St George Girls High School

Year 11

Common Test – 1

2008

Mathematics

Time Allowed: 75 minutes

Instructions

1. All questions should be attempted.
2. Show all working.
3. START EACH QUESTION ON A NEW PAGE.
4. Marks will be deducted for careless work or poorly presented solutions.
5. On the cover sheet of the answer booklet clearly show:

   a) your name
   b) your mathematics class and teacher

Question 1 (10 marks)   **Marks**

a)   Evaluate $\dfrac{(0.5-2.3)^2}{6.7+1.4}$ 1

b)   Simplify $\left(x\sqrt{y}\right)^2$ 1

c)   Which are irrational?   1

     $\sqrt{2}, \quad 1.414, \quad \sqrt[3]{27}, \quad \pi, \quad 0.8\dot{9}$

d)   Rationalise the denominator of $\dfrac{3}{\sqrt{7}-1}$.   3

e)   Expand $(1-2x)^2$ then simplify   2

f)   Factorise fully   2

     i) $-10x-5y$

     ii) a^2-25

Question 2   (10 marks)   Marks

a)   Express $0.35\overset{\bullet}{7}$ as a fraction in simplified form.   3

b)   Factorise $x^3 + 27$.   2

c)   Simplify $9\sqrt{7} + 2\sqrt{75} - 2\sqrt{63} + 4\sqrt{3}$ 2

d)   If $W = \sqrt{\dfrac{g}{h}}$ calculate h when $W = 0.2$ and $g = 9.8$.   3

Question 3   (10 marks)   Marks

a)   Expand and simplify   7

   i) $\left(\sqrt{13} - \sqrt{11}\right)\left(\sqrt{13} + \sqrt{11}\right)$

   ii) $a(a + b) - b(a - b)$

   iii) $\left(2 - 3\sqrt{2}\right)^2 - \left(1 + 3\sqrt{2}\right)^2$

b)   Rationalise the denominator in $\dfrac{6 - 2\sqrt{3}}{6 + 2\sqrt{3}}$.   3

Question 4   (10 marks)   Marks

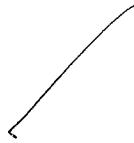a) Solve $2x^2 - 3x = 0$.   2

b) Simplify

   i) $\dfrac{3}{2\sqrt{7} - \sqrt{5}} - \dfrac{4}{2\sqrt{7} + \sqrt{5}}$ 3

   ii) $(a+2)(a^2 - 2a + 4)$

c) Make c the subject of $(a+b)(a-c) = ab$ 3

d) Simplify $\dfrac{3a^2 x^5 y}{15a^6 x^3 y}$ 2

Question 5   (10 marks)   Marks

a) Solve simultaneously   3

   $$5x + 4y = 18$$
   $$2x - 5y = -6$$

b) Factorise fully   4

   i) $2x^2 - x - 15$

   ii) $(x-y)^2 - (x+y)^2$

c) Simplify $\dfrac{(1-x^2)(x+2)^2}{(x+2)(x-1)}$ 3

Question 6   (10 marks)

Marks

a)   Solve $5x^2 + 13x - 7 = 0$ giving answers in exact form   2

b)   Factorise fully and simplify   6

   i) $(1-d) + (1-d)(1-x)$

   ii) $\dfrac{3x^2 + 13x - 10}{x^2 + 5x}$

   iii) $\dfrac{x^3 - 1}{x^3} \times \dfrac{2x^2 + 2x}{x^2 - 1}$

c)   Solve $\dfrac{2x-3}{2} - 5x = 3$ 2

Question 7   (10 marks)

Marks

a)   Solve $3x^2 - 6x - 2 = 0$ by completing the square.   3

b)   Factorise fully   4

   i) $16 - 2a^3$

   ii) $a^2 b - ab - a^2 + a$

c)   Simplify $\dfrac{2}{x^2 - 3x - 4} \div \dfrac{4}{x^2 - 4x}$ 3

Question 8   (10 marks)   **Marks**

a)   Factorise fully $c(x-b)^3 + a^3 c$ 2

b)   A town's population is found using the formula $P = A_0 2.7^{0.15t}$
      where A_0 is the initial population at the start of 2000 and t is the number of
      years after 2000. If $A_0 = 25000$, what is the town's population at the start of 2008.   2

c)   Simplify   6

      i) $\dfrac{x+1}{x-1} - \dfrac{1}{(x-1)^2}$

      ii) $\dfrac{a-4}{a^2+a-6} - \dfrac{6a-18}{a^2-5a+6}$

End of Paper

YEAR 11 MATHEMATICS

2 UNIT COMMON #1

SOLUTIONS

Q1 — Karen

Q2 — Jenny

Q3 — Inna

Q4 — Michelle

Q5 — John

Q6 — Mark

Q7 — Chris

Q8 — Phil

QUESTION 1:

