{3}$. (2)

(ii) $|2x-1| = 5$

$2x-1 = \pm 5$

$2x = 1 \pm 5$

$= -4, 6$

$\therefore x = -2, 3.$ (2)

(iii) $x^2 \le 3x+4$

$x^2 - 3x - 4 \le 0$

$(x-4)(x+1) \le 0$.

LHS $= 0 \Rightarrow x = -1, 4.$

test $x=0$ ✓

$\therefore -1 \le x \le 4$ (3)

(or graphically)

(iv) $\frac{2x-1}{x-3} > 1$

× both sides by $(x-3)^2$

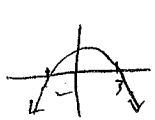
$\Rightarrow (2x-1)(x-3) > (x-3)^2$

ie $(x-3)^2 - (x-3)(2x-1) < 0$

$(x-3)(x-3 - (2x-1)) < 0$

$(x-3)(-x-2) < 0.$

ie $x > 3, \quad x < -2$ (3)



or Critical Value Method
(ie hole & test method)

or graphical method.

b) $x^3 \ge x^2$

ie $x^2 - x^3 \le 0$

$y = x^2 - x^3$

$= x^2(1-x)$

ie $y \le 0$ for given graph

ie $x \ge 1, \quad x = 0.$ (2)

Q4

a) $y = \log_2(4-x).$

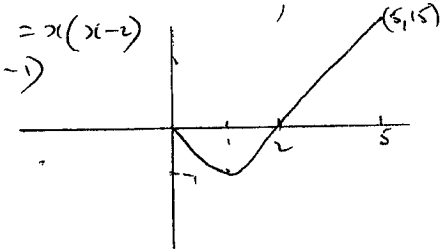
We require $4-x > 0$ (1)

ie D: $x < 4.$

b) $y = x^2 - 2x \quad , \quad 0 \le x \le 5.$

$= x(x-2)$

$V(1,-1)$



R: $-1 \le y \le 15$ (3)

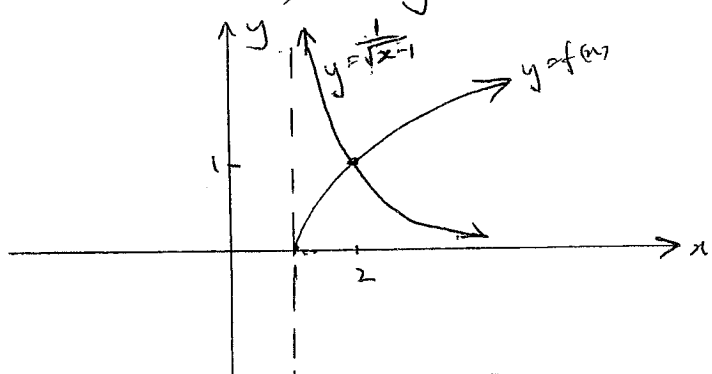
c) (i) $y = \sqrt{x-1}$

D: $x \ge 1$, R: $y \ge 0$

(ii) $y = \frac{1}{\sqrt{x-1}}$ as $x \to \infty, y \to 0$; as $x \to 1^+, y \to \infty$

D: $x > 1$

R: $y > 0$



(4)

(ii) the graph of $y = \sqrt{x-1}$ satisfies the horizontal line test ie each y value has

at most one x-value. ①

(iv) $y = \sqrt{x-1}$ (ii) $y \geq 0$

$\therefore$ inverse is : $x = \sqrt{y-1}$ ①

ie $x^2 = y-1$

$\therefore y = x^2+1 \quad , x \geq 0$

d) Let t = time in hours before trains meet.

Time Distance travelled by first train in this time is $80t$ & distance travelled by second train is $65t$

$\therefore 80t + 65t = 750$ ②

ie $145t = 750$

$\therefore t = \frac{750}{145}$

$\doteq 5.1724$ hours

$\doteq$ 5h 10 min (to nearest min)

Q5

a) $g(x) = \frac{x^3}{3x^2-1}$

$\therefore g(-x) = \frac{(-x)^3}{3(-x)^2-1}$ (2)

$= \frac{-x^3}{3x^2-1}$

$= -g(x)$

$\therefore g(x)$ is odd.

b) $12-\sqrt{5} - (3+3\sqrt{5})$

$= 9 - 4\sqrt{5}$ ②

$= \sqrt{81} - \sqrt{80}$.

> 0

$\therefore 12-\sqrt{5} > 3+3\sqrt{5}$

c) $y = 3^{1-x}$

$= 3^{-x} \times 3$

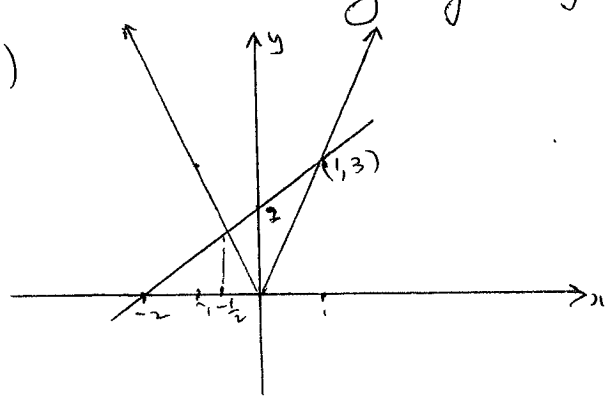
ie $\frac{y}{3} = 3^{-x}$

or $y = 3^x \to y = 3^{-x} \to 3^{-(x-1)}$

ie reflect in y-axis, followed by translating 1 unit to right. ②

ie reflected $y = 3^x$ in y-axis, then stretch vertically by a factor of 3.

d) (i) ③



(ii) $|3x| = x+2$

$\therefore \pm 3x = x+2$

i.e. $3x = x+2$ or $-3x = x+2$

$2x = 2$ $\quad -4x = 2$

$x = 1$ $\quad x = -\frac{1}{2}$

Check solns: $x=1$; $|3| = 3$ ✓

$x = -\frac{1}{2}$; $|-3/2| = -\frac{1}{2} + 2$

$= 3/2$ ✓

$\therefore \quad x = -\frac{1}{2}, 1.$

$|3x| > x+2$ has solution $x < -\frac{1}{2}, x > 1$