(a) 0.4

(b) $(x\sqrt{y})^2 = x^2 y$

(c) $\sqrt{2}$, π

(d) $\dfrac{3}{\sqrt{7}-1} = \dfrac{3}{\sqrt{7}-1} \times \dfrac{\sqrt{7}+1}{\sqrt{7}+1}$

$$= \frac{3\sqrt{7}+3}{6}$$

$$= \frac{\sqrt{7}+1}{2}$$

(e) $(1-2x)^2 = 1 - 4x + 4x^2$

(f) (i) $5(-2x-y)$ OR $-5(2x+y)$

(ii) $a^2 - 25 = (a+5)(a-5)$

QUESTION 2:

(a)   let $x = 0.35777\ldots$ ——— ①

$100x = 35.777\ldots$ ——— ②

$1000x = 357.777\ldots$ ——— ③

③ − ②: $900x = 322$

$\therefore x = \dfrac{322}{900}$

$$= \frac{161}{450}$$

$\therefore 0.35\dot{7} = \dfrac{161}{450}$

(b) $x^3 + 27 = (x+3)(x^2 - 3x + 9)$

(c) $9\sqrt{7} + 2\sqrt{75} - 2\sqrt{63} + 4\sqrt{3}$

$= 9\sqrt{7} + 2(5\sqrt{3}) - 2(3\sqrt{7}) + 4\sqrt{3}$

$= 3\sqrt{7} + 14\sqrt{3}$

(d) $W = \sqrt{\frac{g}{h}}$

$0.2 = \sqrt{\frac{9.8}{h}}$

$\Rightarrow \quad 0.04 = \frac{9.8}{h}$

$0.04h = 9.8$

$\therefore h = \frac{9.8}{0.04}$

$= 245$

QUESTION 3:

(a) (i) $13 - 11 = 2$

(ii) $a(a+b) - b(a-b) = a^2 + ab - ab + b^2$

$= a^2 + b^2$

(iii) $(2 - 3\sqrt{2})^2 - (1 + 3\sqrt{2})^2 = 4 - 12\sqrt{2} + 18 - [1 + 6\sqrt{2} + 18]$

$= 22 - 12\sqrt{2} - 19 - 6\sqrt{2}$

$= 3 - 18\sqrt{2}$

(b) $\frac{6 - 2\sqrt{3}}{6 + 2\sqrt{3}} = \frac{6 - 2\sqrt{3}}{6 + 2\sqrt{3}} \times \frac{6 - 2\sqrt{3}}{6 - 2\sqrt{3}}$

$= \frac{36 - 24\sqrt{3} + 12}{36 - 12}$

$= \frac{48 - 24\sqrt{3}}{24}$

QUESTION 4:

(a) $2x^2 - 3x = 0$

$x(2x - 3) = 0$

$x = 0, \frac{3}{2}$

(b) (i) $\frac{3}{2\sqrt{7} - \sqrt{5}} - \frac{4}{2\sqrt{7} + \sqrt{5}} = \frac{3(2\sqrt{7} + \sqrt{5}) - 4(2\sqrt{7} - \sqrt{5})}{(2\sqrt{7} - \sqrt{5})(2\sqrt{7} + \sqrt{5})}$

$= \frac{7\sqrt{5} - 2\sqrt{7}}{28 - 5}$

$= \frac{7\sqrt{5} - 2\sqrt{7}}{23}$

(ii) $(a+2)(a^2 - 2a + 4) = a^3 - 2a^2 + 4a + 2a^2 - 4a + 8$

$= a^3 + 8$

(c) $(a+b)(a-c) = ab$

$\Rightarrow a - c = \frac{ab}{a+b}$

$\therefore c = a - \frac{ab}{a+b}$

$= \frac{a^2 + ab - ab}{a+b}$

$= \frac{a^2}{a+b}$

(d) $\frac{3a^2 x^5 y}{15 a^6 x^3 y} = \frac{x^2}{5a^4}$

QUESTION 5:

(a)
$$5x + 4y = 18 \quad \text{——①}$$
$$2x - 5y = -6 \quad \text{——②}$$

①× 5: $\quad 25x + 20y = 90 \quad$ ——③
②× 4: $\quad 8x - 20y = -24 \quad$ ——④

③ + ④: $\quad 33x \quad = 66$
$$x = 2 \quad \text{sub in ①}$$
$$10 + 4y = 18$$
$$4y = 8$$
$$y = 2$$

$$\therefore \begin{cases} x = 2 \\ y = 2 \end{cases}$$

(b) (i) $\quad 2x^2 - x - 15 = (x-3)(2x+5)$

(ii) $(x-y)^2 - (x+y)^2 = x^2 - 2xy + y^2 - \left[x^2 + 2xy + y^2\right]$
$$= x^2 - 2xy + y^2 - x^2 - 2xy - y^2$$
$$= -4xy$$

(c) $\dfrac{(1+x)(1-x)^{-1}(x+2)^{\cancel{2}}}{(x+2)(x-1)} = -(1+x)(x+2)$

QUESTION 6:

(a) $\quad 5x^2 + 13x - 7 = 0$
$$x = \frac{-13 \pm \sqrt{13^2 - 4(5)(-7)}}{10} \quad \text{——①}$$
$$= \frac{-13 \pm \sqrt{309}}{10} \quad \text{——①}$$

(b) (i) $(1-\alpha) + (1-\alpha)(1-x) = (1-\alpha)\left[1 + 1 - x\right]$
$$= (1-\alpha)(2-x) \quad \text{——①}$$

(ii) $\dfrac{3x^2 + 13x - 10}{x^2 + 5x} = \dfrac{(x+5)(3x-2)}{x(x+5)}$ ——①
$$= \frac{3x-2}{x} \quad \text{——①}$$

(iii) $\dfrac{x^3 - 1}{x^3} \times \dfrac{2x^2 + 2x}{x^2 - 1} = \dfrac{(x-1)(x^2+x+1)}{x^3} \times \dfrac{2x(x+1)}{(x+1)(x-1)}$ ——①
$$= \frac{2(x^2 + x + 1)}{x^2} \quad \text{——①}$$

(c) $\dfrac{2x-3}{2} - 5x = 3$
$$\Rightarrow 2x - 3 - 10x = 6 \quad \text{——①}$$
$$-8x = 9$$
$$\therefore x = -\frac{9}{8} \quad \text{——①}$$

QUESTION 7:

(a) $\quad 3x^2 - 6x - 2 = 0$
$$\Rightarrow 3x^2 - 6x = 2$$
$$\Rightarrow x^2 - 2x = \frac{2}{3}$$
$$\Rightarrow x^2 - 2x + 1 = \frac{5}{3}$$
$$\therefore (x-1)^2 = \frac{5}{3}$$
$$\Rightarrow x - 1 = \pm\sqrt{\frac{5}{3}}$$
$$= \pm\frac{\sqrt{15}}{3}$$
$$\therefore x = 1 \pm \frac{\sqrt{15}}{3}$$
$$= \frac{3 \pm \sqrt{15}}{3}$$

(b) (i) $16 - 2a^3 = 2(8 - a^3)$
$$= 2(2-a)(4 + 2a + a^2)$$

(ii) $a^2b - ab - a^2 + a = ab(a-1) - a(a-1)$
$$= (a-1)(ab - a)$$
$$= a(a-1)(b-1)$$

OR $a^2b - ab - a^2 + a = a(ab - b - a + 1)$
$$= a[b(a-1) - 1(a-1)]$$
$$= a(a-1)(b-1)$$

(c) $\dfrac{2}{x^2 - 3x - 4} \times \dfrac{x^2 - 4x}{4} = \dfrac{2}{(x-4)(x+1)} \times \dfrac{x(x-4)}{4}$
$$= \dfrac{x}{2(x+1)}$$

QUESTION 8:

(a) $c(x-b)^3 + a^3c = c[(x-b)^3 + a^3]$
$$= c[(x-b) + a][(x-b)^2 - a(x-b) + a^2]$$
$$= c(x-b+a)(x^2 - 2bx + b^2 - ax + ab + a^2)$$

(b) $P = A_0 \, 2.7^{0.15t}$

$\left. \begin{array}{l} A_0 = 25\,000 \\ t = 8 \end{array} \right\} \Rightarrow P = 25\,000 \times 2.7^{0.15 \times 8}$
$$= 82\,333.490\cdots$$
$$= 82\,333 \text{ (correct to nearest integer)}$$

$\therefore$ Population is $82\,333$

(c) (i) $\dfrac{x+1}{x-1} - \dfrac{1}{(x-1)^2} = \dfrac{(x+1)(x-1) - 1}{(x-1)^2}$
$$= \dfrac{x^2 - 1 - 1}{(x-1)^2}$$
$$= \dfrac{x^2 - 2}{(x-1)^2}$$

(ii) $\dfrac{a-4}{a^2 + a - 6} - \dfrac{6a - 18}{a^2 - 5a + 6} = \dfrac{a-4}{(a+3)(a-2)} - \dfrac{6a-18}{(a-3)(a-2)}$
$$= \dfrac{(a-4)(a-3) - (6a-18)(a+3)}{(a+3)(a-2)(a-3)}$$
$$= \dfrac{a^2 - 7a + 12 - [6a^2 - 54]}{(a+3)(a-2)(a-3)}$$
$$= \dfrac{-5a^2 - 7a + 66}{(a+3)(a-2)(a-3)}$$
$$= - \dfrac{5a^2 + 7a - 66}{(a+3)(a-2)(a-3)}$$